Learning Objectives:

1. Understand the basic concepts of matrices.
2. Solve linear systems using augmented matrices

Examples:

For Problems 1–6, refer to the following matrices:

\[ A = \begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 6 & -3 \\ 9 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 5 & 4 & -3 & 6 \end{bmatrix} \]

1. Find the dimensions of A, B, and C.
2. For matrix A, find \( a_{12} \); for matrix B, find \( b_{22} \); and for matrix C, find \( c_{14} \).
3. Is there a row matrix?
4. Is there a column matrix?
5. Find the elements on the principal diagonal of matrix A.
6. Find the elements on the principal diagonal of matrix B.

For Problems 7–10, set up the augmented matrix that corresponds to system of linear equations.

7. \[ \begin{cases} x + y = 20 \\ x - y = 12 \end{cases} \]
8. \[ \begin{cases} x + y = 15 \\ x - y = 9 \end{cases} \]

9. \[ \begin{cases} x + 2y = 16 \\ x - y = 1 \end{cases} \]
10. \[ \begin{cases} x + 3y = 16 \\ x - y = 8 \end{cases} \]

Solve the systems of linear equations in 7 – 10 by using row operations [page 185].

Solve the following systems of linear equations by augmented matrix methods.

11. \[ \begin{cases} x + 2y = 8 \\ 3x + 6y = 24 \end{cases} \]
12. \[ \begin{cases} x + 3y = 7 \\ -2x - 6y = 4 \end{cases} \]
Teaching Notes:

- Emphasize that the dimensions of a matrix is \( \text{# of rows} \times \text{# of columns} \), the subscripts that describe the placement of an element tells which row and which column the element is in, and that a matrix does not have to be square to have a principal diagonal (subscripts must match).
- To set up an augmented matrix, the equations in the system must be in standard form \((Ax + By = C)\).
- When working problems 11 and 12 point out to students the possible matrix forms for linear system of two equations in two variables on the top of page 190. Point out the similarities when solving a system that does not have a unique solution, by graphing, algebraic, and then by matrix methods.

<table>
<thead>
<tr>
<th>Solutions:</th>
<th>Exactly one solution</th>
<th>Infinitely many solutions</th>
<th>No solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Consistent and Independent</td>
<td>Consistent and Dependent</td>
<td>Inconsistent</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graphing:</th>
<th>Intersecting Lines</th>
<th>Collinear Lines</th>
<th>Parallel Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic:</td>
<td>Unique value for variables</td>
<td>Produces a true statement (with no variables)</td>
<td>Produces a false statement (with no variables)</td>
</tr>
</tbody>
</table>
| Matrix:       | \[
\begin{bmatrix}
1 & 0 & m \\
0 & 1 & n
\end{bmatrix}
\]
|               | \[
\begin{bmatrix}
1 & m & n \\
0 & 0 & 0
\end{bmatrix}
\]
|               | (last row is a true statement) | \[
\begin{bmatrix}
1 & m & n \\
0 & 0 & p
\end{bmatrix}
\]
|               | (last row is a false statement) |

- Students will need to practice setting up solutions when parameters are needed (see Example 3, page 188).

Answers:

1. \( A: 2 \times 2; \quad B: 3 \times 2; \quad C: 1 \times 4 \)
2. 2; 1; 6
3. yes C
4. no
5. 3 and 7
6. 6 and 1
7. \[
\begin{bmatrix}
1 & 1 & 20 \\
1 & -1 & 12
\end{bmatrix} \]  \( (16, 4) \)
8. \[
\begin{bmatrix}
1 & 1 & 15 \\
1 & -1 & 9
\end{bmatrix} \]  \( (12, 3) \)
9. \[
\begin{bmatrix}
1 & 2 & 16 \\
1 & -1 & 1
\end{bmatrix} \]  \( (6,5) \)
10. \[
\begin{bmatrix}
1 & 3 & 16 \\
1 & -1 & 8
\end{bmatrix} \]  \( (10,2) \)

\[
\begin{align*}
x &= 8 - 2t \\
y &= t \\
t &\in \mathbb{R}
\end{align*}
\]

11. No Solution
12. No Solution