HKUST

MATH1003 Calculus and Linear Algebra

Final exam (Version C) Name:________________________

14th December 2016 Student ID:________________________

12:30 - 14:30 Seat Number: ________________________

S H Ho Sports Hall Lecture Section: ________________________

Directions:

- Do NOT open the exam until instructed to do so.
- Please turn off all phones and pagers, and remove headphones.
- Please write your name, student ID, Seat number and Lecture Section in the space provided above.
- When instructed to open the exam, please check that you have 10 pages in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each problem. If you do not show enough work, you will get only partial credit.
- Any forms of calculators are NOT allowed.
- This is a closed book examination.
- **Cheating is a serious offense. Students caught cheating will receive a zero score for the midterm exam, and will also be subjected to further penalties imposed by the University.**

Question No.	**Points**	**Out of**
Q. 1-9		45
Q. 9		20
Q. 10		20
Q. 11		20
Total Points		105

Part I: Answer the following multiple choice questions.

Put your MC question answers in CAPTICAL letters in the following boxes.

Question	1	2	3	4	5	Total
Answer						

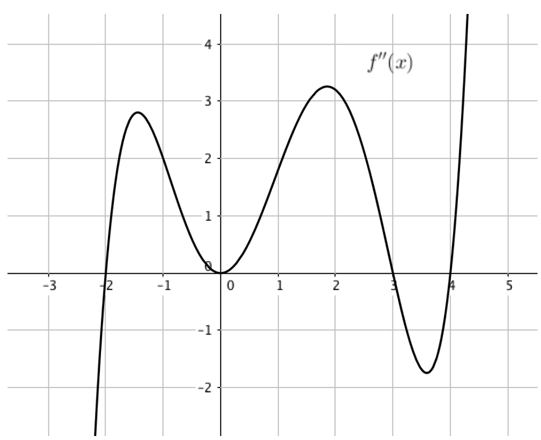
Question	6	7	8	9	Total
Answer					

Each of the following MC questions is worth 5 points. No partial credit.

1. Air is pumped into a spherical balloon at the rate of 8 cubic centimeters per minute. What is the rate of change of the surface area per minute when the radius of the balloon is 2 centimeters? (The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$ and the surface area is $S = 4\pi r^2$.)

   (a) 8. (b) 8π. (c) 4π. (d) 2π. (e) 4.

2. The following is a plot of $f''(x)$, the second derivative of a function $f(x)$. Find ALL the inflection points of $f(x)$.



   (a) $x = 0, 3.5$. (b) $x = -1.5, 1.8$. (c) $x = -1.5, 0, 1.8, 3.5$. (d) $x = -2, 3, 4$. (e) $x = -2, 0, 3, 4$.

3. A candy box is to be made out of a piece of cardboard that measures 8 by 8 inches. Squares of equal size will be cut out of each corner, and then the ends and sides will be folded up to form a rectangular box. What size square should be cut from each corner to obtain a maximum volume?

   (a) 4. (b) $\frac{4}{3}$. (c) $\frac{2}{3}$. (d) 2. (e) None of the above.

4. At which point of x is the tangent line of the graph $y = e^{2x} - 2x + 1$ horizontal?

   (a) $x = 0$. (b) $x = \frac{\ln 2}{2}$ (c) $x = 1$ (d) $x = \frac{\ln 3}{2}$ (e) None of the above

5. What is $f''(0)$ for $f(x) = \ln(1 + e^x)$?

   (a) 0. (b) $\frac{1}{2}$. (c) $\frac{1}{4}$. (d) e. (e) None of the above

6. Which of the following number is the slope of the tangent line to the curve given by

$$\ln(xy) = y^2 - 1$$

   at the point $(x, y) = (1, 1)$?

   (a) 0. (b) $\frac{1}{2}$. (c) 1. (d) 2. (e) None of the above.

7. What value of A would make the function

$$f(x) = \begin{cases} Axe^{\frac{x}{2}} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   a probability density function?

   (a) $\frac{1}{4}$. (b) 2. (c) $\frac{1}{2}$. (d) 4. (e) 1

8. The shelf life (in years) of a laser pointer battery is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{2}{(x+2)^2} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that a randomly selected laser pointer battery has a shelf life of from 1 to 4 years?

(a) $\frac{1}{4}$. (b) $\frac{1}{6}$. (c) $\frac{1}{3}$. (d) $\frac{2}{5}$. (e) None of the above.

9. Which of the following is the value of the definite integral

$$\int_1^2 \ln\left(xe^{2x}\right)\,dx?$$

(a) $2\ln 2 + 1$. (b) $\ln 2 + 3$. (c) $\ln 2 + 1$. (d) $2\ln 2 + 2$. (e) None of the above.

Part II: Answer each of the following 3 long questions. Unless otherwise specified, numerical answers should be either exact or correct to 2 decimal places.

10. Consider the graph of the function $f(x) = \dfrac{x^2 + x + 2}{x - 1}$ (five sub-problems).

(1). What is the domain of $f(x)$? What are the vertical and horizontal asymptotes (if there are any)? What are the x- and y-intercepts (if there are any)?

(2). List all critical numbers if there is any. Find the intervals on which $f(x)$ is increasing, and those on which $f(x)$ is decreasing.

(3). List all inflection points if there is any. Find the intervals on which $f(x)$ is concave upward, and those on which $f(x)$ is concave downward.

(4) Find the local maximum and local minimum of $y = f(x)$. Are they absolute maximum and absolute minimum of $y = f(x)$? Why?

(5) Use the above information to sketch the graph $y = f(x)$.

11. Calculate the indicated integrations (four sub-problems)

(1).

$$\int \left(x^3 + \frac{1}{x} + e^x\right) dx.$$

(2).

$$\int x \left(e^x + e^{x^2}\right) dx.$$

(3).

$$\int \left(\ln x + \frac{1}{x} \right) dx.$$

(4).

$$\int \left(\ln x + \frac{1}{x} \right)^2 dx.$$

12. Set-up the integral for computation
Instruction: just set-up the integral without explicitly computing it. For example, the area bounded by $y = x$ and the x axis over the interval $[1, 2]$ is given by $\int_1^2 x dx$. No need to compute it.

(1). Find the area between the graph of $f(x) = x^2 - 1$ and the x axis over the interval $[0, 3]$.

(2). Find the area bounded by the graphs of $f(x) = x^2 - 1, g(x) = -x - 3, x = -1$ and $x = 2$.

(3). Find the area of the finite region bounded by the graphs of $f(x) = 5 - x^2$ and $g(x) = 2 - 2x$.

(4). Find the area of the finite region bounded by the graphs of $f(x) = x^3 + 5x^2 + 5x$ and $g(x) = x$.

Scratch paper

*** END OF PAPER ***