HKUST

MATH1003 Calculus and Linear Algebra

Midterm Exam (Version A) Name:_____

8th October 2016 Student ID:_____

10:30-12:00 Lecture Section: _____

Directions:

- Do NOT open the exam until instructed to do so.

- Please turn off all phones and pagers, and remove headphones.

- Please write your name, ID number, and Tutorial Section in the space provided above.

- When instructed to open the exam, please check that you have <u>7 pages</u> in addition to the cover page.

- Answer all questions. Show an appropriate amount of work for each problem. If you do not show enough work, you will get only partial credit.

- You may use an ordinary scientific calculator, but calculators with graphical, or symbolic calculation functions are NOT allowed.

- This is a closed book examination.

- **Cheating is a serious offense. Students caught cheating will receive a zero score for the midterm exam, and will also be subjected to further penalties imposed by the University.**

Question No.	Points	Out of
Q. 1-10		40
Q. 11		10
Q. 12		15
Q. 13		15
Total Points		80

Part I: Answer the following multiple choice questions.

Put your MC question answers in the following boxes.

Question	1	2	3	4	5	Total
Answer						

Question	6	7	8	9	10	Total
Answer						

For each of the following MC questions, from the choices (a)(b)(c)(d) and (e), choose *exactly* one correct answer. Each MC question is worth 4 points. No partial credit.

1. An online bank listed a 1-year CD that earns 1.25% compounded monthly. Find the APY as a percentage, rounded to three decimal places.

   (a) 1.257%   (b) 1.246%   (c) 1.345%   (d) 1.250%   (e) 1.251%

2. Some friends tell you that they paid $25,000 down on a new house and are to pay $525 per month for 30 years. If interest is 7.9% compounded monthly, what was the selling price of the house?

   (a) $92987.25   (b) $85679.38   (c) $92349.19   (d) $85789.19   (e) $97233.92

3. At the time they retire, a couple has $200,000 in an account that pays 8.4% compounded monthly. If the couple decides to withdraw $3,000 a month as long as it takes, and then make a final withdraw which may be a smaller amount than $3,000 to reduce the account balance to zero. What is the amount of the last withdraw?

   (a) $300.56   (b) $323.38   (c) $311.27   (d) $346.60   (e) $350.87

4. A promissory note will pay $27,000 at maturity 10 years from now. How much money should you be willing to pay now if money is worth 5.5% compounded monthly?

   (a) $15748.23   (b) $14568.48   (c) $15597.23   (d) $16625.46   (e) $17729.95

5. Which of the following matrices is in reduced form?

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}.$$

(a) A, B and C are all in reduced form.

(b) Only A and B are in reduced form.

(c) Only B is in reduced form.

(d) Both B and C are in reduced form.

(e) None of them is in reduced form.

6. Which of the following matrix operations are well-defined? (Only one is correct!)

$$A = \begin{bmatrix} 1 & 4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 7 \end{bmatrix}, \quad D = \begin{bmatrix} -100 \\ 205 \\ 300 \end{bmatrix}.$$

(a) $BA + C$.

(b) $C^2 - 3B$.

(c) $-2C + AD$.

(d) $ACD + 5$.

(e) $BDA + 10$.

7. Let $A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 0 & 1 & -2 \\ -3 & 0 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$. Find the entries in the third row of AB.

(a) $[5, 1]$.

(b) $[7, 2]$

(c) $[15, 36]$

(d) $[1, -3]$

(e) $[2, 3]$

8. For what value h the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & h & 0 \end{bmatrix}$ is not invertible?

   (a) $h = 0$ (b) $h = 1$ (c) $h = 2$ (d) $h = 3$ (e) $h = 4$

9. If the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ satisfies

$$A \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

what is a_{21}?

   (a) 1   (b) 0.1   (c) -1 (d) -0.1 (e) 4

10. The economy of a country is based on two sectors, agriculture and oil. Production of a dollar's worth of agriculture requires an input of $0.40 from agriculture and $0.35 from oil. Production of a dollar's worth of oil requires an input of $0.20 from agriculture and $0.05 from oil. The final demand is $250 million for oil and $40 million for agriculture. What is the output from agriculture?

   (a) $135 million

   (b) $459 million

   (c) $176 million

   (d) $460 million

   (e) $550 million

Part II: Answer each of the following 3 long questions. Unless otherwise specified, numerical answers should be either exact or correct to 2 decimal places.

11. [*10 pts*] A person purchased a house 20 years ago by paying 20% down and signing a 30-year mortgage at 13.2% compounded monthly. Now interest rates have dropped and the owner is able to refinance the unpaid balance by signing a new 10-year mortgage at 5% compounded monthly, which reduces the original monthly payment by $500.

   (a) What was the original monthly payment before refinancing (i.e., monthly payment for the original 30-year mortgage)?

   (b) How much did the person pay for the house 20 years ago?

   (c) How much total interest will the person save by refinancing the mortgage?

12. The sum of three numbers is 14. The largest is 4 times the smallest, while the sum of the largest and twice the smallest is the remaining number plus 8. Find the three numbers.

(1). Let x_1, x_2, x_3 be the largest, the middle, and the smallest numbers respectively. Write down the linear system of equations in terms of x_1, x_2, x_3.

(2). Solve the linear system in (1) using the Gauss-Jordan elimination method. Show all steps.

13. The rush-hour traffic flow (in vehicles per hour) for a network of four one-way streets is shown in the figure.
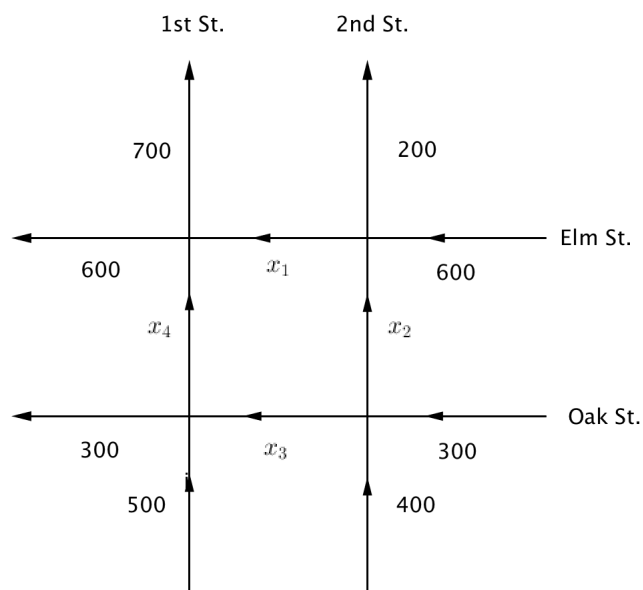
    (1). Write down the system of equations determined by the flow of traffic through the four intersections.

    (2). Find the solution of the system in part (1). (You may write on the back.)

(3). What is the maximum number of vehicles per hour that can travel from Oak Street to Elm Street on 1st Street? What is the minimum number?