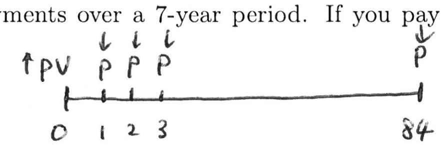
MATH1003 Calculus and Linear Algebra, Tutorial

Week 02 — Worksheet: Mathematics of Finance II

1. **(Demonstration, Amortization)** (p. 162, Q 32) (A) A car costs \$80,000. You pay 10% down and amortize the rest with equal monthly payments over a 7-year period. If you pay 9.25% compounded monthly, what is your monthly payment?



$7 \times 12 = 84$

$i = \frac{0.0925}{12}$

$PV = 80000 \times 0.9$

$$80000 \times 0.9 = P \frac{1-\left(1+\frac{0.0925}{12}\right)^{-84}}{\frac{0.0925}{12}}$$

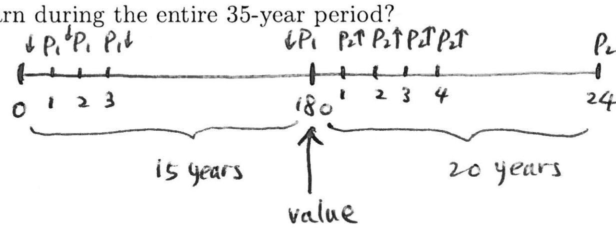
solve $P = 1167.57$

(B) How much interest will you pay?

$$84 \times P - 80000 \times 0.9 = 26075.88$$

2. **(Demonstration, Annuity)** (p. 163 , Q. 48)

(a) An ordinary annuity pays 6.48% compounded monthly. A person wants to make equal monthly deposits into the account for 15 years in order to them make equal monthly withdraws of \$1,500 for the next 20 years, reducing the balance to 0. How much should be deposited each month for the first 15 years? How much interest does the person earn during the entire 35-year period?



$15 \times 12 = 180$

$20 \times 12 = 240$

$P_2 = 1500$

$$P_1 \frac{\left(1+\frac{0.0648}{12}\right)^{180}-1}{\frac{0.0648}{12}} = 1500 \times \frac{1-\left(1+\frac{0.0648}{12}\right)^{-240}}{\frac{0.0648}{12}}$$

$\boxed{P_1 = 664.99}$

(b) If the person makes monthly deposits of \$1,000 for the first 15 years, how much can be withdrawn monthly for the next 20 years?

→ Total interest $= 240 \times 1500 - 180 \times P_1 = \underline{240301.8}$

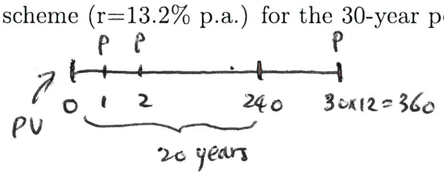
(b). $$1000 \times \frac{\left(1+\frac{0.0648}{12}\right)^{180}-1}{\frac{0.0648}{12}} = P_2 \frac{1-\left(1+\frac{0.0648}{12}\right)^{-240}}{\frac{0.0648}{12}}$$

$\boxed{P_2 = 2255.69}$

3. **(Demonstration, Refinance)** (p.163 , Q.52) A person purchased a house 20 years ago for \$ 200,000 by paying 20% down and signing a 30-year mortgage at 13.2% compounded monthly. Interest rates have dropped and the owner wants to refinance that unpaid balance by signing a new 10-year mortgage at 8.2% compounded monthly.

(a) What is the monthly payment under the old scheme (r=13.2% p.a.) for the 30-year period?

$PV = 200000 \times 0.8$



$$200000 \times 0.8 = P \frac{1-\left(1+\frac{0.132}{12}\right)^{-360}}{\frac{0.132}{12}}$$

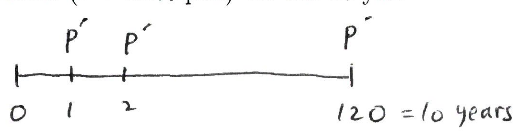
$$P = 1794.97$$

(b) What is the unpaid balance now (after 240 payments)?

$$P \frac{1-\left(1+\frac{0.132}{12}\right)^{-120}}{\frac{0.132}{12}} = 119272.89$$

(c) What is the new monthly payment under the new refinance scheme ($\hat{r}$ = 8.2% p.a.) for the 10-year period?

$$119272.89 = P' \frac{1-\left(1+\frac{0.082}{12}\right)^{-120}}{\frac{0.082}{12}}$$



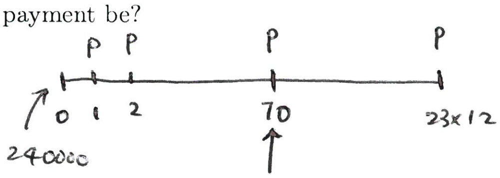
$$P' = 1459.74$$

(d)How much interest will refinancing save?

$$1794.97 \times 120 - 1459.74 \times 120 = \boxed{40227.6}$$

4. **(WebWork problem)** Dave takes out a 23-year mortgage of 240000 dollars for his new house. Dave gets an interest rate of 15.6 percent compounded monthly. He agrees to make equal monthly payments, the first coming in one month. After making the 70th payment, Dave wants to buy a boat, so he wants to refinance his house to reduce his monthly payment by 400 dollars, and to get a better interest rate. In particular, he negotiates a new rate of 7.2 percent compounded monthly, and agrees to make equal monthly payments (each 400 dollars less than his original payments) for as long as necessary, followed by a single smaller payment. How large will Dave's final loan payment be?

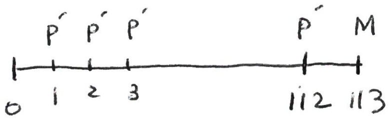
① $$240000 = P \frac{1-\left(1+\frac{0.156}{12}\right)^{-23\times12}}{\frac{0.156}{12}}$$



$$P = 3210.87$$

② After the 70th payment, he still owe money to the bank, the amount is

$$T = P \frac{1-\left(1+\frac{0.156}{12}\right)^{-(23\times12-70)}}{\frac{0.156}{12}} = 229726.31$$



③ $$229726.31 = (3210.87-400) \frac{1-\left(1+\frac{0.072}{12}\right)^{-n}}{\frac{0.072}{12}} \qquad n = 112.68$$

④ $$229726.31 = (3210.87-400) \frac{1-\left(1+\frac{0.072}{12}\right)^{-112}}{\frac{0.072}{12}} + \frac{M}{\left(1+\frac{0.072}{12}\right)^{113}}$$

$$\boxed{M = 1915.36}$$