MATH1003 Calculus and Linear Algebra, 2017-18 Fall

Week 03 — Worksheet: Matrices

1. Which one or ones of the following matrices is in the reduced form? Try to reduce the ones which are not in the reduced form to the reduced form.

(a) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (d) $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 4 \end{array}\right]$

$3R_2+R_1 \to R_1$

(c) is in the reduced form

$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 13 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (b)

$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & \frac{1}{2} & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 4 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

2. Write down the augmented matrices of the following systems, and then try to solve the system:

(a)
$$\begin{cases} x_1 + 2x_2 - 3x_3 = 0 \\ 2x_1 - x_2 + x_3 = 0 \\ 3x_1 + 3x_2 + x_3 = 0 \end{cases}$$

(b)
$$\begin{cases} x_1 + x_2 = 0 \\ x_2 + x_3 = 1 \\ x_3 + x_4 = 2 \end{cases}$$

(a) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & -1 & 1 & 0 \\ 3 & 3 & 1 & 0 \end{array}\right] \xrightarrow[-3R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & -5 & 7 & 0 \\ 0 & -3 & 10 & 0 \end{array}\right] \xrightarrow{-\frac{1}{5}\times R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & -\frac{7}{5} & 0 \\ 0 & -3 & 10 & 0 \end{array}\right]$.

$\xrightarrow[3R_2+R_3 \to R_3]{-2R_2+R_1 \Rightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{5} & 0 \\ 0 & 1 & -\frac{7}{5} & 0 \\ 0 & 0 & \frac{29}{5} & 0 \end{array}\right] \xrightarrow{\frac{5}{29}R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{7}{5} & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[\frac{7}{5}R_3+R_2 \to R_2]{-\frac{1}{5}R_3+R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

(b) $\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \xrightarrow{-R_2+R_1 \to R_1} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$

$\xrightarrow[R_3+R_1 \to R_1]{-R_3+R_2 \to R_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$

$x_1 = 1 - x_4$
$x_2 = -1 + x_4$
$x_3 = 2 - x_4$

3. A chemical manufacturer wants to lease a fleet of 24 railroad tank cars with a combined carrying capacity of 520,000 gallons. Tank cars with three different carrying capacities are available: 8000 gallons, 16000 gallons, and 24000 gallons. How many of each type of tank car should be leased?

x_1, x_2, x_3

$$\begin{cases} x_1 + x_2 + x_3 = 24 \\ 8000x_1 + 16000x_2 + 24000x_3 = 520000 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 8000 & 16000 & 24000 & 520000 \end{array}\right] \xrightarrow{-8000R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 0 & 8000 & 16000 & 328000 \end{array}\right]$$

$$\xrightarrow{\frac{R_2}{8000}\to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 0 & 1 & 2 & 41 \end{array}\right] \xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -17 \\ 0 & 1 & 2 & 41 \end{array}\right]$$

$x_1 = -17 + t$

$x_2 = 41 - 2t$

$x_3 = t$

$17 \le t \le 20$

4. In a laboratory experiment, rats are to be fed 5 packets of food containing a total of 80 units of vitamin E. There are four different brands of food packets that can be used. A packet of brand A contains 5 units of vitamin E, a packet of brand B contains 10 units of vitamin E, A packet of brand C contains 15 units of vitamin E, A packet of brand D contains 20 units of vitamin E.

(a) How many packets of each brand should be mixed and fed to the rats?

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 5 \\ 5x_1 + 10x_2 + 15x_3 + 20x_4 = 80 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 5 & 10 & 15 & 20 & 80 \end{array}\right] \xrightarrow{-5R_1+R_2\to R_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 5 & 10 & 15 & 55 \end{array}\right] \xrightarrow{\frac{1}{5}R_2\to R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 3 & 11 \end{array}\right] \xrightarrow{-R_2+R_1\to R_1} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & -6 \\ 0 & 1 & 2 & 3 & 11 \end{array}\right]$$

$x_1 = -6 + t_1 + 2t_2$

$x_2 = 11 - 2t_1 - 3t_2$

$x_3 = t_1$

$x_4 = t_2$

(b) The cost of the four brands are: A, \$1.50; B, \$3.00; C, \$3.75; and D, \$2.25. Which of the solutions to the previous question would minimize the cost of the rat food? (You may assume the number of packets used for each brand is an non-negative integer in this part.)

$A = 1.5x_1 + 3x_2 + 3.75x_3 + 2.25x_4$

$= -9 + 1.5x_3 + 3x_4 + 33 - 6x_3 - 9x_4 + 3.75x_3 + 2.25x_4$

$= 24 - 0.75x_3 - 3.75x_4$

$$\begin{cases} 6 \le t_1 + 2t_2 \le 11 \\ 6 \le 2t_1 + 3t_2 \le 11 \\ 0 \le t_1 \le 5 \\ 0 \le t_2 \le 5 \end{cases} \to$$

$t_1 = 0 \Rightarrow t_2 = 3 \to A = 12.75$

$t_1 = 1 \Rightarrow t_2 = 3 \to A = 12$

$t_1 = 2 \Rightarrow t_2 = 2 \to A = 15$

$t_1 = 3 \Rightarrow t_2$ X

$t_1 = 4 \Rightarrow t_2 = 1 \to A = 17.25$

$t_1 = 5 \Rightarrow t_2$ X

$\begin{cases} x_1 = 1 \\ x_2 = 0 \\ x_3 = 1 \\ x_4 = 3 \end{cases}$