MATH1003 Calculus and Linear Algebra, 2017-18 Fall

Week 04 — Worksheet: Matrices

Let

$$A = \begin{bmatrix} 7 & 12 \\ 5 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -1 \\ 7 \end{bmatrix}, \quad D = [2 \;\; 4].$$

1. Compute $A - 3B =$

$$\begin{bmatrix} 7 & 12 \\ 5 & -3 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -1 & -6 \end{bmatrix}$$

2. Compute $AB = \begin{bmatrix} 7 & 12 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} =$ $\begin{bmatrix} 31 & 26 \\ -1 & 7 \end{bmatrix}$

3. Compute $BA = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 7 & 12 \\ 5 & -3 \end{bmatrix} =$ $\begin{bmatrix} 17 & 6 \\ 19 & 21 \end{bmatrix}$

4. Compute $CD = \begin{bmatrix} -1 \\ 7 \end{bmatrix}[2 \;\; 4] =$ $\begin{bmatrix} -2 & -4 \\ 14 & 28 \end{bmatrix}$

5. Compute $DC = [2 \;\; 4]\begin{bmatrix} -1 \\ 7 \end{bmatrix} =$ 26

6. Find B^{-1}.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \end{array}\right]$$

$$\to \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \end{array}\right]$$

7. Compute $B^{-1}C$.

$$\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 7 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

8. Find two numbers m, n such that $B\begin{bmatrix} m \\ n \end{bmatrix} = C$.

$$\begin{bmatrix} m \\ n \end{bmatrix} = B^{-1} \cdot C = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

9. In a local election, the public relations firm ABC promoted its candidate in three ways: telephone calls (W_1), house calls(W_2), and letters(W_3). The cost per contact is given in matrix M, and the number of contacts of each type made in three adjacent cities Berkeley(C_1), Oakland(C_2) and San Francisco (C_3) is given in matrix N.

$$M = \begin{bmatrix} 1.2 \\ 3 \\ 1.45 \end{bmatrix} \quad N = \begin{bmatrix} 1000 & 500 & 5000 \\ 2000 & 800 & 8000 \\ 3000 & 1000 & 6000 \end{bmatrix}$$

That is, the i^{th} entry of M tells the unit cost (in dollars) to contact one person using W_i; while the (ij) entry of N says in city C_i, there are N_{ij} persons to contact using way W_j .

(a) Compute NM and explain the meaning of its entries.

$$\begin{bmatrix} 9950 \\ 16400 \\ 15300 \end{bmatrix}$$

cost in C_1, C_2, C_3

(b) What is the total promotion cost of all 3 cities?

$9950 + 16400 + 15300 = 41650$

(c) Public relations firm XYZ offer another promotion scheme, using the same ways, over the same city, same number recipients, but the total amount of money spent on each city are respectively:
Berkeley: \$10000;
Oakland:\$16440;
Francisco: \$15100.
Find firm XYZ's unit cost for each way of contact, given that

$$N^{-1} = \begin{bmatrix} -\frac{1}{250} & \frac{1}{400} & 0 \\ \frac{3}{200} & -\frac{9}{800} & \frac{1}{400} \\ -\frac{1}{2000} & \frac{1}{1600} & -\frac{1}{4000} \end{bmatrix}.$$

$$NM' = \begin{bmatrix} 10000 \\ 16440 \\ 15100 \end{bmatrix}$$

$$M' = N^{-1} \cdot \begin{bmatrix} 10000 \\ 16440 \\ 15100 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 2.8 \\ 1.5 \end{bmatrix}$$