

MATH1003 Calculus and Linear Algebra, 2017-18 Fall

Week 05 — Worksheet: Mid-term Review

Part I: Multiple Choice Questions.

Each of the following MC questions is worth 10 points, for a total of 50 points. No partial credit. Put your MC question answers in the following table.

- C 1. Solve the system of linear equations

$$\begin{cases} x - 2y + 2z = 3 \\ 3x - 7y + 5z = 4 \\ 2x - 4y + 5z = 9 \end{cases}$$

We find $z = ?$

- (a) $z = 1$ (b) $z = 2$ (c) $z = 3$ (d) $z = 0$ (e) None of the above

- d 2. A note will pay \$15,000 at maturity 5 years from now. How much should you be willing to pay for the note now if money is worth 5.2% compounded quarterly?

- (a) \$13,640.6 (b) \$13,257.8 (c) \$12,305.7 (d) * \$11,585.2 (e) \$10,650.2

- d 3. $A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 0 & 1 & -2 \\ -3 & 0 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$. Find the entries in the third row of AB .

- (a) $[5, 1]$. (b) $[7, 2]$ (c) $[15, 36]$ (d) $[1, -3]$ (e) $[2, 3]$

- C 4. How many days will it take money to double if it is invested at 7.5% compounded daily? (Use a 365-day year.)

- (a) 9.24 (b) 3200 (c) 3374 (d) 10.53 (e) 3650

Part II: Answer each of the following 2 long questions. Unless otherwise specified, numerical answers should be either exact or correct to 2 decimal places. Write all steps.

5. [25 pts] A person purchased a \$250,000 home 10 years ago by paying 20% down and signing a 30-year mortgage at 12% compounded monthly.

(a) Find the monthly payment for the mortgage.

$$250000 \times 0.8 = \text{PMT} \frac{1 - \left(1 + \frac{12\%}{12}\right)^{-30 \cdot 12}}{\frac{12\%}{12}}$$

$$\text{PMT} = 2057.23$$

(b) Find the unpaid balance of the mortgage now (or how much does he still owe to the bank).

$$\text{unpaid balance} = \text{PMT} \frac{1 - \left(1 + \frac{12\%}{12}\right)^{-(30 \cdot 12 + 10 \cdot 12)}}{\frac{12\%}{12}}$$

$$\text{unpaid balance} = 186835.99$$

(c) How much interest has he paid to the bank so far?

$$\text{PMT} \cdot n - \text{principal} + \text{unpaid balance}$$

$$2057.23 \times 120 - 250000 \times 0.8 + 186835.99 = 233703.59$$

(d) Now the owner got a deal from a bank: he paid down 20% of the unpaid balance, and signed a new 20-year mortgage at 8% compounded monthly. What is the monthly payment for the new mortgage?

$$186835.99 \times 0.8 = \text{new PMT} \frac{1 - \left(1 + \frac{8\%}{12}\right)^{-20 \cdot 12}}{\frac{8\%}{12}}$$

$$\text{new PMT} = 1250.22$$

(e) With refinancing, what is the total interest the person will pay during the next 20-year period?

$$\text{Total interest} = \text{new PMT} \cdot 240 - \text{new principal}$$

$$= 1250.22 \times 240 - 186835.99 \times 0.8$$

$$= 150584.01$$

6. [25 pts] An economy is based on two sectors, energy (E) and water (W). To produce one dollar's worth of E requires 0.6 dollar's worth of E and 0.1 dollar's worth of W, and to produce one dollar's worth of W requires 0.2 dollar's worth of E and 0.7 dollar's worth of W.

(a) Find the technology matrix M for the economy.

$$M = \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.7 \end{bmatrix}$$

(b) Find the total output for each sector that is needed to satisfy a final demand of \$40 billion for energy and \$30 billion for water.

$$(I - M) = \begin{pmatrix} 0.4 & -0.2 \\ -0.1 & 0.3 \end{pmatrix} \quad D = \begin{pmatrix} 40 \\ 30 \end{pmatrix}$$

$$(I - M)^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

$$X = (I - M)^{-1} D = \begin{bmatrix} 150 \\ 200 \end{bmatrix}$$

output for energy: 150 output for water: 200

(c) Find the final demand for each sector if the total output of energy is \$60 billion and the total output of water is \$70 billion.

$$D = (I - M) X = \begin{pmatrix} 0.4 & -0.2 \\ -0.1 & 0.3 \end{pmatrix} \begin{pmatrix} 60 \\ 70 \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 15 \end{pmatrix}$$

final demand for energy: 10

final demand for water: 15