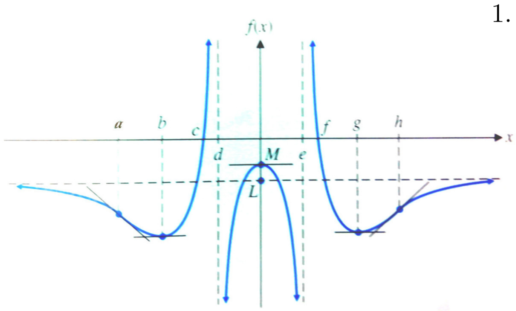
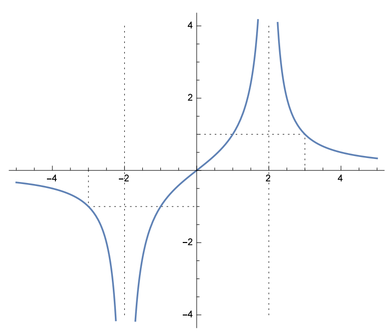
MATH1013 Calculus I, 2012-13 Fall
Week 09 — Graphing Functions

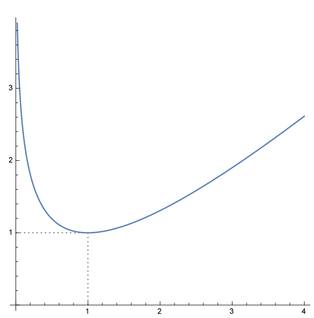


1. (P.667, Q. 1) Identify the following based on the function graph:
   (a) the intervals on which $f'(x) < 0$: $(-\infty, b) \cup (0, e) \cup (e, g)$
   (b) the intervals on which $f'(x) > 0$: $(b, d) \cup (d, 0) \cup (g, +\infty)$
   (c) the intervals on which $f(x)$ is increasing: $(b, d) \cup (d, 0) \cup (g, +\infty)$
   (d) the intervals on which $f(x)$ is decreasing: $(-\infty, b) \cup (0, e) \cup (e, g)$
   (e) the x coordinates of local maximum: 0
   (f) the x coordinates of local minimum: b, g
   (g) the intervals on which $f''(x) < 0$: $(-\infty, a) \cup (d, e) \cup (h, +\infty)$
   (h) the intervals on which $f''(x) > 0$: $(a, d) \cup (e, h)$
   (i) the intervals on which $f(x)$ is concave upward: $(a, d) \cup (e, h)$
   (j) the intervals on which $f(x)$ is concave downward: $(-\infty, a) \cup (d, e) \cup (h, +\infty)$
   (k) the x coordinates of the inflection point: a, d, e, h
   (l) the horizontal asymptotes: $y = L$
   (m) the vertical asymptotes: $x = d, x = e$

2. (P. 668, Q.9) Sketch a graph of f satisfying the following conditions:
   (a) Domain of f includes all real numbers except $x = 2$ and $x = -2$;
   (b) $f(-3) = -1, f(0) = 0, f(3) = 1$;
   (c) $f'(x) < 0$ on $(-\infty, -2)$ and $(2, +\infty)$;
   (d) $f'(x) > 0$ on $(-2, 2)$;
   (e) $f''(x) < 0$ on $(-\infty, -2)$ and $(-2, 0)$;
   (f) $f''(x) > 0$ on $(0, 2)$ and $(2, +\infty)$;
   (g) vertical asymptotes: $x = 2$ and $x = -2$;
   (h) horizontal asymptotes: $y = 0$.



3. (P. 668, Q.27) Sketch $f(x) = x - \ln x$.



Domain: $x > 0$.
First-order Derivative:

$$f'(x) = 1 - \frac{1}{x} \begin{cases} < 0 & \text{when } 0 < x < 1; \text{ (decreasing)} \\ = 0 & \text{when } x = 1; \text{ (critical number \& local minimum)} \\ > 0 & \text{when } x > 1. \text{ (increasing)} \end{cases}$$

Second-order Derivative:

$$f''(x) = \frac{1}{x^2} > 0. \text{ (concave upward)}$$

Vertical Asymptote: $x = 0$.

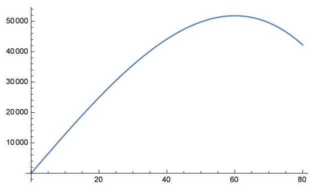
4. (P. 669, Q. 73)(Revenue) The marketing research department for a computer company used a large city to test the market for the firm's now laptop. The department found that the relationship between unit price p (in dollars) and demand x (units sold per week) was approximately given by

$$p(x) = 1296 - 0.12x^2 \qquad 0 \leqslant x \leqslant 80$$

Therefore the weekly revenue is given by

$$R(x) = 1296x - 0.12x^3 \qquad 0 \leqslant x \leqslant 80$$

Graph R.



Domain: $0 \leq x \leq 80$.
First-order Derivative:

$$R'(x) = 1296 - 0.36x^2 \begin{cases} > 0 & \text{when } 0 < x < 60; \text{ (increasing)} \\ = 0 & \text{when } x = 60; \text{ (critical number \& local maximum)} \\ > 0 & \text{when } x > 60. \text{ (decreasing)} \end{cases}$$

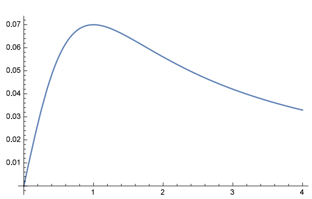
Second-order Derivative:

$$R''(x) = -0.72x < 0. \text{ (concave downward)}$$

5. (P. 670, Q. 83)(Medicine) A drug is injected into a patient's bloodstream through her right arm. The drug concentration (in milligrams per cubic centimeter) in blood stream of the left arm t hours after the injection is given by

$$C(t) = \frac{0.14t}{t^2 + 1}.$$

Graph C.



Domain: $t \geq 0$.
First-order Derivative:

$$C'(x) = \frac{-0.14(t^2 - 1)}{(t^2 + 1)^2} \begin{cases} > 0 & \text{when } 0 < t < 1; \text{ (increasing)} \\ = 0 & \text{when } t = 1; \text{ (critical number \& local maximum)} \\ > 0 & \text{when } t > 1. \text{ (decreasing)} \end{cases}$$

Second-order Derivative:

$$C''(x) = \frac{0.28t(t^2 - 3)}{(t^2 + 1)^3} \begin{cases} < 0 & \text{when } 0 < t < \sqrt{3}; \text{ (concave downward)} \\ = 0 & \text{when } t = \sqrt{3}; \text{ (inflection point)} \\ > 0 & \text{when } t > \sqrt{3}. \text{ (concave upward)} \end{cases}$$