MATH1003 Calculus and Linear Algebra, 2017-18 Fall

Week 10— Optimization

1. **(Maximum Revenue and Profit)** (P. 689, Q. 12) A company manufactures and sells x digital cameras per week. The weekly price-demand and cost equations are, respectively

$$p(x) = 400 - 0.4x \text{ and } C(x) = 2000 + 160x.$$

(A) How many cameras the company should produce every week so to maximize the weekly revenue?
(B) How many cameras the company should produce every week so to maximize the weekly profit?

Solution

(A) Maximize weekly revenue

$$R(x) = p(x) \cdot x = 400x - 0.4x^2$$

subject to $x > 0$.
$R'(x) = 400 - 0.8x \Longrightarrow$ Critical point: $x = 500$.
$R''(x) = -0.8 < 0 \Longrightarrow R(500) = 100000$ is the absolute maximum (the only critical point).

(B) Maximize weekly profit

$$P(x) = R(x) - C(x) = -2000 + 240x - 0.4x^2$$

subject to $x > 0$.
$P'(x) = 240 - 0.8x \Longrightarrow$ Critical point: $x = 300$.
$P''(x) = -0.8 < 0 \Longrightarrow P(300) = 34000$ is the absolute maximum (the only critical point).

2. **(Hotel Rental Income)** (P. 690, Q. 20) A 300-room hotel is filled to capacity at \$ 80 per room per night. For each \$ 1 increasing in room rate, 3 fewer rooms are rented. If each rental room costs \$ 10 to service per day, how much should the management charge for each room so to maximize the profit (assuming all rooms are identical)?

Solution

Charge per room: x; Number of room rented: $n(x) = 300 - 3(x - 80) = 540 - 3x$;
Revenue: $R(x) = n(x) \cdot x = 540x - 3x^2$; Cost: $C(x) = n(x) \cdot 10 = 5400 - 30x$.
Maximize profit

$$P(x) = R(x) - C(x) = -5400 + 570x - 3x^2$$

subject to $x \geq 80$.
$P'(x) = 570 - 6x \Longrightarrow$ Critical point: $x = 95$.
$P''(x) = -6 < 0 \Longrightarrow P(95) = 21675$ is the absolute maximum (the only critical point).

3. **(Construction Cost)** (P. 690, Q. 25) A fence is to be built to enclose a rectangular region of 800 square feet. The fence along three sides is to be built using material that costs \$6 per foot while the material for the fourth side cost \$18 per foot. Find the dimensions of the rectangle that will minimize the construction

Solution

Area: $800 = xy \Rightarrow y = \frac{800}{x}$;
Cost: $C = 6y + 6y + 6x + 18x = 24x + 12y = 24x + \frac{9600}{x}$.
To minimize

$$C(x) = 24x + \frac{9600}{x}$$

subject to $x > 0$.
$C'(x) = 24 + \frac{9600}{x^2} \Longrightarrow$ Critical point: $x = 20$.
$C''(x) = \frac{19200}{x^3} > 0 \Longrightarrow C(20) = 960$ is the absolute minimum (the only critical point).

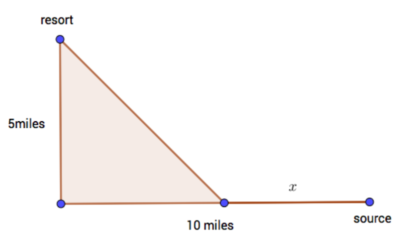
4. **(Inventory Control)** (P. 690, Q. 28) A pharmacy has uniform annual demand for 200 bottles of a certain antibiotic. It costs \$10 to store 1 bottle for 1 year and \$40 to place an order. How many times during the year the pharmacy should order that antibiotic in order to minimize the total storage and ordering cost?

   Solution

   Assume x is the number of bottles bought each time; y is the number of orders.
   Annual demand: $200 = xy \Rightarrow y = \frac{200}{x}$; Total Cost $C = 40y + 5x = 5x + \frac{8000}{x}$.
   To minimize
   $$C(x) = 5x + \frac{8000}{x}$$
   subject to $0 < x \le 200$.
   $C'(x) = 5 + \frac{8000}{x^2} \Longrightarrow$ Critical point: $x = 40$.
   $C''(x) = \frac{16000}{x^3} > 0 \Longrightarrow C(40) = 400$ is the absolute minimum (the only critical point).

5. **(Construction Cost)** (P. 690, Q. 31) A fresh water pipeline is to be run from a source on the edge of a lake to a small resort on an island 5 miles offshore. The distance between the nearest point of the shore to the resort and the source is 10 miles. If it costs 1.4 times as much as to lay the pipe in the lake as it does on land, where the pipe should land in order to minimize the constructional cost?

   Solution



   Minimize total cost
   $$C(x) = x + 1.4\sqrt{5^2 + (10-x)^2} = x + 1.4\sqrt{125 - 20x + x^2}$$
   subject to $0 \le x \le 10$.
   $C'(x) = 1 + \frac{1.4x - 14}{\sqrt{125 - 20x + x^2}} \Longrightarrow$ Critical point: $x = 10 - \frac{25\sqrt{6}}{12} = 4.897$.
   $C''(x) = \frac{35}{(125 - 20x + x^2)^{\frac{3}{2}}} > 0 \Longrightarrow C(4.897) = 14.899$ is the absolute minimum (the only critical point).