HKUST

MATH1003 Calculus and Linear Algebra

Midterm Examination (Version A)

3th October 2015

10:30-12:00

Name:_____

Student ID:_____

Lecture Section: _____

Directions:

- Do NOT open the exam until instructed to do so.
- Please turn off all phones and pagers, and remove headphones.
- Please write your name, ID number, and Tutorial Section in the space provided above.
- When instructed to open the exam, please check that you have 5 pages in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each problem. If you do not show enough work, you will get only partial credit.
- You may use an ordinary scientific calculator, but calculators with graphical, or symbolic calculation functions are NOT allowed.
- This is a closed book examination.
- **Cheating is a serious offense. Students caught cheating will receive a zero score for the midterm exam, and will also be subjected to further penalties imposed by the University.**

Question No.	Points	Out of
Q. 1-7		35
Q. 8		15
Q. 9		15
Q. 10		15
Total Points		80

Part I: Answer the following multiple choice questions.

Put your MC question answers in the following boxes.

Question	1	2	3	4	5	6	7	Total
Answer	A	B	A	B	E	B	A	

Each of the following MC questions is worth 5 points. No partial credit.

1. John has started a saving plan in which a fixed amount of money will be deposited into an account at the end of every month in the coming 36 months. The annual interest rate on the account is 6%, compounded monthly. If John wants to have a total sum of \$36,000 at the end of the 36 month period, what is the amount of the monthly deposit John would need to make, rounded to the nearest dollar?

   (a) \$915 (b) \$241 (c) \$765 (d) \$890 (e) \$2341

2. You can afford monthly deposits of \$200 into an account that pays 5.7% compounded monthly. How long will it be until you have \$7,000? (Round to the next-higher month if not exact.)

   (a) 30 (b) 33 (c) 41 (d) 52 (e) 20

3. Given the augmented matrix $A = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & m & n \end{array}\right]$, for what values of m and n, the corresponding system of linear equations is consistent and has infinitely many solutions?

   (a) $m = n = 0$ (b) $m \neq 0, n \neq 0$ (c) $m = 0, n \neq 0$

   (d) $m \neq 0, n \neq 0$ (e) None of the previous

4. Among the four matrices below:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right], \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right], \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right], \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 4 \end{array}\right],$$

how many are in reduced form?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. Let $M_1 = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$, $M_2 = \begin{bmatrix} -1 & 1 \end{bmatrix}$. Then compute $3M_2M_1^{-1}$

(a) $[-3,-3]$ (b) $[3,-3]$ (c) $[2,3]$ (d) $[1,-1]$ (e) None of the previous

6. For the matrix equation $XA = B$ where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}, \quad B = [1,1,0], \quad X = [x_1, x_2, x_3].$$

If it is known that $A^{-1} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$, what is the value of x_2?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

7. Given the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, what is second row of A^{-1}?

(a) $[0,1,-1]$ (b) $[0,1,1]$ (c) $[1,1,1]$ (d) $[-1,1,-1]$ (e) $[0,1,0]$

Part II: Answer each of the following 3 long questions. Unless otherwise specified, numerical answers should be either exact or correct to 2 decimal places.

8. [*15 pts*] A person purchased a \$200,000 home by paying 20% down and signing a 30-year mortgage at 13.2% compounded monthly.

(a) Find the monthly payment for the mortgage.

4' $0.8 \times 200{,}000 = PMT \dfrac{1-\left(1+\frac{0.132}{12}\right)^{-360}}{\frac{0.132}{12}}$

$PMT = 1794.97$

(b) At the end of the 29th year, he sold the house for \$450,000. How much money will he has after paying off the remaining loan?

4' $PV_0 = PMT \dfrac{1-(1+i)^{-12}}{i}$ $\quad i = \dfrac{0.132}{12}$

$= 20075.41$

3' Remaining loan $= 450{,}000 - PV_0 = 429924.59$

(c) How much total interest has he paid during these 29 years?

4' Total interest = total payment − reduced loan

$= PMT \times 29 \times 12 - (0.8 \times 200{,}000 - PV_0)$

$= 484723.42$

9. [*15 pts*] A corporation has a taxable income of \$7,650,000. At this income level, the federal income tax rate is 50%, the state tax rate is 20%, and the local tax rate is 10%. If each tax rate is applied to the total taxable income, the resulting tax liability for the corporation would be 80% of taxable income. However, it is customary to deduct taxes paid to one agency before computing taxes for the other agencies. Assume that the federal taxes are based on the income that remains after the state and local taxes are deducted, and that state and local taxes are computed in a similar manner.

   (a) Let the federal tax, state tax and local tax be x_1, x_2, x_3 respectively. Set up the system of linear equations. (Hint: as for federal income tax, the equation is $x_1 = 0.5(7650000 - x_2 - x_3)$. What are the other two equations?)

2'+2' $\begin{cases} x_2 = 0.2\,(7{,}650{,}000 - x_1 - x_3) \\ x_3 = 0.1\,(7{,}650{,}000 - x_1 - x_2) \end{cases}$

8'

   (b) Solve the equations by Gauss-Jordan elimination. Show all steps. You may write on the back if there is not enough space.

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 0.5 & 0.5\times 7{,}650{,}000 \\ 0.2 & 1 & 0.2 & 0.2\times 7{,}650{,}000 \\ 0.1 & 0.1 & 1 & 0.1\times 7{,}650{,}000 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 2 & 1 & 1 & 7{,}650{,}000 \\ 1 & 5 & 1 & 7{,}650{,}000 \\ 1 & 1 & 10 & 7{,}650{,}000 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 10 & 7{,}650{,}000 \\ 1 & 19 & & 7{,}650{,}000 \\ 4 & -9 & & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -9 & 0 \\ 1 & 19 & & 7{,}650{,}000 \\ 0 & -85 & & -4\times 7{,}650{,}000 \end{array}\right]$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & & & 3240{,}000 \\ & 1 & & 810{,}000 \\ & & 1 & 360{,}000 \end{array}\right) \longrightarrow \begin{cases} x_1 = 3{,}240{,}000 \\ x_2 = 810{,}000 \\ x_3 = 360{,}000 \end{cases}$$

   (c) What is the tax liability of the corporation, as a percentage of taxable income if these deductions are taken into consideration?

3' $\dfrac{x_1 + x_2 + x_3}{7{,}650{,}000} = 57.65\%$

10. [*15 pts*] The economy of a country is based on two sectors, agriculture and oil. Production of a dollar's worth of agriculture requires an input of \$0.40 from agriculture and \$0.35 from oil. Production of a dollar's worth of oil requires an input of \$0.20 from agriculture and \$0.05 from oil.

(3') (a) Write down the technology matrix of the economy based on agriculture (A) and oil (O).

$$M = \begin{bmatrix} 0.4 & 0.2 \\ 0.35 & 0.05 \end{bmatrix}$$

(7') (b) Find the total output from each sector that is needed to satisfy a final demand of \$40 million for agriculture and \$250 million for oil.

$$D = \begin{bmatrix} 40 \\ 250 \end{bmatrix} \qquad I-M = \begin{pmatrix} 0.6 & -0.2 \\ -0.35 & 0.95 \end{pmatrix}$$

4': $(I-M)^{-1} = \begin{bmatrix} 1.9 & 0.4 \\ 0.7 & 1.2 \end{bmatrix}$

2': $X = (I-M)^{-1}D = \begin{bmatrix} 176 \\ 328 \end{bmatrix}$ → 1' { output for agriculture : 176 M; output for oil : 328 M }

(5') (c) Suppose the final demand for agriculture and oil in (b) are increased by 10% and 20% respectively. Find the percentage increase in the corresponding total output for each sector.

1' → $D_1 = \begin{bmatrix} 40 \times 1.1 \\ 250 \times 1.2 \end{bmatrix}$

2': $X_1 = (I-M)^{-1} D_1 = \begin{bmatrix} 203.6 \\ 390.8 \end{bmatrix}$

⇒ 2'' increase in Agriculture : $\frac{203.6-176}{176} = 15.68\%$

increase in oil : $\frac{390.8-328}{328} = 19.15\%$

*** END OF PAPER ***