HKUST

MATH1003 Calculus and Linear Algebra

Midterm Exam (Version C)

Name:_____

8th October 2016

Student ID:_____

10:30-12:00

Lecture Section: _____

Directions:

- Do NOT open the exam until instructed to do so.
- Please turn off all phones and pagers, and remove headphones.
- Please write your name, ID number, and Tutorial Section in the space provided above.
- When instructed to open the exam, please check that you have 7 pages in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each problem. If you do not show enough work, you will get only partial credit.
- You may use an ordinary scientific calculator, but calculators with graphical, or symbolic calculation functions are NOT allowed.
- This is a closed book examination.
- **Cheating is a serious offense. Students caught cheating will receive a zero score for the midterm exam, and will also be subjected to further penalties imposed by the University.**

Question No.	Points	Out of
Q. 1-10		40
Q. 11		10
Q. 12		15
Q. 13		15
Total Points		80

Part I: Answer the following multiple choice questions.

Put your MC question answers in the following boxes.

Question	1	2	3	4	5	Total
Answer	a	e	d	c	c	

Question	6	7	8	9	10	Total
Answer	d	d	c	b	c	

For each of the following MC questions, from the choices (a)(b)(c)(d) and (e), choose *exactly one* correct answer. Each MC question is worth 4 points. No partial credit.

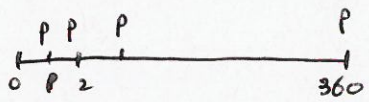
1. An online bank listed a 1-year CD that earns 1.25% compounded monthly. Find the APY as a percentage, rounded to three decimal places.

   (a) 1.257% (b) 1.246% (c) 1.345% (d) 1.250% (e) 1.251%

   $APY = \left(1+\frac{0.0125}{12}\right)^{12}-1$

2. Some friends tell you that they paid \$25,000 down on a new house and are to pay \$525 per month for 30 years. If interest is 7.9% compounded monthly, what was the selling price of the house?

   (a) \$92987.25 (b) \$85679.38 (c) \$92349.19 (d) \$85789.19 (e) \$97233.92

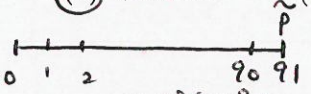
   

   $\frac{525\left[1-\left(1+\frac{0.079}{12}\right)^{-360}\right]}{\frac{0.079}{12}} + 2500$

3. At the time they retire, a couple has \$200,000 in an account that pays 8.4% compounded monthly. If the couple decides to withdraw \$3,000 a month as long as it takes, and then make a final withdraw which may be a smaller amount than \$3,000 to reduce the account balance to zero. What is the amount of the last withdraw?

   (a) \$300.56 (b) \$323.38 (c) \$311.27 (d) \$346.60 (e) \$350.87

   $200000 = 3000\,\frac{1-\left(1+\frac{0.084}{12}\right)^{-n}}{\frac{0.084}{12}}$ $\quad n = 90.12$

   $200000 = 3000\,\frac{1-\left(1+\frac{0.084}{12}\right)^{-90}}{0.084/12} + \frac{\tilde{P}}{(1+0.084/12)^{91}}$ $\quad \tilde{P} = ?$

4. A promissory note will pay \$27,000 at maturity 10 years from now. How much money should you be willing to pay now if money is worth 5.5% compounded monthly?

   (a) \$15748.23 (b) \$14568.48 (c) \$15597.23 (d) \$16625.46 (e) \$17729.95

   $P\left(1+\frac{0.055}{12}\right)^{120} = 27000 \qquad P = 15597.23.$

5. Which of the following matrices is in reduced form?

$$A = \begin{bmatrix} \underline{1} & 2 & 0 & 0 & 1 \\ 0 & \underline{1} & 0 & 0 & 1 \\ 0 & 0 & 0 & \underline{1} & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \underline{1} & 2 & 0 & 0 & 0 \\ 0 & 0 & \underline{1} & 0 & 0 \\ 0 & 0 & 0 & \underline{1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} \underline{1} & 0 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 \\ 0 & 0 & \underline{2} & 0 \end{bmatrix}.$$

(a) A, B and C are all in reduced form.

(b) Only A and B are in reduced form.

(c) Only B is in reduced form.

(d) Both B and C are in reduced form.

(e) None of them is in reduced form.

6. Which of the following matrix operations are well-defined? (Only one is correct!)

$$A = \begin{bmatrix} 1 & 4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 7 \end{bmatrix}, \quad D = \begin{bmatrix} -100 \\ 205 \\ 300 \end{bmatrix}.$$

1×3 $\quad 2 \times 3$ $\quad 3 \times 3$ $\quad 3 \times 1$

(a) $BA + C$.

(b) $C^2 - 3B$.

(c) $-2C + AD$.

(d) $ACD + 5$.

(e) $BDA + 10$.

7. Let $A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 0 & 1 & -2 \\ -3 & 0 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$. Find the entries in the third row of AB. $= \begin{bmatrix} c_{31} & c_{32} \end{bmatrix}$

(a) $[5, 1]$.

(b) $[7, 2]$

(c) $[15, 36]$

(d) $[1, -3]$

(e) $[2, 3]$

$$c_{31} = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix} = 1$$

$$c_{32} = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} -6 \\ 1 \\ 2 \end{bmatrix} = -3$$

8. For what value h the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & h & 0 \end{bmatrix}$ is not invertible?

(a) $h = 0$ (b) $h = 1$ (c) $h = 2$ (d) $h = 3$ (e) $h = 4$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & h & 0 \end{bmatrix} \xrightarrow{-2R1+R3 \to R3} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 2+h & -2 \end{bmatrix} \xrightarrow{\frac{R2}{2} \to R2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 2+h & -2 \end{bmatrix} \xrightarrow{-(2+h)R2+R3 \to R3}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & * \end{bmatrix}$$

$* = \frac{1}{2}(2+h) - 2 = 0$

$\Rightarrow 2+h = 4 \Rightarrow h = 2$

9. If the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ satisfies

$$A \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

what is a_{21}?

(a) 1 (b) 0.1 (c) -1 (d) -0.1 (e) 4

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}^{-1} = \frac{1}{3 \cdot 2 + 4} \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix}$$

$\uparrow$ a_{21}

10. The economy of a country is based on two sectors, agriculture and oil. Production of a dollar's worth of agriculture requires an input of \$0.40 from agriculture and \$0.35 from oil. Production of a dollar's worth of oil requires an input of \$0.20 from agriculture and \$0.05 from oil. The final demand is \$250 million for oil and \$40 million for agriculture. What is the output from agriculture?

(a) \$135 million
(b) \$459 million
(c) \$176 million
(d) \$460 million
(e) \$550 million

$$M = \begin{matrix} \\ A \\ O \end{matrix} \begin{matrix} \begin{matrix} A & O \end{matrix} \\ \begin{bmatrix} 0.4 & 0.2 \\ 0.35 & 0.05 \end{bmatrix} \end{matrix}$$

$X = (I - M)^{-1} D$

$$I - M = \begin{bmatrix} 0.6 & -0.2 \\ -0.35 & 0.95 \end{bmatrix}$$

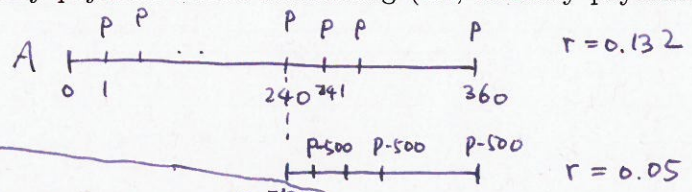
$$(I - M)^{-1} = \frac{1}{0.6 \times 0.95 - 0.35 \times 0.2} \begin{bmatrix} 0.95 & 0.2 \\ 0.35 & 0.6 \end{bmatrix}$$

$$D = \begin{matrix} \begin{bmatrix} 40 \\ 250 \end{bmatrix} \end{matrix} \begin{matrix} A \\ O \end{matrix}$$

Part II: Answer each of the following 3 long questions. Unless otherwise specified, numerical answers should be either exact or correct to 2 decimal places.

11. [*10 pts*] A person purchased a house 20 years ago by paying 20% down and signing a 30-year mortgage at 13.2% compounded monthly. Now interest rates have dropped and the owner is able to refinance the unpaid balance by signing a new 10-year mortgage at 5% compounded monthly, which reduces the original monthly payment by \$500.

(a) What was the original monthly payment before refinancing (i.e., monthly payment for the original 30-year mortgage)?



(3) $P \dfrac{1-\left(1+\frac{0.132}{12}\right)^{-120}}{\frac{0.132}{12}} = (P-500)\dfrac{1-\left(1+\frac{0.05}{12}\right)^{-120}}{\frac{0.05}{12}}$

(2)(1) $P = 1693.70$

(b) How much did the person pay for the house 20 years ago?

(2) $PV = 1693.70 \times \dfrac{1-\left(1+\frac{0.132}{12}\right)^{-360}}{\frac{0.132}{12}}$

$= 150973.37$

(2) The house is worth $\dfrac{150973.37}{0.8} = 188716.72$

(c) How much total interest will the person save by refinancing the mortgage?

(2) Original interest $= 1693.70 \times 360 - 150973.37 = A$

B = New interest $= 1693.70 \times 240 + (1693.70 - 500) \times 120 - 150973.37$

So interest saved $= A - B = 500 \times 120 = 60000$.

12. The sum of three numbers is 14. The largest is 4 times the smallest, while the sum of the largest and twice the smallest is the remaining number plus 8. Find the three numbers.

(1). Let x_1, x_2, x_3 be the largest, the middle, and the smallest numbers respectively. Write down the linear system of equations in terms of x_1, x_2, x_3.

$$\begin{cases} x_1 + x_2 + x_3 = 14 \\ x_1 - 4x_3 = 0 \\ x_1 - x_2 + 2x_3 = 8 \end{cases}$$

(2). Solve the linear system in (1) using the Gauss-Jordan elimination method. Show all steps.

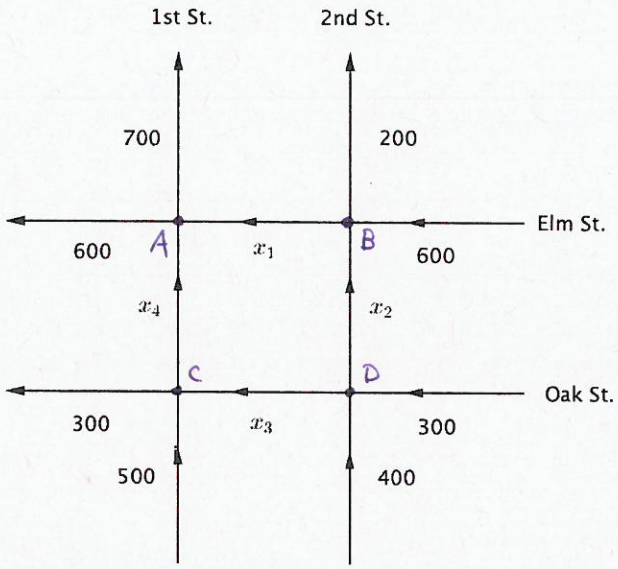
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 14 \\ 1 & 0 & -4 & 0 \\ 1 & -1 & 2 & 8 \end{array}\right] \xrightarrow[-R1+R3 \to R3]{-R1+R2 \to R2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 14 \\ 0 & -1 & -5 & -14 \\ 0 & -2 & 1 & -6 \end{array}\right] \xrightarrow{R2/(-1) \to R2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 14 \\ 0 & 1 & 5 & 14 \\ 0 & -2 & 1 & -6 \end{array}\right]$$

$$\xrightarrow{2R2+R3 \to R3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 14 \\ 0 & 1 & 5 & 14 \\ 0 & 0 & 11 & 22 \end{array}\right] \xrightarrow{R3/11 \to R3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 14 \\ 0 & 1 & 5 & 14 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow[-R3+R1 \to R1]{-5R3+R2 \to R2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 12 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{-R2+R1 \to R1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{cases} x_1 = 8 \\ x_2 = 4 \\ x_3 = 2 \end{cases}$$

13. The rush-hour traffic flow (in vehicles per hour) for a network of four one-way streets is shown in the figure.



(1). Write down the system of equations determined by the flow of traffic through the four intersections.

A: $x_1 + x_4 = 600 + 700 = 1300$ $\quad x_1 + x_4 = 1300$

B: $600 + x_2 = 200 + x_1$ $\quad x_1 - x_2 = 400$

C: $x_3 + 500 = 300 + x_4$ $\quad x_3 - x_4 = -200$

D: $x_2 + x_3 = 300 + 400$ $\quad x_2 + x_3 = 700$

(2). Find the solution of the system in part (1). (You may write on the back.)

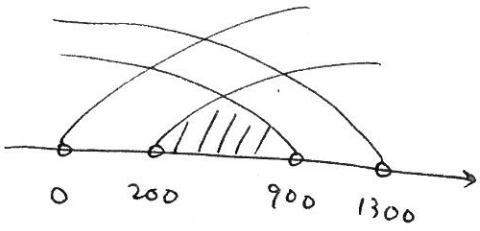
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1300 \\ 1 & -1 & 0 & 0 & 400 \\ 0 & 0 & 1 & -1 & -200 \\ 0 & 1 & 1 & 0 & 700 \end{array}\right] \xrightarrow{-R1+R2 \to R2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1300 \\ 0 & -1 & 0 & -1 & -900 \\ 0 & 0 & 1 & -1 & -200 \\ 0 & 1 & 1 & 0 & 700 \end{array}\right]$$

$$\xrightarrow{-R2 \to R2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1300 \\ 0 & 1 & 0 & 1 & 900 \\ 0 & 0 & 1 & -1 & -200 \\ 0 & 1 & 1 & 0 & 700 \end{array}\right] \xrightarrow{-R2+R4 \to R4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1300 \\ 0 & 1 & 0 & 1 & 900 \\ 0 & 0 & 1 & -1 & -200 \\ 0 & 0 & 1 & -1 & -206 \end{array}\right]$$

$$\xrightarrow{-R3+R4 \to R4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1300 \\ 0 & 1 & 0 & 1 & 900 \\ 0 & 0 & 1 & -1 & -200 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{cases} x_1 + x_4 = 1300 \\ x_2 + x_4 = 900 \\ x_3 - x_4 = -200 \end{cases} \qquad \begin{cases} x_1 = -t + 1300 \geq 0 \\ x_2 = -t + 900 \geq 0 \\ x_3 = t - 200 \geq 0 \\ x_4 = t \quad \geq 0 \end{cases}$$

(3). What is the maximum number of vehicles per hour that can travel from Oak Street to Elm Street on 1st Street? What is the minimum number?

Continue : so $\begin{cases} t \leq 1300 \\ t \leq 900 \\ t \geq 200 \\ t \geq 0 \end{cases}$



we have $200 \leq t \leq 900$

(3) : The question is $x_4 = t$, the max is 900, the min is 200.

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