Solution

HKUST

MATH1003 Calculus and Linear Algebra

Midterm 2 (Version A)

12th November 2016

10:30-12:00

Name:____________________

Student ID:____________________

Lecture Section: ____________________

Directions:

- Do NOT open the exam until instructed to do so.
- Please turn off all phones and pagers, and remove headphones.
- Please write your name, ID number, and Tutorial Section in the space provided above.
- When instructed to open the exam, please check that you have 7 pages in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each problem. If you do not show enough work, you will get only partial credit.
- Any forms of calculators are NOT allowed.
- This is a closed book examination.
- **Cheating is a serious offense. Students caught cheating will receive a zero score for the midterm exam, and will also be subjected to further penalties imposed by the University.**

Question No.	Points	Out of
Q. 1-8		40
Q. 9		15
Q. 10		16
Q. 11		9
Total Points		80

version A.C

Part I: Answer the following multiple choice questions.

Put your MC question answers in the following boxes.

Question	1	2	3	4	5	6	7	8	Total
Answer	C	A	E	C	B	C	D	A	40

Each of the following MC questions is worth 5 points. No partial credit.

1. Which function(s) below is not continuous at the point $x = 1$?

   (a) $f(x) = \frac{1}{x+1}$.

   (b) $f(x) = x - 1$

   (c) $f(x) = \begin{cases} x+1, & \text{if } x < 1; \\ x-1, & \text{if } x \geq 1 \end{cases}$

   (d) $f(x) = e^x - 1$

   (e) $f(x) = \ln(x+1)$

2. What is $f'(0)$ for $f(x) = e^{x^2} + x^e$?

   (a) 0. (b) 1. (c) $1+e$. (d) $2e$. (e) None of the above

3. Which of the following statements are true? Given $f(x) = x^3 - 12x + 20$,

   (a) $f(x)$ is increasing in the interval $(0, 2)$.

   (b) $f(x)$ is decreasing in the interval $(-\infty, 2)$.

   (c) $f(x)$ is increasing in the interval $(-2, 2)$.

   (d) $f(x)$ is decreasing in the interval $(-2, +\infty)$.

   (e) $f(x)$ is decreasing in the interval $(-1, 1)$.

4. Which of the following points are the inflection points of the graph of the function

$$f(x) = 4x^3 - 6x^2 - 24x?$$

(a) 0. (b) -1. (c) $\frac{1}{2}$. (d) 2. (e) None of the above.

5. Which of the following numbers are the slopes of the tangent lines to the graph of

$$y - xy^2 + x^2 + 1 = 0$$

at the points where $x = 1$?

(a) 1. (b) $-\frac{1}{3}$. (c) $\frac{1}{3}$. (d) $\frac{2}{5}$. (e) -1.

6. A point is moving on the graph $xy = 1$. When the point is at $(1, 1)$, its x coordinate is decreasing at 4 units per second. How fast is the y coordinate changing at that moment?

(a) 1. (b) -1. (c) 4. (d) -4. (e) None of the above

7. Which following answer is correct for $\lim_{x \to 1^-} \dfrac{x-2}{x-1}$?

(a) 0. (b) 2. (c) $-\infty$. (d) $+\infty$. (e) None of above.

8. Which value is the absolute maximum of $f(x) = x^3 + 3x^2 - 9x - 7$ over the interval $[-2, 2]$?

(a) 15. (b) -12. (c) -5. (d) 20. (e) None of the above.

Part II: Answer each of the following 3 long questions. Unless otherwise specified, numerical answers should be either exact or correct to 2 decimal places.

9. Calculate the indicated derivatives (three sub-problems)

(1). Find $f'(x)$ where $f(x) = e^{\sqrt{x}} + \sqrt{\ln x}$.

$$f'(x) = (e^{\sqrt{x}} + \sqrt{\ln x})' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$$

(2). Find $\frac{dy}{dx}$ where y is an implicit function of x given by $xe^y - y = x^2 - 1$.

$$(xe^y - y)' = (x^2-1)'$$

$$1 \cdot e^y + x \cdot e^y \cdot y' - y' = 2x$$

$$(xe^y - 1)y' = 2x - e^y$$

$$y' = \frac{2x - e^y}{xe^y - 1}$$

(3). Find the second derivative $f''(x)$ where $f(x) = \ln(x^2 + 1)$

$$f'(x) = \frac{2x}{x^2+1}$$

$$f''(x) = \left(\frac{2x}{x^2+1}\right)' = \frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$$

$$= \frac{2 - 2x^2}{(x^2+1)^2}$$

10. Consider the function $f(x) = x^4 - 2x^3 + 1$ (four sub-questions).

+4 (1). Find critical numbers of f, the intervals on which f is increasing, and those on which f is decreasing.

$f'(x) = 4x^3 - 6x^2$ $f'(x) = 0$. $x = 0$ & $x = 3/2$.

$= 2x^2(2x-3)$

Critical numbers: $x = 0$. $x = \frac{3}{2}$ +2

f' <0 $f'<0$ $f'>0$

$f'(-1)<0$ 0 $f(1)<0$. $3/2$ $f(2)>0$

f is decreasing over $(-\infty, 3/2)$ +1

f is increasing over $(\frac{3}{2}, +\infty)$ +1

(2). Find the local extrema points and absolute extrema points if there is any.

+4

~~$f''=(4\times3)x^2+(12)x=(12)x(x+1)$.~~

Since f is decreasing over $(-\infty, 3/2)$

and is increasing over $(3/2, +\infty)$,

$x = 3/2$ is a local minimum point. +2

Since $x = 3/2$ is the only critical point,

it is also the absolute minimum. +2

(3). Find the inflection points of f if there is any, the intervals on which f is concave upward, and those on which f is concave downward.

+4

$f''(x) = 12x^2 - 12x = 12x(x-1)$.

$f''(x) = 0$, $x = 0$ or $x = 1$

sign chart: $f'' > 0$ | $f'' < 0$ | $f'' > 0$

$f''(-1) > 0$ 0 $f''(\frac{1}{2}) < 0$ 1 $f''(2) > 0$

Inflection points: $x = 0$ and $x = 1$. +2

$(-\infty, 0)$: concave up

$(0, 1)$: concave down +1

$(1, +\infty)$: concave up

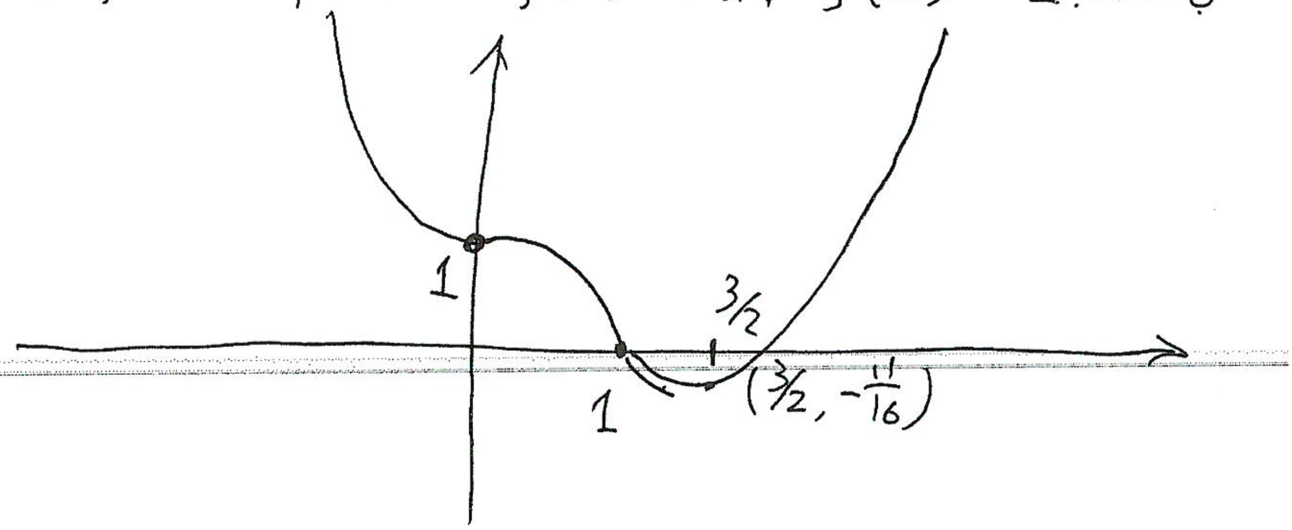
(4). Sketch the graph of f.

+4

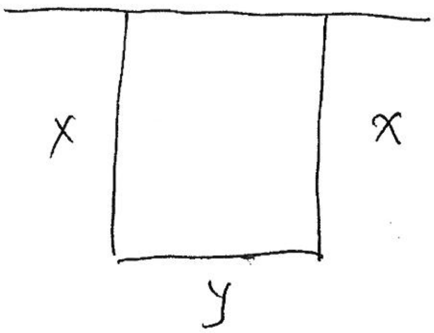
critical points: $f(\frac{3}{2}) = (\frac{3}{2})^4 - 2 \times (\frac{3}{2})^3 + 1$

$= \frac{81}{16} - \frac{2 \times 27}{8} + 1 = \frac{81 - 108 + 16}{16} = -\frac{11}{16}$ $= -0.688$

inflection points: $f(0) = 1$, $f(1) = 1 - 2 + 1 = 0$



11. A homeowner has \$80 to spend on building a fence around a rectangular garden. Three sides of the fence will be constructed with wire fencing at a cost of \$4 per linear foot. The fourth side was already constructed by the previous owner. Thus there will be no cost for the fourth side. Find the dimensions and the area of the largest garden that can be enclosed with \$80 woth of fencing.



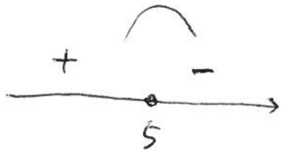
$$80 = 4x + 4x + 4y$$
$$= 8x + 4y$$
$$0 \leq x \leq 10$$
maximize: xy

$$20 = 2x + y$$
$$y = 20 - 2x$$
$$A(x) = xy = x(20 - 2x) = 20x - 2x^2$$
$$A'(x) = 20 - 4x. \quad A'(x) = 0. \quad x = \frac{20}{4} = 5.$$
$$A(5) = 5 \times (20 - 2 \times 5) = 50.$$

If $x = 0$, $A(0)$; if $x = 10$, $A(10) = 10$

Thus the maximum area is 50 when $x = 5$ and $y = 10$.

or 1st derivative test: $A'(x) = 4(5 - x)$



2nd derivative test: $A''(x) = -4 < 0$

so $x = 5$ is the local maximum, and hence absolute maximum

Scratch paper

*** END OF PAPER ***