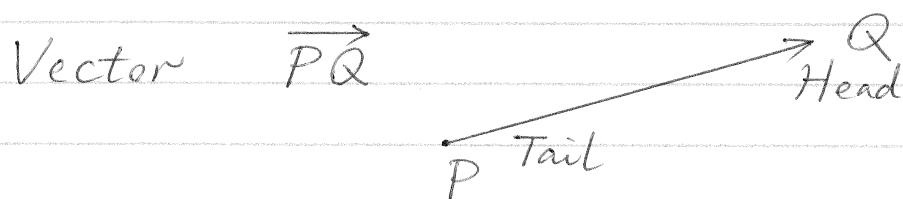


Vectors and Vector-Valued Functions

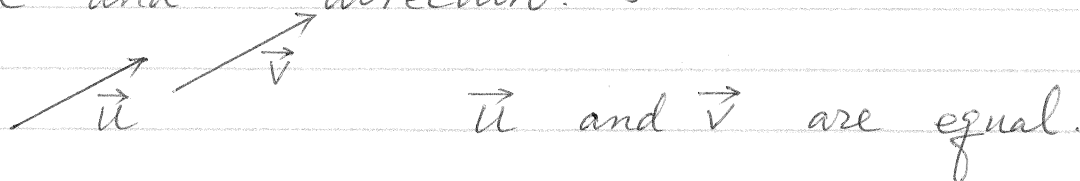
I Vectors in the plane

Basic vector operations

Vectors are quantities that have both length and direction.



Two vectors are equal if they have the same magnitude and direction.



Scalars : Quantities having magnitude but no direction.

Zero vector $\vec{0}$: Vector with zero length and no direction.

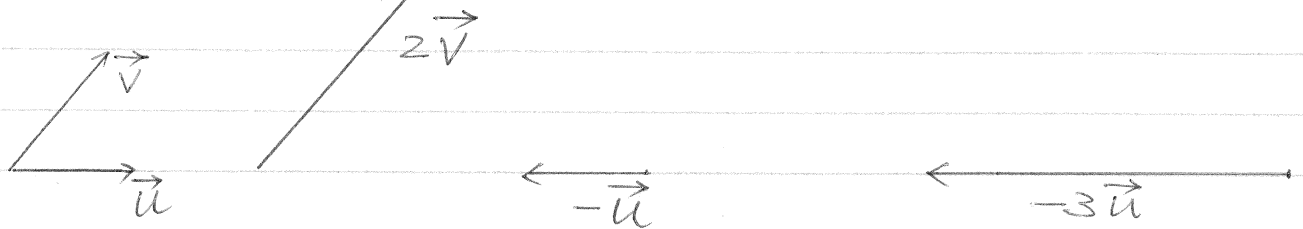
▲ For a vector, if its length becomes zero, then direction becomes meaningless.

Scalar multiplication

Given a scalar c and a vector $\vec{v}$, the scalar multiple $c\vec{v}$ is a vector whose length is the length of $\vec{v}$ multiplied by $|c|$.

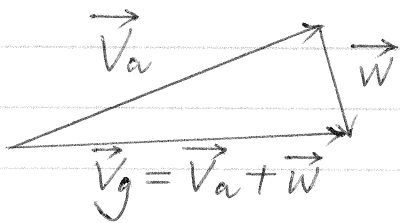
As to the direction, if $c > 0$, then $c\vec{v}$ has the same direction as $\vec{v}$. If $c < 0$, then $c\vec{v}$ and $\vec{v}$ point in opposite directions.

Two vectors are parallel if they are scalar multiples of each other.



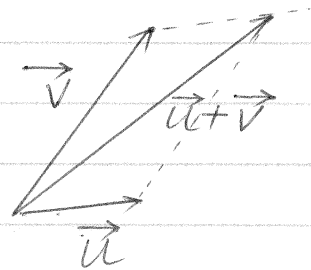
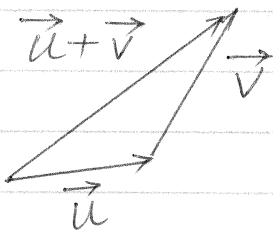
Vector addition and subtraction

The physical idea and picture

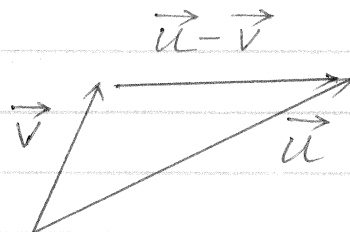
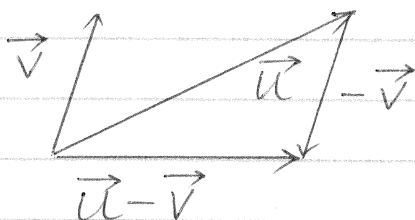


$\vec{v}_a$: Plane velocity relative to air
 $\vec{w}$: wind velocity
 $\vec{v}_a + \vec{w} = \vec{v}_g$: Plane velocity relative to the ground.
 Parallelogram Rule

Triangle rule



The difference $\vec{u} - \vec{v}$ is defined by $\vec{u} + (-1)\vec{v}$.

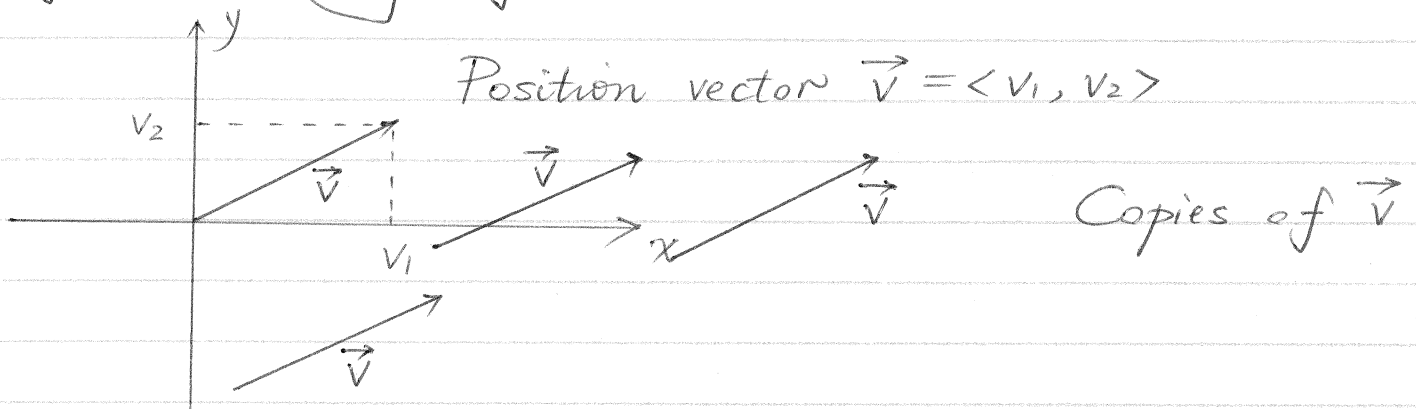


Vector components

Position vector : a vector $\vec{v}$ with its tail at origin and its head at the point (v_1, v_2) .
 $\vec{v}$ is written as $\langle v_1, v_2 \rangle$

v_1 and v_2 are the x - and y components of $\vec{v}$.

$\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ are equal if and only if $u_1 = v_1$ and $u_2 = v_2$.



Magnitude

$P(x_1, y_1)$ and $Q(x_2, y_2)$ are given.
 $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$

Length of $\vec{PQ}$, denoted by $|\vec{PQ}|$, is the distance between P and Q .

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\vec{v} = \langle v_1, v_2 \rangle, \quad |\vec{v}| = \sqrt{v_1^2 + v_2^2}$$

Vector operations

$$\vec{u} = \langle u_1, u_2 \rangle \quad \text{and} \quad \vec{v} = \langle v_1, v_2 \rangle$$

c is a scalar

$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

$$\vec{u} - \vec{v} = \langle u_1 - v_1, u_2 - v_2 \rangle$$

$$c\vec{u} = \langle cu_1, cu_2 \rangle$$

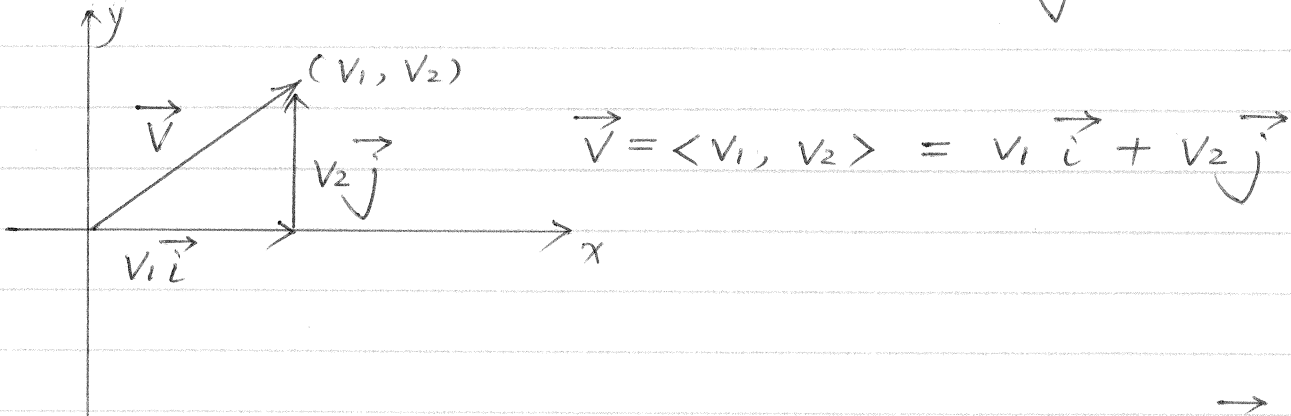
Unit vectors

A unit vector is a vector with length 1.

$$\vec{i} = \langle 1, 0 \rangle, \quad \vec{j} = \langle 0, 1 \rangle$$

$$\begin{aligned} \vec{u} = \langle u_1, u_2 \rangle &= u_1 \langle 1, 0 \rangle + u_2 \langle 0, 1 \rangle \\ &= u_1 \vec{i} + u_2 \vec{j} \end{aligned}$$

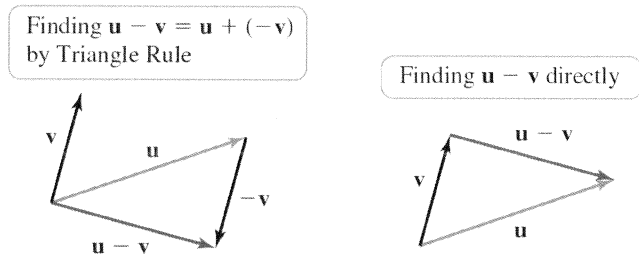
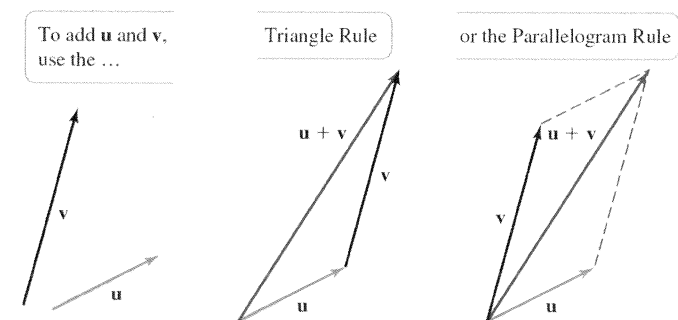
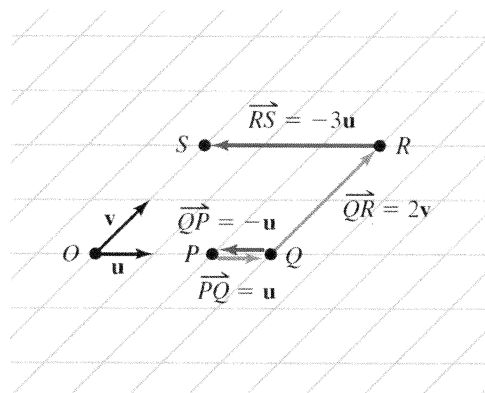
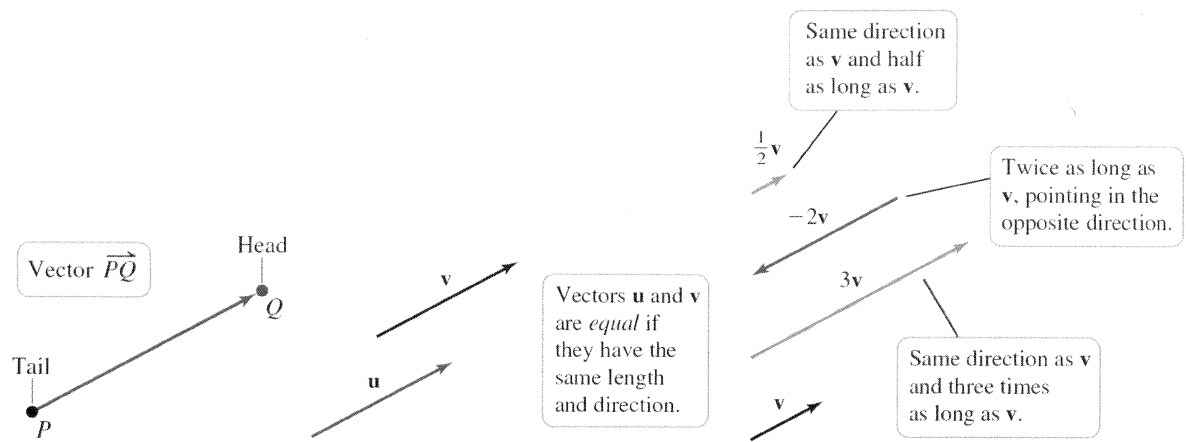
coordinate unit vectors $\vec{i}$ and $\vec{j}$.

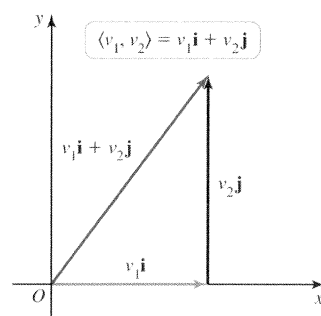
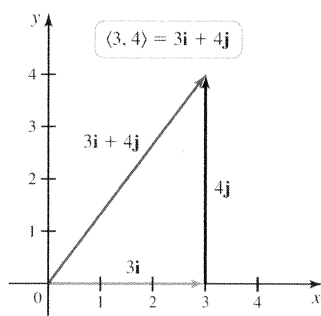
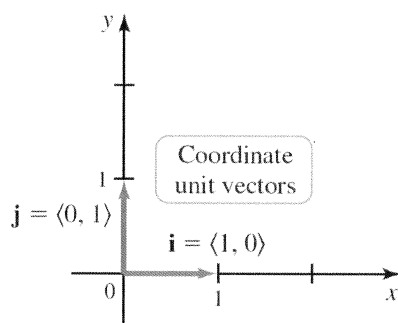
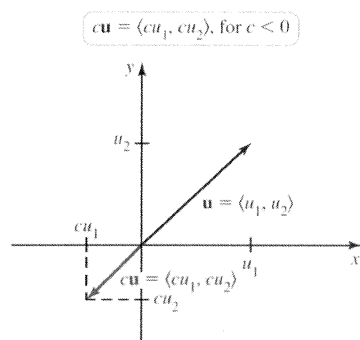
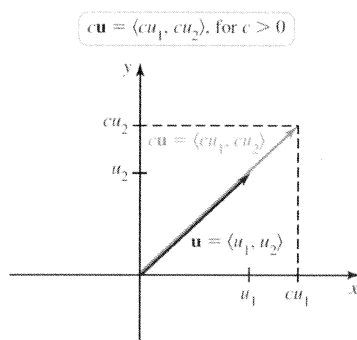
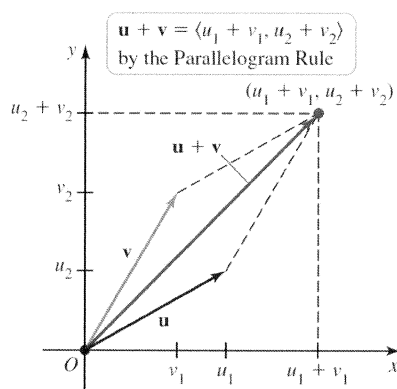
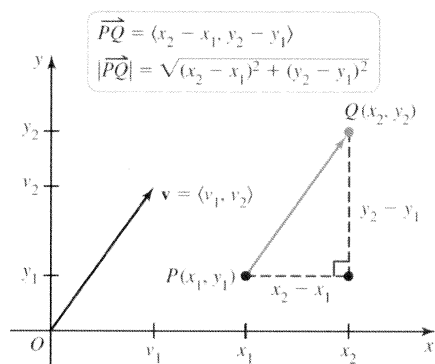


Given a Nonzero vector $\vec{v}$, $\pm \frac{\vec{v}}{|\vec{v}|}$ are unit vectors parallel to $\vec{v}$.

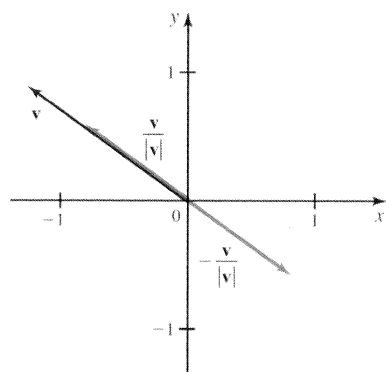
$$\left| \pm \frac{\vec{v}}{|\vec{v}|} \right| = \frac{1}{|\vec{v}|} |\vec{v}| = 1$$

12.1



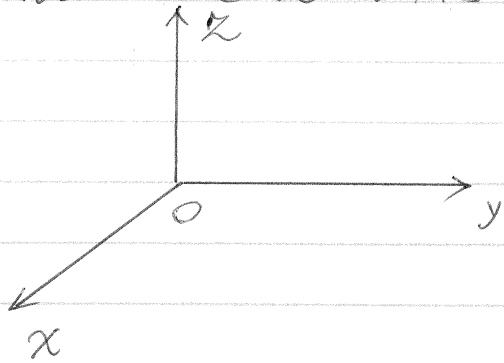


$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$ and $-\mathbf{u} = -\frac{\mathbf{v}}{|\mathbf{v}|}$ have length 1.



2 Vectors in three dimensions

Add the z axis : Right-handed system



xy plane, yz plane, and xz plane.

These three planes divide the space into eight regions called octants.

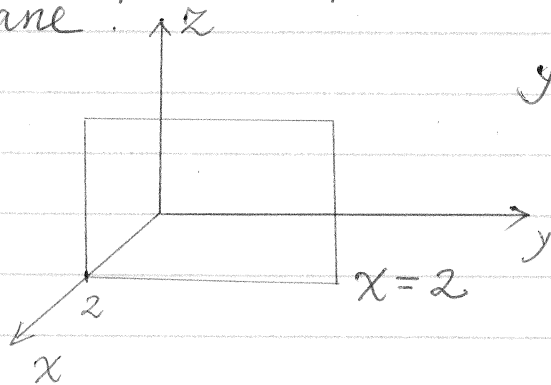
From 4 quadrants in 2D to 8 octants in 3D.

Equations of simple planes

$$x = a, \quad y = b, \quad z = c.$$

Each defines a plane

$x=2$ is a plane parallel to and 2 units from the yz plane.



yz plane : $x=0$

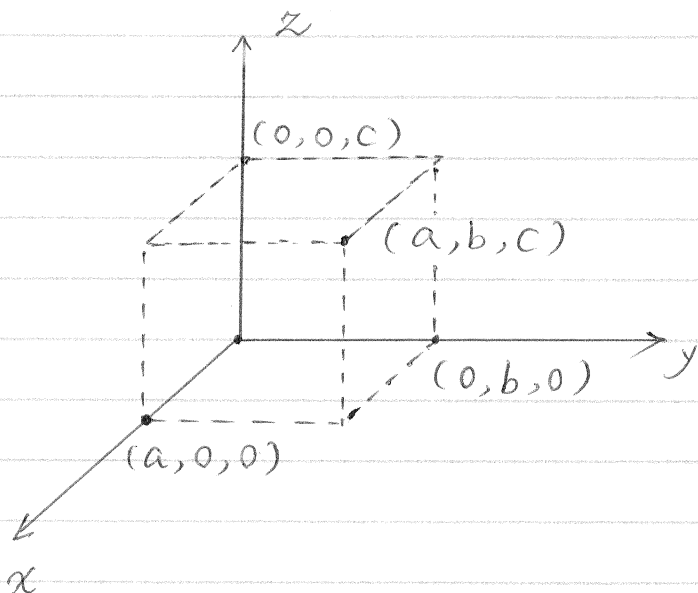
Distances in xyz system

Distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

Figure for a point of coordinates (a, b, c)

Figure for the computation of distance
(By constructing two triangles)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



A rectangular box with one vertex at the origin and the opposite vertex at the point (a, b, c) .

Equation of a sphere

A sphere centered at (a, b, c) with radius r is the set of points satisfying

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

Distance between (a, b, c) and $(x, y, z) = r$.

Vectors in $\mathbb{R}^3$

The position vector $\langle v_1, v_2, v_3 \rangle$ has its tail at the origin and its head at the point (v_1, v_2, v_3)

Vectors having the same magnitude and direction ^{are} equal.

$P(x_1, y_1, z_1)$

$Q(x_2, y_2, z_2)$

vector $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

Vector operations

Let c be a scalar, $\vec{u} = \langle u_1, u_2, u_3 \rangle$,
and $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$$

$$\vec{u} - \vec{v} = \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle$$

$$c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$$

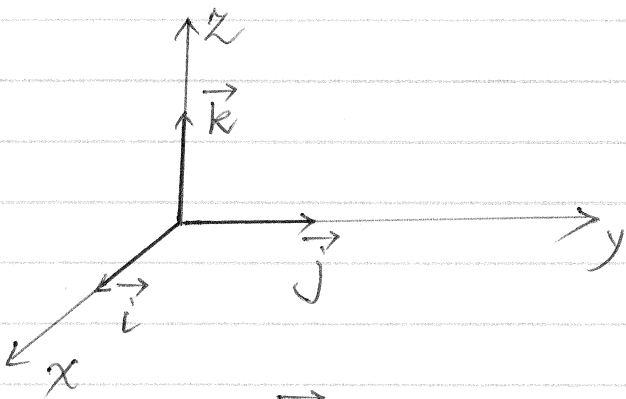
$$\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Coordinate unit vectors $\vec{i} = \langle 1, 0, 0 \rangle$

$$\vec{j} = \langle 0, 1, 0 \rangle$$

$$\vec{k} = \langle 0, 0, 1 \rangle$$



$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$

$$= v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}$$

12.2

