

13.2 Graphs and level curves

Functions of two variables

$$z = f(x, y) \quad \text{or} \quad F(x, y, z) = 0$$

x and y : two independent variables.

Function, domain and range.

$z = f(x, y)$ is a function.

Each point (x, y) in the domain $D \xrightarrow{z = f(x, y)}$

A unique number z

z varies in the range of f as (x, y) vary in the domain.

See figure.

Ex. Find the domain and range of $z = g(x, y) = \sqrt{4 - x^2 - y^2}$.

Domain: $\{(x, y): x^2 + y^2 \leq 4\}$

such that z is real

Range: $\{z: 0 \leq z \leq 2\}$

Graphs of $z = f(x, y)$

The point $(x, y, f(x, y))$ lies on the graph of $z = f(x, y)$.

Every line parallel to z axis intersects the graph of $z = f(x, y)$ at most once.

(x, y) in $D \xrightarrow{f}$ unique z

See figure

Graphing

a. $f(x, y) = 2x + 3y - 12$

b. $g(x, y) = x^2 + y^2$

c. $h(x, y) = \sqrt{1 + x^2 + y^2}$

a: A plane, b: Paraboloid, c: hyperboloid.
See figures.

Level curves

Contour curve: Intersection of $z = f(x, y)$
and $z = z_0$.

Projected onto xy plane: $f(x, y) = z_0$,
Level curve.

See figure.

Level curves of

a) $f(x, y) = y - x^2 - 1$

$$z_0 = y - x^2 - 1, \quad y = x^2 + 1 + z_0$$

$$z_0 = e^{x^2 - y^2}, \quad \text{b) } f(x, y) = e^{x^2 - y^2}$$

$$x^2 + y^2 = -\ln z_0 = \ln \frac{1}{z_0}$$

See figures.

Functions of more than two variables ($n > 2$)

$y = f(x_1, x_2, \dots, x_n)$
 $(x_1, x_2, \dots, x_n)$ in the Domain $\xrightarrow{f}$ a unique y in the range

Ex. $g(x, y, z) = \sqrt{16 - x^2 - y^2 - z^2}$

Domain defined by $x^2 + y^2 + z^2 \leq 16$, a ball.

Range: $[0, 4]$

Graphs of functions of more than two variables.

$$W = f(x, y, z)$$

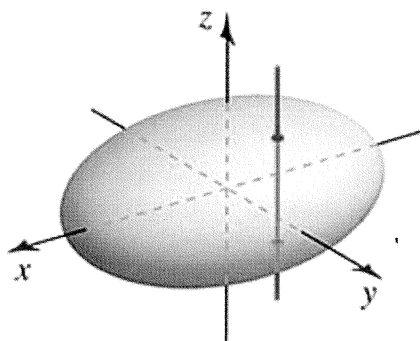
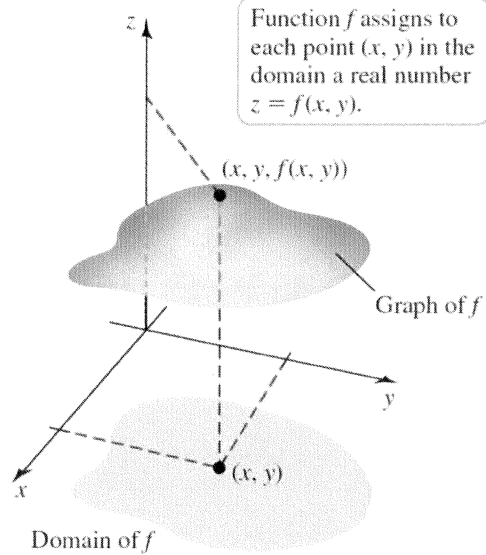
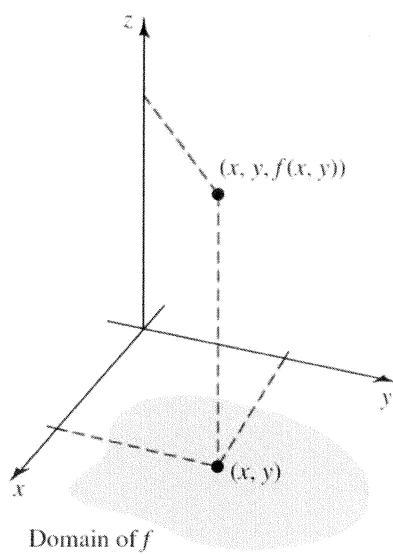
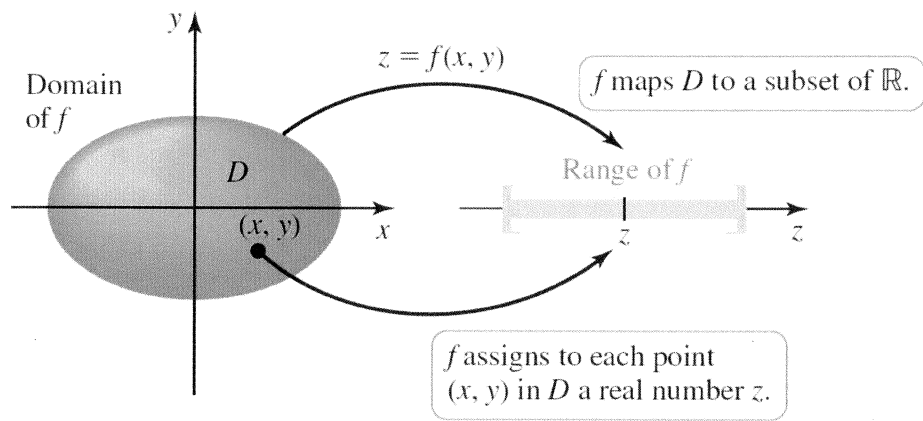
Level surfaces defined by $w_0 = f(x, y, z)$

$$W = f(x, y, z) = \sqrt{z - x^2 - 2y^2}$$

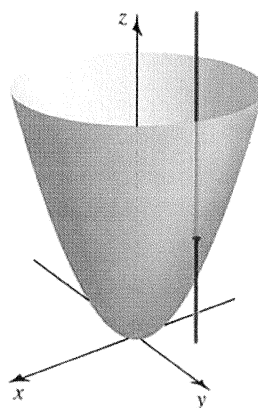
$$w_0 = \sqrt{z - x^2 - 2y^2}, \quad z = x^2 + 2y^2 + w_0^2$$

Each level surface is an elliptic paraboloid.

See figure.



An ellipsoid does not pass the vertical line test: not the graph of a function.



This elliptic paraboloid passes the vertical line test: graph of a function.

