MATH333: Tutorial 2 (18-2-00)

1 MATLAB

Similar to FORTRAN, I have put one set of note, Element Of MATLAB, in my web. You can download it and have a look. Besides this note, there are several ways to learn MATLAB commands:

1. *demo* :
2. *help function* :
3. *helpdesk* :

Several examples are put in my webpage:

$$http://home.ust.hk/\sim matim/tutorial2/matlab/$$

2 ODE

To solve the ODE problem

$$\frac{dy}{dx} = f(x,y)\,,$$

with the initial condition $y(x_0) = y_0$, you have learn the Euler method from the course MATH 231 last semester:

2.1 Euler Method

The solution can be found by

$$y_{n+1} = y_n + hf(x_n, y_n) \text{ for } n = 0, 1, 2, \cdot\ .$$

2.2 RK4

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\,,$$

where

$$k_1 = hf(x_n, y_n)\,,\ k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)\,,$$
$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)\,,\ k_4 = hf(x_n + h, y_n + k_3)\,.$$

2.3 Generalize RK4

In order to solve a system of first order ODE,

$$\frac{dx}{dt} = f(t, x, y)\,,\ \frac{dy}{dt} = g(t, x, y)\,,$$

we can generalize the previous idea to:

$$x_{n+1} = x_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\,, y_{n+1} = y_n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)\,,$$

where

$$\begin{aligned}
k_1 = dtf(t_n, x_n, y_n) \quad &, \quad l_1 = dtg(t_n, x_n, y_n)\,,\\
k_2 = dtf(t_n + \frac{dt}{2}, x_n + \frac{k_1}{2}, y_n + \frac{l_1}{2}) \quad &, \quad l_2 = dtg(t_n + \frac{dt}{2}, x_n + \frac{k_1}{2}, y_n + \frac{l_1}{2})\,,\\
k_3 = dtf(t_n + \frac{dt}{2}, x_n + \frac{k_2}{2}, y_n + \frac{l_2}{2}) \quad &, \quad l_3 = dtg(t_n + \frac{dt}{2}, x_n + \frac{k_2}{2}, y_n + \frac{l_2}{2})\,,\\
k_4 = dtf(t_n, x_n + k_3, y_n + l_3) \quad &, \quad l_4 = dtg(t_n, x_n + k_3, y_n + l_3)\,.
\end{aligned}$$

3 Exercise

Please solve this problem by writing a MATLAB code.

The following differential equation describe the motion of a spaceship in orbit about the earth and the moon.

$$\frac{d^2x}{dt^2} = 2\frac{dy}{dt} + x - \frac{\mu_1(x+\mu_0)}{r_1^3} - \frac{\mu_0(x+\mu_1)}{r_2^3} - f\frac{dx}{dt}\,,$$

$$\frac{d^2y}{dt^2} = -2\frac{dx}{dt} + y - \frac{\mu_1 y}{r_1^3} - \frac{\mu_0 y}{r_2^3} - f\frac{dy}{dt}\,,$$

with

$$\mu_0 = \frac{1}{82.45}\,,\ \mu_1 = 1 - \mu_0\,,\ r_1^2 = (x+\mu_0)^2 + y^2\,,\ r_2^2 = (x-\mu_1)^2 + y^2\,.$$

1. By plotting the orbit $y(t)$ vs. $x(t)$ with the initial conditions

   $$x(0) = 1.2\,,\ \frac{dx(0)}{dt} = 0\,,\ y(0) = 0\,,\ \frac{dy(0)}{dt} = -1.04935751\,,$$

   show that when $f = 0$ the orbit is periodic with period $T = 6.19216933$.

2. With the same initial conditions as (1) but with $f = 1$, integrate the differential equations from $0 \le t \le 5$. Plot the orbit to show that the spaceship eventually crashes.

3. Repeat the computation of (2) with $f = 0.1$ and plot it's path.