MATH333: Tutorial 3 (25-2-00)

1 MATLAB

Using MATLAB to plot a curve, $y = f(x)$, a surface, $z = f(x, y)$, and the contour plot of the surfce.

2 Numerical Recipe

This provides a lot of useful subroutine for different numerical methods:

$$http://www.nr.com$$

This is an online book for explaining how they work and how to use these subroutines. To include a subroutine from Numerical Recipe in FORTRAN, say, we execute

$$f77 \; -o program \; program.f \; \$ust\_NRF$$

Note that, we have to goto $user - setup$ to set an enviornment such that we can call numerical recipe first.

Examples:

1. root-finding methods,
2. integrations,
3. RK4 to solve
$$y'' + \sin(x) = 0\,, y(0) = 0\,, y'(0) = 1\,.$$

3 Exercise

1. Download a FORTRAN program call $bugs.f$ from:

$$http://home.ust.hk/\sim matim/tutorial3/bugs.f$$

The program is pretended to calculate

$$\sum_{i=1}^{n} \frac{i^5}{(i+1)^2}$$

for n=10. The answer should be 2389.54. Please remove all those bugs and obtain a correct result.

2. Please solve this problem by writing a FORTRAN code.

   Consider the Fresnel integrals, defined by

$$C(\omega) = \int_0^\omega \cos(\frac{1}{2}\pi x^2)dx \, , \; S(\omega) = \int_0^\omega \sin(\frac{1}{2}\pi x^2)dx \, .$$

   By using the subroutine *qromb*, provided by the numerical recipe in FORTRAN (section 4.3, p.134), plot a smooth curve of $S(\omega)$ vs. $C(\omega)$ for the range $0 \leq \omega \leq 5$. The resulting curve is called a Cornu spiral.

3. Please solve this problem by writing a FORTRAN code.

   The motion of a damped pendulum with external periodic driving force can be modelled by the differential equation

$$\frac{d^2\theta}{dt^2} + \frac{1}{q}\frac{d\theta}{dt} + \sin\theta = g\cos\omega t \, ,$$

   where q is a damping parameter, g is the forcing amplitude and ω is tha driving frequency.

   (a) By calling Numerical Recipe, solve the above ODE by choosing suitable value of the parameters to obtain a graph describing the following situations:

      i. An unforced and undamped pendulum;
      ii. An unforced and underdamped pendulum;
      iii. An unforced and overdamped pendulum;
      iv. An unforced and critically damped pendulum;
      v. A forced and undamped pendulum at resonance, and
      vi. A damped-driven "chaotic" pendulum.

   (b) (Bonus) Compare the above solutions by writing your own RK4 subroutine.

   (c) (Bonus) Compare the above solutions by using MATLAB to solve the ODE's.