KMV Model

Expected default frequency

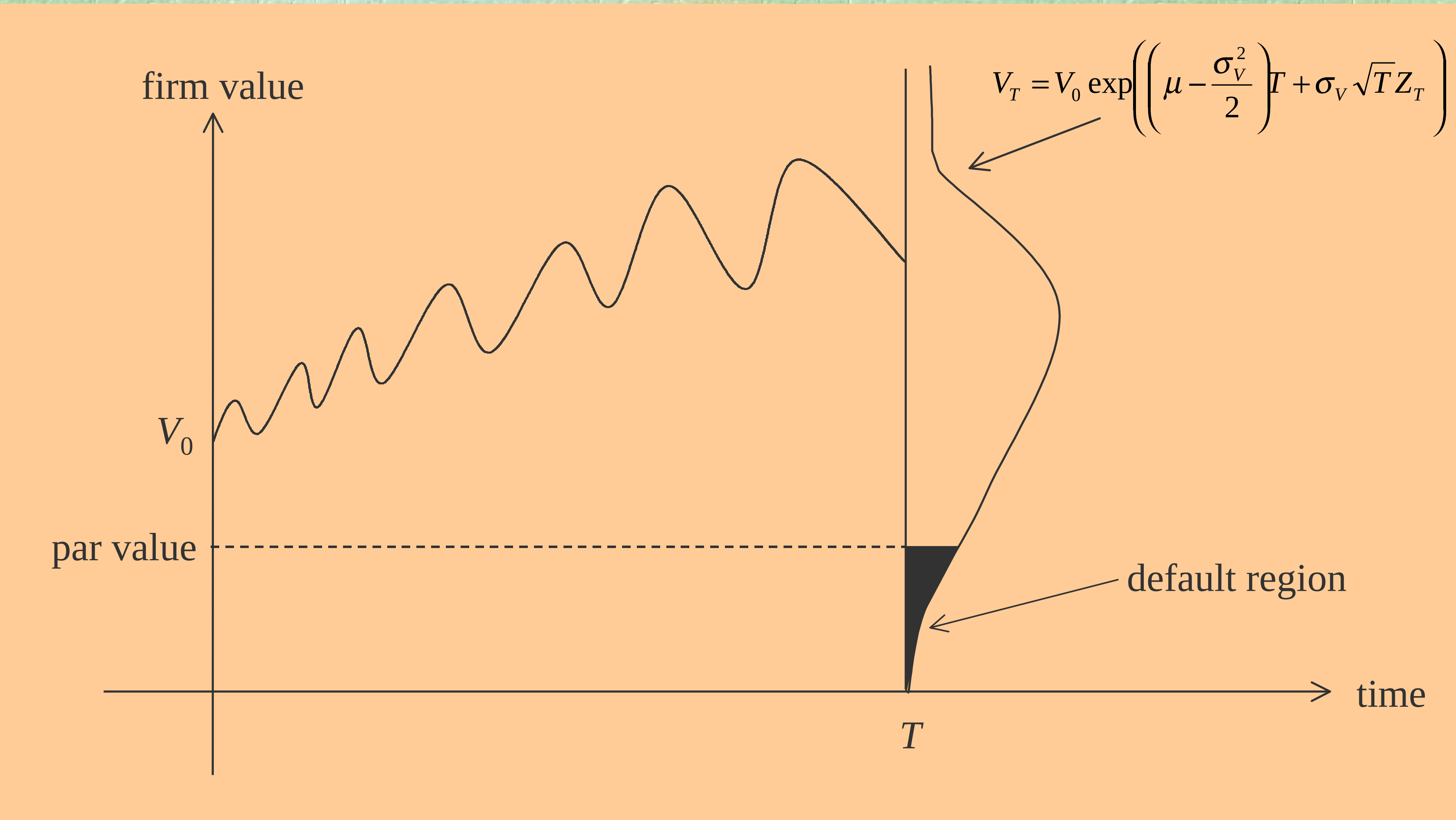
- *Expected default frequency* (EDF) is a forward-looking measure of actual probability of default. EDF is firm specific.

- KMV model is based on the *structural approach* to calculate EDF (credit risk is driven by the firm value process).

  – It is best when applied to publicly traded companies, where the value of equity is determined by the stock market.

  – The market information contained in the *firm's stock price* and *balance sheet* are translated into an *implied risk of default*.

Distribution of terminal firm value at maturity of debt



- According to KMV's empirical studies, log-asset returns confirm quite well to a normal distribution, and σ_V stays relatively constant.

Three steps to derive the actual probabilities of default:

1. Estimation of the market value and volatility of the firm’s asset.

2. Calculation of the distance to default, an index measure of default risk.

3. Scaling of the distance to default to actual probabilities of default using a default database.

Estimation of firm value V and volatility of firm value σ_V

- Usually, only the price of equity for most public firms is directly observable, and in some cases, part of the debt is directly traded.

- Using option pricing approach:

  equity value, $E = f(V, \sigma_V, K, c, r)$

  and

  volatility of equity, $\sigma_E = g(V, \sigma_V, K, c, r)$

  where K denotes the leverage ratio in the capital structure, c is the average coupon paid on the long-term debt, r is the riskfree rate.

- Solve for V and σ_E from the above 2 equations.

Distance to default

default point, $d^* =$ short-term debt + $\frac{1}{2} \times$ long-term debt

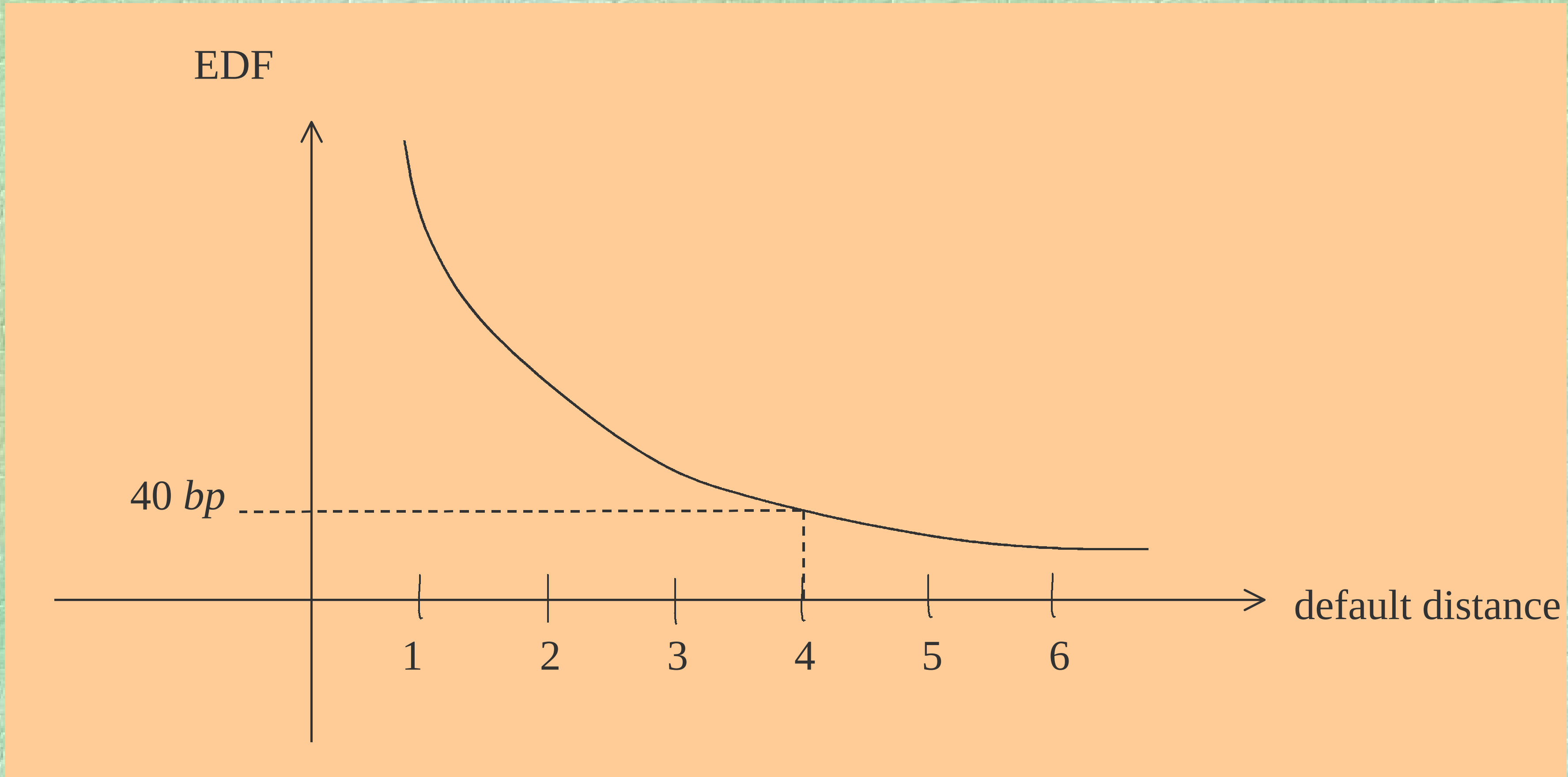
distance to default,

$$d_f = \frac{E(V_T) - d^*}{\sigma_V}$$

$$= \frac{\ln \frac{V_0}{d^*} + \left(\mu - \frac{\hat{\sigma}_V^2}{2}\right)T}{\hat{\sigma}_V \sqrt{T}},$$

where V_0 is the current market value of firm, μ is the expected net return on firm value and $\hat{\sigma}_V$ is the annualized firm value volatility.

Probabilities of default from the default distance



Based on historical information on a large sample of firms, for each time horizon, one can estimate the proportion of firms of a given default distance (say, $d_f = 4.0$) which actually defaulted after one year.

Example

Current market value of assets	$V_0 = 1{,}000$
Net expected growth of assets per annum*	$\mu = 20\%$
Expected asset value in one year	$V_T = 1{,}200$
Annualized asset volatility	$\sigma_V = 100$
Default point	$d^* = 800$

default distance, $$d_f = \frac{1{,}200 - 800}{100} = 4.$$

Among the population of all the firms with $d_f = 4$ at one point in time, say 5,000 firms, 20 defaulted in one year. Then

$$EDF_{1\text{-}yr} = \frac{20}{5{,}000} = 0.004 = 40bp.$$

* KMV Credit Monitor uses a constant asset growth assumption for all firms in the same market.

Example *Federal Express* (dollars in billions of US$)

	November 1997	*February 1998*
Market capitalization (price × shares outstanding)	\$7.9	\$7.3
Book liabilities	\$4.7	\$4.9
Market value of assets	\$12.6	\$12.2
Asset volatility	15%	17%
Default point	\$3.4	\$3.5
Default distance	$\frac{12.6-3.4}{0.15\times 12.6}=4.9$	$\frac{12.2-3.5}{0.17\times 12.2}=4.2$
EDF	$0.06\%(6\,bp)\equiv AA^-$	$0.11\%(11\,bp)\equiv A^-$

The causes of changes for an EDF are due to variations in the *stock price*, *debt level* (leverage ratio), and *asset volatility*.

Key features in KMV model

1. Dynamics of EDF comes mostly from the *dynamics of the equity values*.

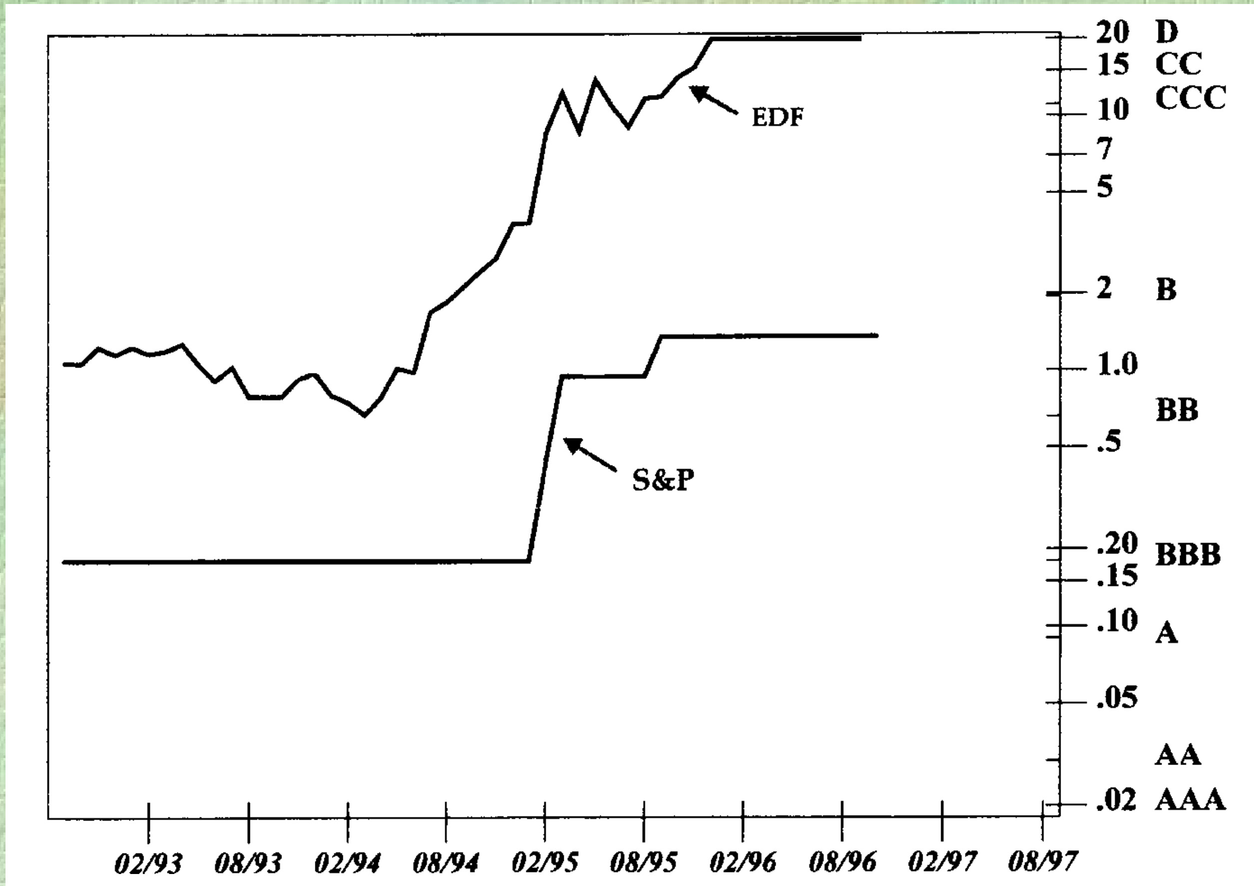
2. *Distance to default ratio* determines the level of default risk.
   - This key ratio compares the firm's net worth to its volatility.
   - The net worth is based on values from the equity market, so it is both timely and superior estimate of the firm value.

3. Ability to adjust to the *credit cycle* and ability to quickly reflect any *deterioration in credit quality*.

4. Work best in highly *efficient liquid market conditions*.

Strength of KMV approach

- Changes in EDF tend to anticipate at least one year earlier than the downgrading of the issuer by rating agencies like Moody's and S & P's.



- EDF provides a cardinal rather than ordinal ranking of credit quality.

- Accurate and timely information from the *equity market* provides a *continuous* credit monitoring process that is difficult and expensive to duplicate using traditional credit analysis.

- Annual reviews and other traditional credit processes cannot maintain the same degree of vigilance that EDFs calculated on a monthly or a daily basis can provide.

Weaknesses of KMV approach

- It requires some *subjective estimation* of the input parameters.
- It is difficult to construct theoretical EDF's without the *assumption of normality* of asset returns.
- *Private firms' EDFs* can be calculated only by using some comparability analysis based on accounting data.
- It does not *distinguish* among different types of long-term bonds according to their seniority, collateral, covenants, or convertibility.

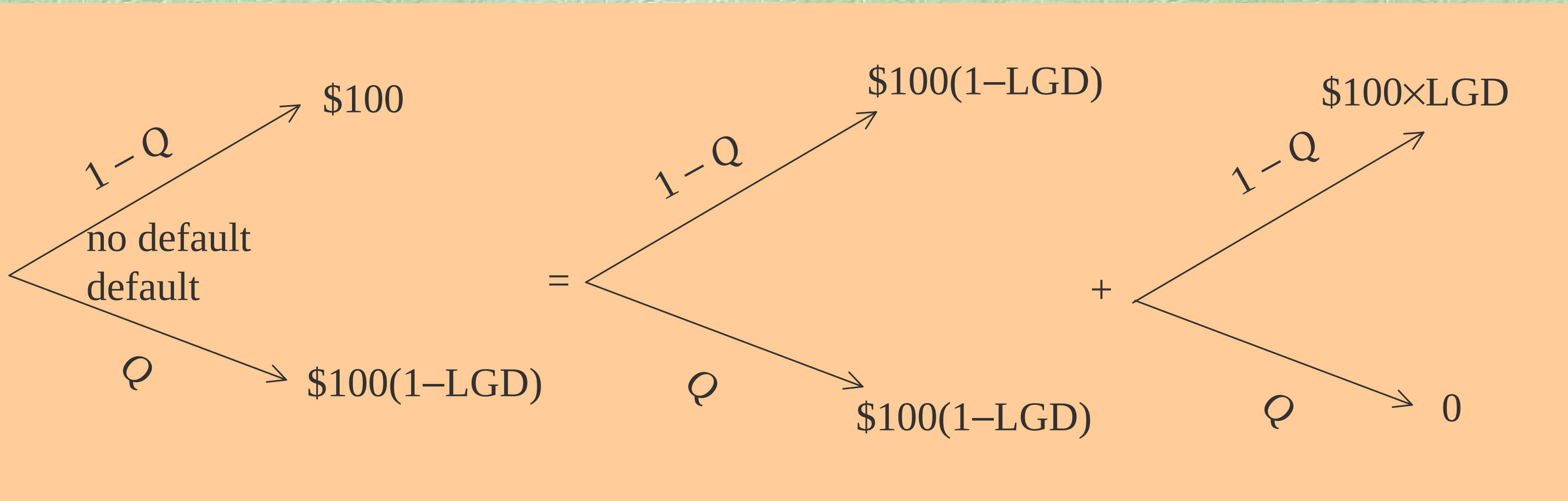
Example

Valuation of a zero coupon bond with a promised payment in a year of \$100, with a recovery of (1 – LGD) upon issuer's default.

* LGD = loss given default (assumed to be 40% here)

Q = risk neutral probability that the issuer defaults in one year from now (assumed to be 20% here)

The expectation is calculated using the risk neutral probabilities but not the actual probabilities as they can be observed in the market from historical data or EDFs.



risky bond

default free component

risky component

PV_1 = PV(risk-free cash flow) = \$100 (1 – LGD)/(1 + r) = \$54.5
r = risk free rate (assumed to be 10% here)
PV_2 = PV (risky cash flow) = E_Q (discounted risky cash flow)

$$= \frac{100\text{LGD}(1-Q)+0\times Q}{1+r} = \$29.1.$$

Let s denote the credit spread

$$\frac{100(1-LGD)}{1+r} + \frac{100LGD(1-Q)}{1+r} = \frac{100}{1+r+s}$$

so that

$$s = \frac{\text{LGD}\times Q}{1-L\text{GD}\times Q}(1+r) = 9.6\%.$$

Derivation of the risk neutral EDFs

Let V_T^* be the firm value process at T under the modified risk neutral process.

$$\frac{dV_t^*}{V_t^*} = r\,dt + \sigma dZ_t$$

$$Q = P_r[V_T^* \le DPT_T]$$

$$= P_r\left[\ln V_0 + \left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}Z_T \le \ln DPT_T\right]$$

$$= P_r\left[Z_T \le -\frac{\ln\left[\frac{V_0}{DPT_T} + \left(r - \frac{\sigma^2}{2}\right)T\right]}{\sigma\sqrt{T}}\right] = N(-d_2^*)$$

where

$$d_2^* = \frac{\ln\frac{V_0}{DPT_T} + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}.$$

On the other hand, the expected default frequency under the actual process is given by

$$EDF_T = N(-d_2)$$

where

$$d_2 = \frac{\ln \frac{V_0}{DPT_T} + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}.$$

Hence, the risk neutral *EDF*

$$Q = N\left[N^{-1}(EDF_T) + \frac{\mu - r}{\sigma}\sqrt{T}\right].$$

From CAPM, we have

$$\mu - r = \beta(\mu_M - r) = \frac{\rho_{FM}\sigma}{\sigma_M}(\mu_M - r)$$

where β = beta of the asset with the market portfolio

$\mu_M - r$ = market risk premium for one unit of beta risk (to be estimated by a separate statistical process).