Caps and Swaps

Floating rate securities

Coupon payments are reset periodically according to some *reference rate*.

reference rate + index spread

e.g. 1-month LIBOR + 100 basis points (positive index spread)
5-year Treasury yield – 90 basis points (negative index yield)

- Reference rate can be some financial index

e.g. return on the S & P 500 or non-financial index

e.g. price of a commodity or inflation index

(in 1997, US government begin issuing such bonds)

Caps

Restriction on the *maximum* coupon rate-cap.

The bondholder effectively sold an option to the bond issuer; coupon rate taken to be min (r_{float}, r_{cap}).

Floors

Minimum coupon rate specified for a floating rate security-floor.

The bond issuer sold an option to the bond holder;

coupon rate taken to be max (r_{float}, r_{floor}).

Cap and floor provisions are embedded options in fixed income securities.

Range notes

Coupon rate is equal to the reference rate as long as the reference rate is within a certain range at the reset date. If the reference rate is outside of the range, the coupon rate is zero for that period.

Inverse floaters

Coupon rates are reset periodically according to

$$K - L \times \text{reference rate.}$$

To prevent the coupon rate to fall below zero, a floor value of zero is usually imposed.

In general, an inverse floater is created from a fixed rate security – called collateral. Actually, from the collateral, two bonds are created: a floater and an inverse floater.

Consider a 10-year 7.5% coupon semi-annual pay bond. $100 million of the bond is used as a collateral to create a floater with a par value of $50 million and an inverse floater with a par value of $50 million.

Floater coupon rate:	reference rate + 1%
Inverse floater coupon rate:	14% – reference rate

The weighted average of the coupon rate of the combination of the two bonds is

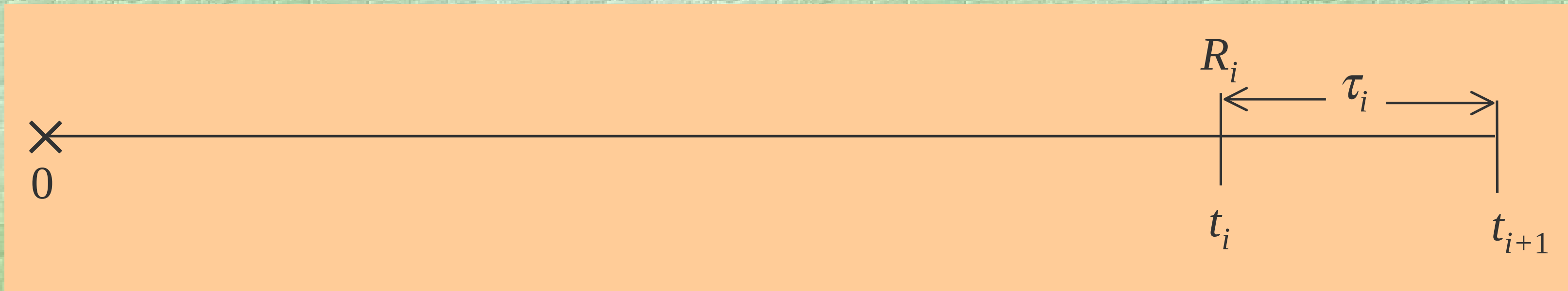
$$0.5(\text{reference rate} + 1\%) + 0.5(14\% - \text{reference rate}) = 7.5\%.$$

If a floor is imposed on the inverse, then correspondingly a cap is imposed on the floater:

$$\text{inverse's price} = \text{collateral's price} - \text{floater's price}$$

Plain vanilla interest rate swap

It is an agreement whereby two parties undertake to exchange, at known dates in the future, a fixed for a floating set of payments.



Let R_i be the τ-period spot rate prevailing at time t_i
(e.g. 3-month or 6-month LIBOR rate for a quarterly or semi-annual swap, respectively);

X be the fixed rate contracted at the outset paid by the fixed-rate payer;
N_i be the notional principal of the swap outstanding at time t_i

τ_i be the frequency or tenor of the swap $= t_{i+1} - t_i$ in years
e.g. $\tau_i = 1/4$ for semi-annual swap.

- *Fixed leg* is made up by payments B_i paid at time t_{i+1}

$$B_i = N_i X \tau_i$$

- *Floating leg* consists of payment A_i at time t_{i+1} where

$$A_i = N_i R_i \tau_i$$

Since the realization at time t_i of the spot rate is not known at time 0, $t < t_i$

$$PV(A_i) = E(N_i R_i \tau_i P(0, t_{i+1}))$$

where $P(t, T)$ is the price at time t of a discount bond maturing at time T.

Let F_i denote the forward rate between $[t_i, t_i+1]$ agreed at time 0. By the compounding rule of discounting

$$P(0, t_{i+1}) = \frac{1}{1 + F_i \tau_i} P(0, t_i) \text{ or } F_i = \frac{\frac{P(0,t_i)}{P(0,t_{i+1})} - 1}{\tau_i}.$$

Consider the portfolio constructed at time 0 which holds one unit of discount bond maturing at time t_i and shorts one unit of discount bond maturing at time t_{i+1}. Value of the portfolio at time t_i is

$$V(t_i) = 1 - \frac{1}{1 + R_i \tau_i} = \frac{R_i \tau_i}{1 + R_i \tau_i}.$$

Consider the payment of amount $R_i \tau_i$ at time t_{i+1}, its present value at time t_i is $\dfrac{R_i \tau_i}{1 + R_i \tau_i}$, which is the same as the present value at time t_i of the above portfolio of two bonds.

Hence, at time 0, the commitment to pay $R_i\tau_i$ at time t_{i+1} and the strategy of holding a bond $P(0, t_i)$ and shorting a bond $P(0, t_{i+1})$ must have the same value, that is,

$$P(0,t_i) - P(0,t_{i+1}) = R_i \tau_i P(0,t_{i+1})$$

or

$$R_i = \frac{\frac{P(0,t_i)}{P(0,t_{i+1})} - 1}{\tau_i}.$$

Note that R_i is the same as the projected forward rate F_i. To avoid arbitrage, the unknown τ-period spot rate τ_i must be set equal to the projected forward rate F_i.

Present value at time 0 of floating leg payments

$$= \sum PV(A_i) = \sum N_i F_i \tau_i P(0, t_{i+1}).$$

Present value at time 0 of fixed leg payments

$$= \sum PV(B_i) = \sum N_i X \tau_i P(0, t_{i+1}).$$

The *equilibrium swap* rate is defined to be the fixed rate X such that the above two present values are the same:

$$X = \frac{\sum N_i X \tau_i P(0, t_{i+1})}{\sum N_i \tau_i P(0, t_{i+1})}.$$

This is the weighted average of the projected forward rates.

By setting $w_i = \dfrac{N_i X \tau_i P(0, t_{i+1})}{\sum N_i \tau_i P(0, t_{i+1})}$, we have $X = \sum w_i F_i$.

For the payer of the fixed rate, the present value of the swap at time t is

$$NPV_{swap}(t) = -\sum N_i X \tau_i P(t, t_{i+1}) + \sum N_i F_i \tau_i P(t, t_{n+1})$$

where F_i are now the forward rates calculated from the discount curve at time t. The second term can be written as $\sum N_i X_t \tau_i P(t, t_{i+1})$,where X_t is the equilibrium swap rate prevailing at time t.

$$NPV_{swap}(t) = (X_t - X_0)\sum N_i \tau_i P(t, t_{i+1}) = (X_t - X_0)\sum B_i.$$

Some simplification Take $N_i = 1$, we obtain

$$\sum_{i=1}^{n} F_i \tau_i P(0, t_{i+1}) = \sum_{i=1}^{n} \tau_i \left[\frac{P(0,t_i) / P(0,t_{i+1}) - 1}{\tau_i} \right] P(0, t_{i+1})$$

$$= \sum_{i=1}^{n} P(0, t_i) - P(0, t_{i+1}) = P(0, t_1) - P(0, t_{n+1}).$$

Use of a currency swap to enhance yield

Direct

US Treasury notes

8.14% (US$)

Investor

Yield pickup
= 8.51% – 8.14%
= 0.37%

Synthetic

German govt bonds

8.45% (DM)

Investor

8.45% (DM)

8.14% (US$)

Swap house

- Instead of buying *10-year US Treasury notes* yielding 8.14%, the investor purchased *10-year German government* bonds yielding 8.45% (denominated and payable in deutshemarks), and simultaneously entered into a *currency swap*.

- Under the swap, the investor agreed to exchange its DM cashflows over the life of the swap for US dollars.

Risks (besides the default risk of the German government)

1. Default risk of the swap counterparty;
2. Over the 10-year life, the investor might have desired to liquidate the investment early and sell the German bonds prior to the maturity of the swap (left with a swap for which it had no obvious use as a hedging instrument).

Combination of swaps

- Combination of two plain vanilla commodity swaps, a plain vanilla currency swap, and a plain vanilla interest rate swap.

Goal To enable an oil-producing nation to obtain a fixed long-term supply of rice in exchange for long-term quantity of oil.

Without the swap

The oil-producing nation was simply to sell oil on the spot market for US dollars, then convert those dollars into Japanese yen and purchase rice in Japan on the spot market.

Structure of the swaps

1. A *commodity swap* was entered into under which the oil producer locked in long-term fixed dollar price for selling future specified oil production.

2. The US dollars were indirectly converted into fixed yen using a combination of (a) fixed-for-floating US dollar denominated *interest rate swap*, then followed by (b) floating-rate dollar for fixed rate yen *currency swap*.

3. The fixed-rate yen were converted through a *commodity swap* into the yen needed to buy a fixed quantity of rice on the spot market.

Counterparty risks of the four swaps!