CreditMetrics

CreditMetrics

- *Methodology* for measuring credit risk in a portfolio context.
- *Datasets* of default probabilities, recovery rates, credit migration likelihoods and correlations.
- *Software system* to calculate and report credit risk for a portfolio of positions.

Key features of CreditMetrics

- It is based on *credit migration analysis*, i.e. the probability of moving from one credit quality to another within a given horizon.
- It models the *full forward distribution* of the values of any bond or loan portfolio, where the changes in values are related to credit migration only, while interest rates are assumed to evolve in a deterministic fashion (limitation: *assuming no market risk*).
- Credit-VaR of a portfolio is derived as the percentile of the distribution corresponding to the desired confidence level.

Challenging difficulties in Credit-VaR

- The portfolio distribution is *far from being normal* (actually it is highly skewed and fat-tailed).
- The correlations in credit quality changes for all pairs of obligors are not directly observable. The evaluation is based on the *joint probability of asset returns*.

Credit-VaR for a bond

1. *Specify a rating system*

   Rating categories, combined with the probabilities of migrating from one credit quality to another over the credit risk horizon. (For example, Moody's or S&P's or a proprietary rating system internal to the bank.)
   Assumption: all issuers are credit-homogeneous within the same rating class.

2. Specify the *forward discount curve* at the risk horizon(s) for each credit category, and the "*recovery rate*".

Translate the above information into the forward distribution of the changes in portfolio value consecutive to credit migration.

Specification of the transition matrix

- The transition probabilities are based on more than 20 years of history of firms across all industries.
- Actual transition and default probabilities vary quite substantially over the years, depending whether the economy is in recession, or in expansion.
- Many banks prefer to rely on their own statistics which relate more closely to the composition of their loan and bond portfolios. They may have to adjust historical values to be consistent with one's assessment of current environment.

Investment grade ratings

S & P and others	***Moody's***	***Interpretation***
AAA	Aaa	Highest quality; Extremely Strong
AA+	Aa1	High quality
AA	Aa2	
AA-	Aa3	
A+	A1	Strong payment capacity
A	A2	
A-	A3	
BBB+	Baa1	Adequate payment capacity
BBB	Baa2	
BBB-	Baa3	

Speculative grade ratings

S & P and others	***Moody's***	***Interpretation***
BB+	Ba1	Likely to fulfill obligations;
BB	Ba2	Ongoing uncertainty
BB-	Ba3	
B+	B1	High risk obligations
B	B2	
B-	B3	
CCC+	Caa1	Current vulnerability to default
CCC	Caa2	
CCC-	Caa3	
CC		
C	Ca	In bankruptcy or default, or other
D		market shortcomings

Transition matrix, probabilities of credit rating migrating from one rating quality to another, within one year.

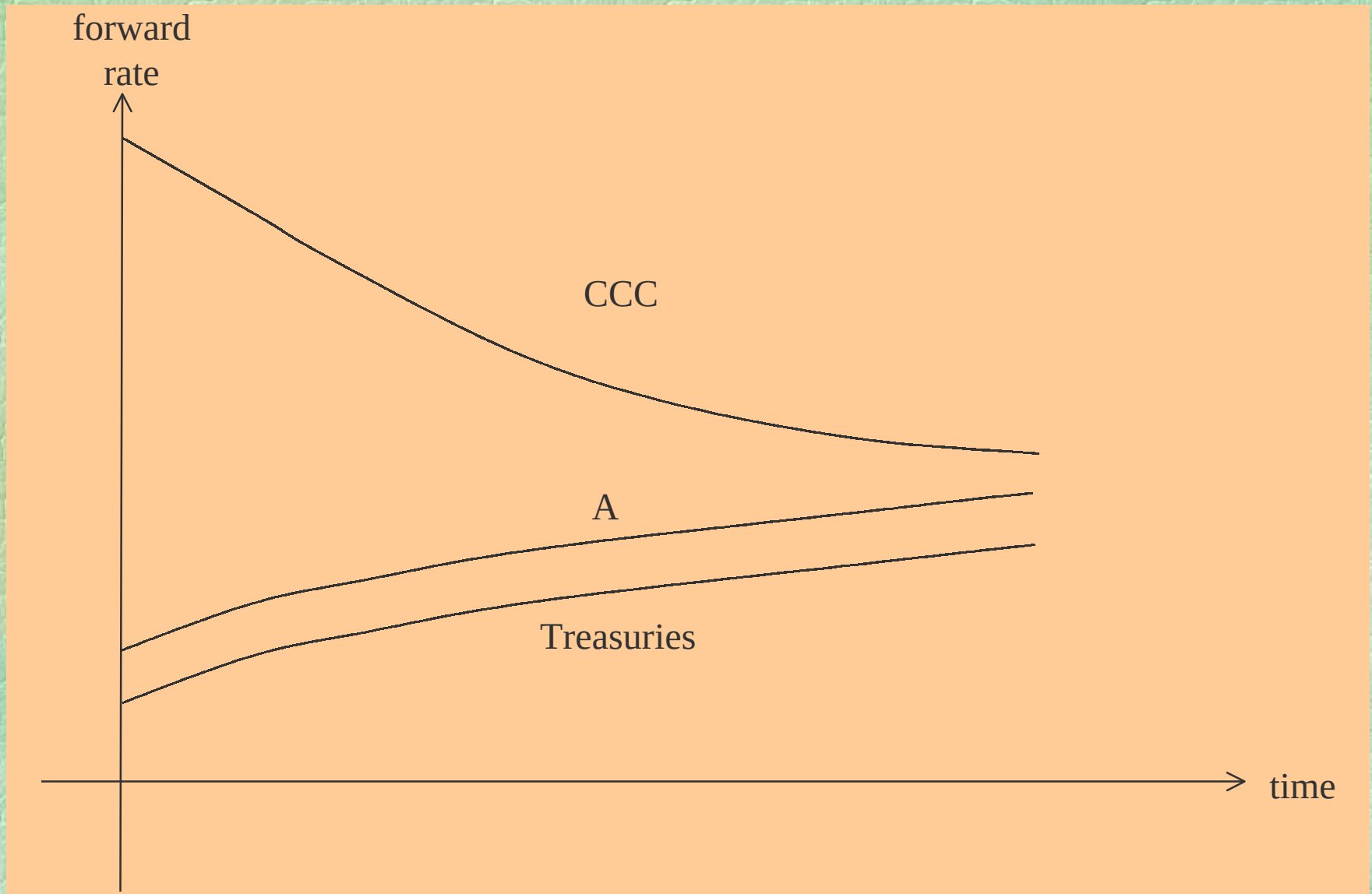
Initial Rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	1.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek (April 15, 1996)

Specify the spread curve

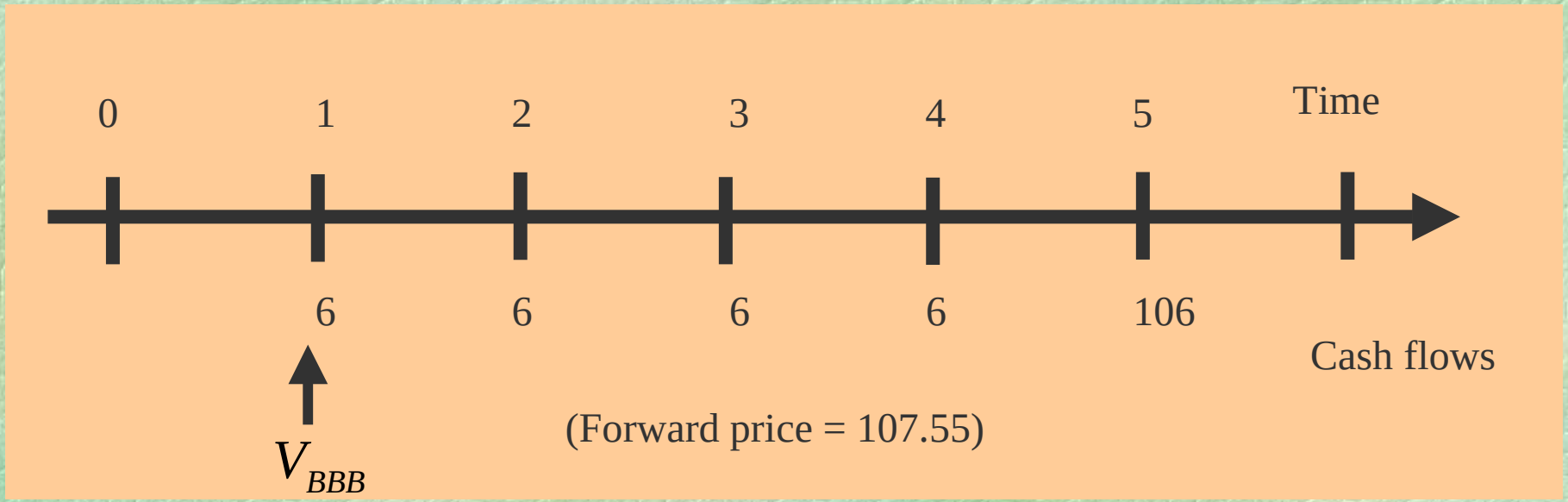
Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

One year forward zero curves for each credit rating (%)



- Spread of high (low) investment grade bonds increases (decreases) with maturity.

Specify the forward pricing model



One-year forward price of the BBB bond

$$V_{BBB} = 6 + \frac{6}{1.041} + \frac{6}{(1.0467)^2} + \frac{6}{(1.0525)^3} + \frac{106}{(1.0563)^4} = 107.55$$

One-year forward values for a BBB bond

Year-end rating	Value ($)
AAA	109.37
AA	109.19
A	108.66
BBB	107.55
BB	102.02
B	98.10
CCC	83.64
Default	51.13

Source: CreditMetrics, J. P. Morgan

Specify the recovery rate

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior subordinated	17.09	10.90

Source: Carty & Lieberman [1996]

Recovery rates by seniority class (% of face value, i.e., "par")

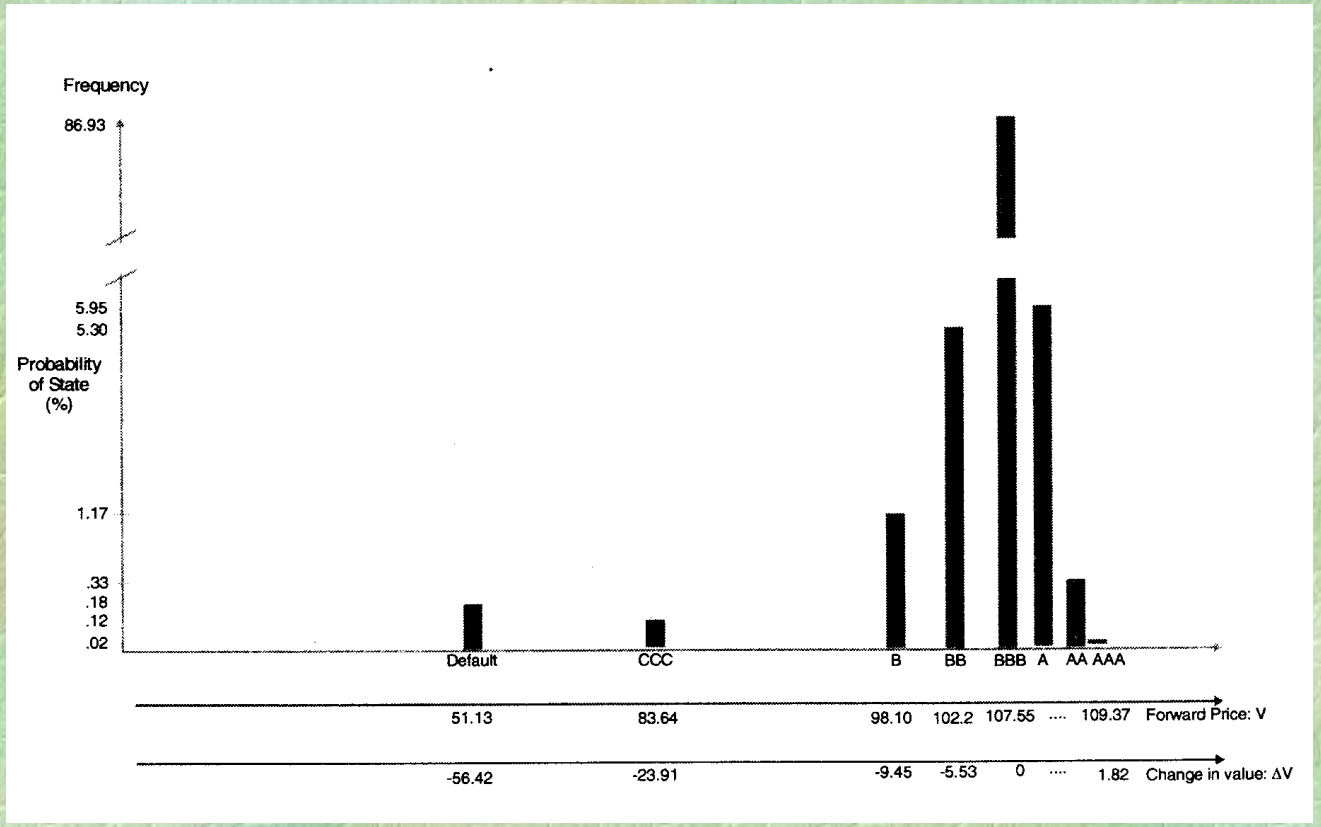
Derive the forward distribution of the changes in bond value

Year-end rating	Probability of state: p (%)	Forward price: V ($)	Change in value: ΔV ($)
AAA	0.02	109.37	1.82
AA	0.33	109.19	1.64
A	5.95	108.66	1.11
BBB	86.93	107.55	0
BB	5.30	102.02	-5.53
B	1.17	98.10	-9.45
CCC	0.12	83.64	-23.91
Default	0.18	51.13	-56.42

Source: CreditMetrics, J. P. Morgan

Distribution of the bond values, and changes in value of a BBB bond, in one year. Note the long downside tail.

Histogram of the one-year forward prices and changes in value of a BBB bond



First percentile of $\Delta V = -23.91$
(credit VaR at the 99% confidence level)

$$\text{Mean}\,(\Delta V) = m = \sum_i p_i \Delta V_i = -0.46$$

$$\text{Variance}\,(\Delta V) = \sigma^2 = \sum_i p_i (\Delta V_i - \text{mean}) = 8.95$$

First percentile of a normal distribution = $m - 2.33\sigma = -7.43$.

(Recall that 98% of observations lie between 2.33σ and -2.33σ from the mean for normal distributions.)

Joint migration probabilities (%) with zero correlation for 2 issuers rated *BB* and *A*

Obligor #1 (BB)		Obligor #2 (single-A)							
		AAA	AA	A	BBB	BB	B	CCC	Default
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.03	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
AA	0.14	0.00	0.00	0.13	0.01	0.00	0.00	0.00	0.00
A	0.67	0.00	0.02	0.61	0.40	0.00	0.00	0.00	0.00
BBB	7.73	0.01	0.28	7.04	0.43	0.06	0.02	0.00	0.00
BB	80.53	0.07	1.83	73.32	4.45	0.60	0.20	0.01	0.05
B	8.84	0.01	0.02	8.05	0.49	0.07	0.02	0.00	0.00
CCC	1.00	0.00	0.02	0.91	0.06	0.01	0.00	0.00	0.00
Default	1.06	0.00	0.02	0.97	0.06	0.01	0.00	0.00	0.00

Assuming zero correlation, each entry is given by the product of the transition probabilities for each obligor. For example,

$$73.32\% = 80.53\% \times 91.05\%$$

Correlations between changes in credit quality

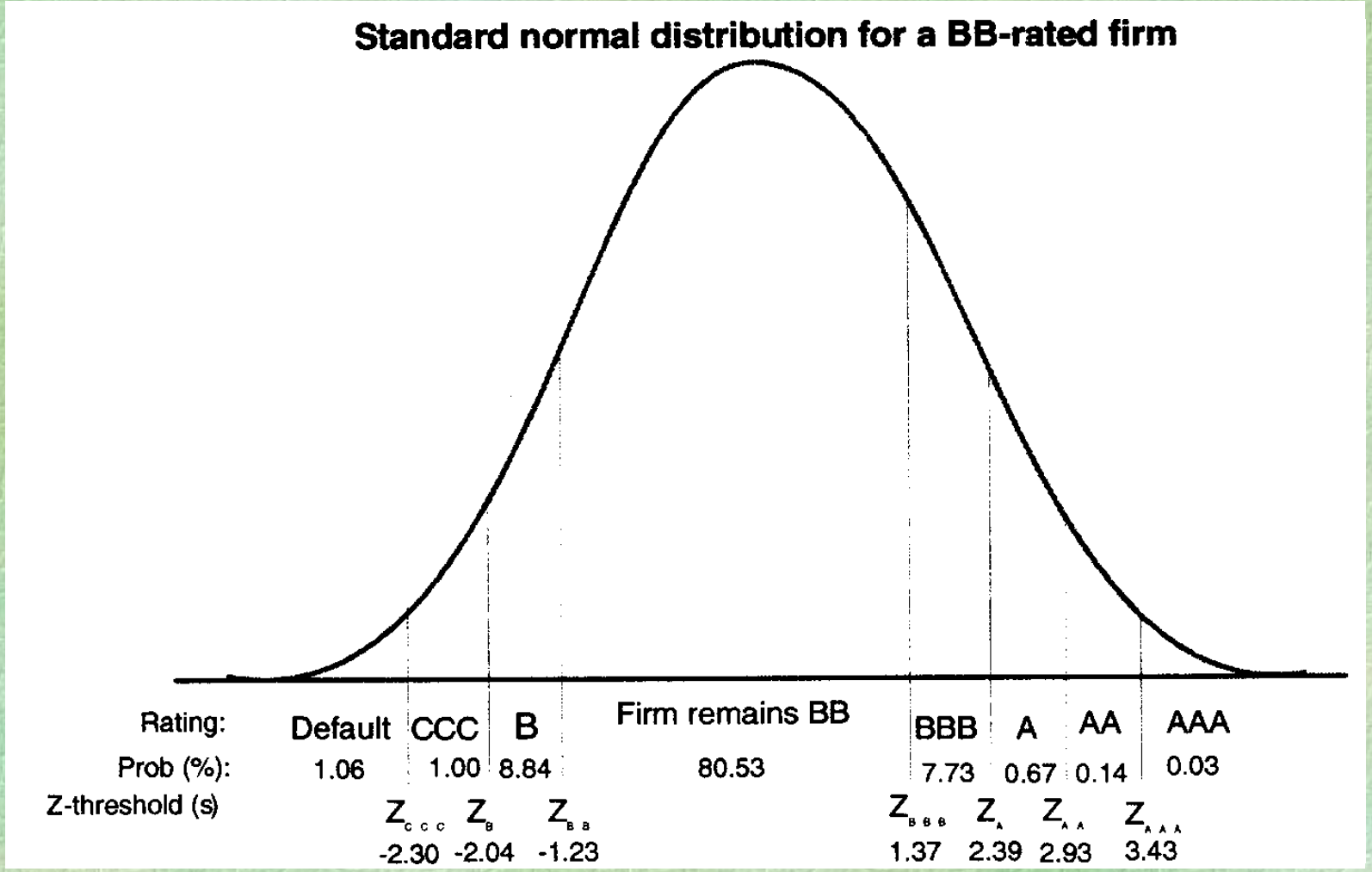
Correlations are expected to be higher for firms within the same industry or in the same region.

Correlations vary with the relative state of the economy in the business cycle. Eg. when the economy is performing well, default correlations go down.

Default and migration probabilities should not stay stationary over time.

Contrary to KMV, CreditMetrics have chosen the equity price as a Proxy for the asset value of the firm that is not directly observed – a very strong assumption that may greatly affect the accuracy.

Derive the credit quality thresholds for each credit rating



- We slice the distribution of asset returns into bands in such a way that if we draw randomly from this distribution, we reproduce exactly the migration frequencies shown in the transition matrix.

- Z_{ccc} is the threshold point to trigger default; Z_{ccc} is the "distance-to-default".

- CreditMetrics estimates the correlations between the equity returns of various obligors, then the model infers the correlations between changes in credit quality directly from the joint distribution of equality returns. The use of equity returns as a proxy is based on the assumption that firm's activities are all equity financed.

Calculation of the joint rating probabilities

$$\Pr(-1.23 < r_{BB} < 1.37, -1.51 < r_A < 1.98) = \int_{-1.23}^{1.37} \int_{-1.51}^{1.98} f(r_{BB}, r_A; \rho) dr_{BB} dr_A = .7365$$

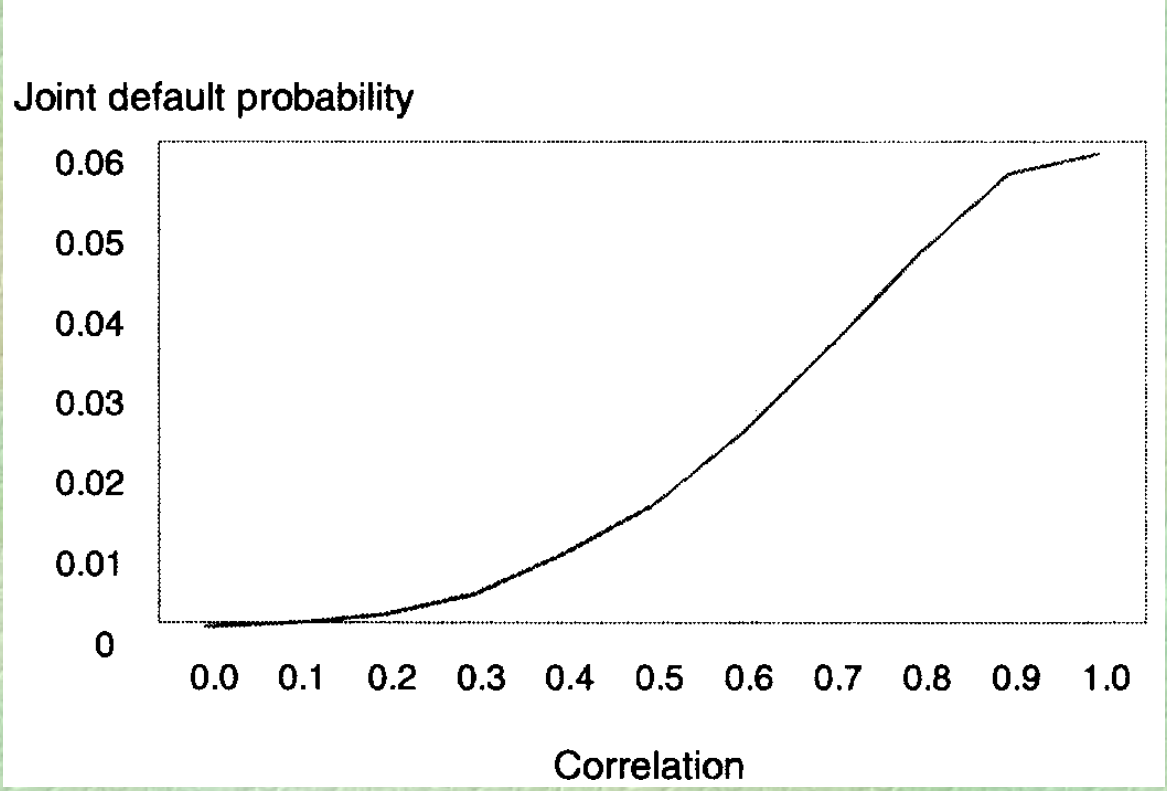
Rating of first company (BB)	Rating of second company (A)								
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Def**	**Total**
AAA	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.03
AA	0.00	0.01	0.13	0.00	0.00	0.00	0.00	0.00	0.14
A	0.00	0.04	0.61	0.01	0.00	0.00	0.00	0.00	0.67
BBB	0.02	0.35	7.10	0.20	0.02	0.01	0.00	0.00	7.73
BB	0.07	1.79	73.65	4.24	0.56	0.18	0.01	0.04	80.53
B	0.00	0.08	7.80	0.79	0.13	0.05	0.00	0.01	8.84
CCC	0.00	0.01	0.85	0.11	0.02	0.01	0.00	0.00	1.00
Def	0.00	0.01	0.90	0.13	0.02	0.01	0.00	0.00	1.06
Total	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06	100

Joint rating probabilities (%) for BB and A rated obligors when correlation between asset returns is 20%.

Probability of joint defaults

$$corr(DEF_1, DEF_2) = \frac{P(DEF_1, DEF_2) - P_1 \cdot P_2}{\sqrt{P_1(1-P_1) \cdot P_2(1-P_2)}}$$

$$P(DEF_1, DEF_2) = \Pr\left[r_1 \le -d_2^1, r_2 \le -d_2^2\right] = N_2(-d_2^1, -d_2^2; \rho)$$



Probability of joint defaults as a function of asset return correlation

Sample calculation

Take ρ = 20%,

$$P(DEF_1, DEF_2) = N_2(-d_2^1, -d_2^2; \rho) = N_2(-3.24, -2.30; 0.20) = 0.000054.$$

$P_1 = 0.06\%$ and $P_2 = 1.06\%$; so

$$\text{corr}\,(DEF_1, DEF_2) = 0.019 = 1.9\%.$$

Credit diversification

Implement a *Monte Carlo simulation* to generate the full distribution of the portfolio values:

1. Derivation of the asset return thresholds for each rating categories.

2. Estimation of the correlation between each pair of obligors' asset returns.

3. Generation of asset return scenarios according to their joint normal distribution (using the Cholesky decomposition). Each scenario is characterized by n standardized asset returns, one for each obligor.

4. Given the spread curves which apply for each rating, the portfolio is revalued.

5. Repeat the procedure a large number of times and plot the distribution of the portfolio values.

Credit-VaR and calculation of economic capital

Economic capital stands as a cushion to absorb unexpected losses related to credit events.

$V(p)$ = value of the portfolio in the worst case scenario at the p% confidence level

FV = forward value of the portfolio = $V_0(1 + PR)$

where PR = promised return on the portfolio

EV = expected value of the portfolio = $V_0(1 + ER)$

where ER = expected return on the portfolio

EL = expected loss = $FV - EV$

$$Capital = EV - V(p)$$

One may query the following assumptions in CreditMetrics

1. Firms within the same rating class are assumed to have the same default rate.

2. The actual default rate (migration probabilities) are set equal to the historical default rate (migration frequencies).

- Default is only defined in a statistical sense (non-firm specific) without explicit reference to the process which leads to default.

Some other credit risk models attempt to make improvements on

- Defaults rates vary with current economic and financial conditions of the firm.
- Default rates change continuously (ratings are adjusted in a discrete fashion).
- Microeconomic approach to default: a firm is in default when it cannot meet its financial obligations.

Empirical studies show:

- Historical average default rate and transition probabilities can deviate significantly from the actual rates.
- Substantial differences in default rates may exist within the same bond rating class.
- Overlapping in default probability ranges may be quite large.