Portfolio credit derivatives

The payoff is related to the credit events in a whole portfolio of risky assets–correlation products.

Reference assets

1. n^{th} to default credit default swaps
2. Collateralized debt obligations
3. Index tranches
4. CDO^2 structure

Pricing considerations

- default intensities of obligors in the portfolio
- recovery rates upon default
- default correlation among the obligors

Basket default swaps

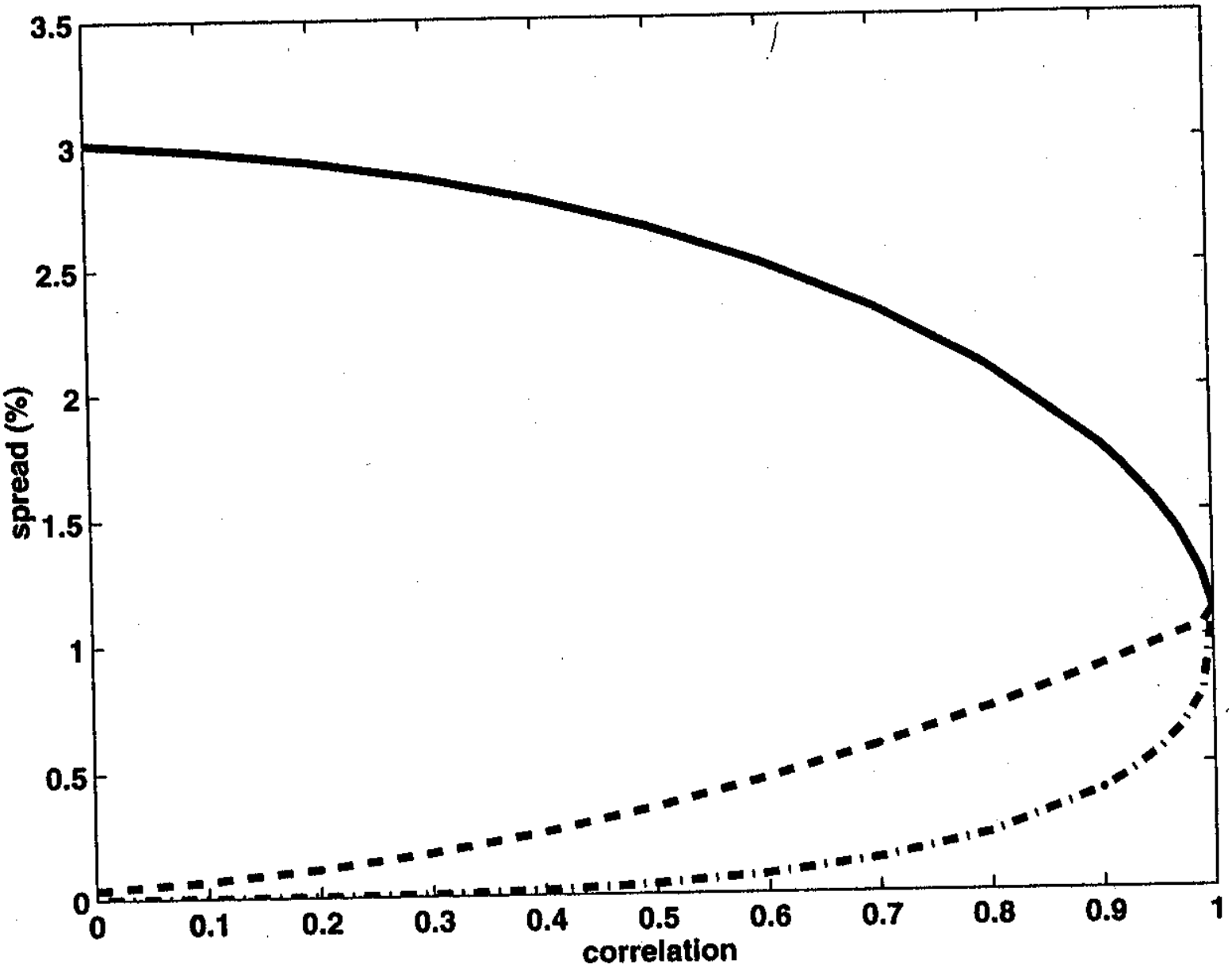
The credit event to insure against is the event of the k^{th} default. A premium or spread s is paid as an insurance fee until maturity or the event of the k^{th} default.

If the k^{th} default occurs before swap's maturity, the Protection Buyer can present the defaulting bond to the Protection Seller in exchange for the face value of the bond.

Sum of the k^{th}-to-default swap spreads is greater than the sum of the individual spreads

$$\sum_{k=1}^{n} s^k > \sum_{i=1}^{n} s_i.$$

Why? Both sides insure exactly the same risk, but at the time of the first default, one stops paying the huge spread s^1 on one side but on the plain-vanilla side one stops just paying the spread s_i of the first default i.



kth-to-default spread versus correlation for a basket with three underlyings: (solid) s^{1st}, (dashed) s^{2nd}, (dashed-dotted)

In the figure, we show the k^{th}-to-default spreads for 3 underlyings with

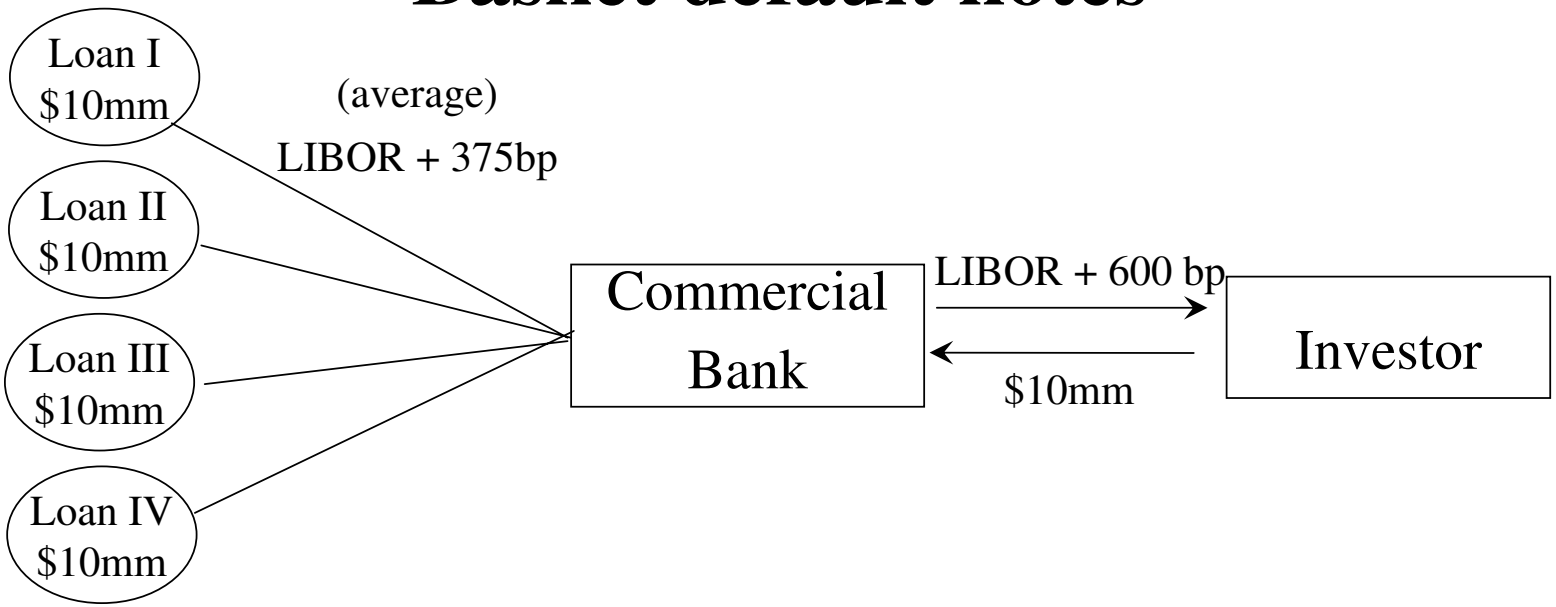
$$s_1 = 0.009, \quad s_2 = 0.010 \quad \text{and} \quad s_3 = 0.001$$

and we assume pair-wise equal correlation.

- If the credit events are independent, the fair spread s^1 is close to the sum of the fair default swap spreads.

- If they are dependent, then $s^1 = \max_i s_i$.

Basket default notes



In the event that any of the loans defaults, the note is terminated. The bank keeps the $10 million of note proceeds, and the defaulted loan is put to the investor.

Effectively, the investor bears the *risk of the first default*. However, any subsequent defaults are the sole responsibility of the bank.

Spread to buy 5 year protection for the n^{th} default from the basket of 10 names. All firms have the same probability of default. For $\lambda_i = 0.01$, this means about 1% of defaulting each year. The correlation between each pair of names is 0.3. The spread is in the basis points per annum.

	Default Intensity for all Firms		
n	0.01	0.02	0.03
1	440	814	1165
2	139	321	513
3	53	149	263
4	21	71	139
5	8	34	72
6	3	15	36
7	1	6	16
8	0	2	6
9	0	1	2
10	0	0	0

The cost of protection increases at a decreasing rate for low n and at an increasing rate for high n.

Spread to buy protection for the n^{th} default from the basket of 10 names. All pairs of firms have the same correlation. The default intensity for each firm is 0.01. The spread is in the basis points per annum.

	Pairwise correlation		
n	0	0.3	0.6
1	603	440	293
2	98	139	137
3	12	53	79
4	1	21	49
5	0	8	31
6	0	3	19
7	0	1	12
8	0	0	7
9	0	0	3
10	0	0	1

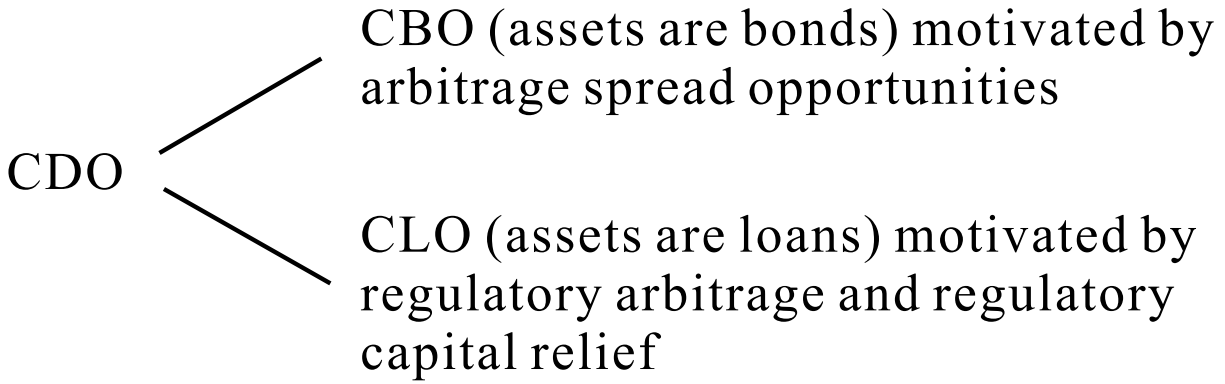
Impact of correlations

Increasing the pairwise correlations between all firms while holding the default intensity constant lowers the cost of protection in an n^{th} to default CDS if n is small and increases it if n is large.

- When the correlation is zero, the cost of default protection is a sharply declining function of n.
- In the limit when the default times are perfectly correlated all entities default at the same time and the cost of n^{th} to default protection is the same for all n.
- As correlations increase we are progressing from the first case to the second case so that the cost of protection decreases for low n and increases for high n.

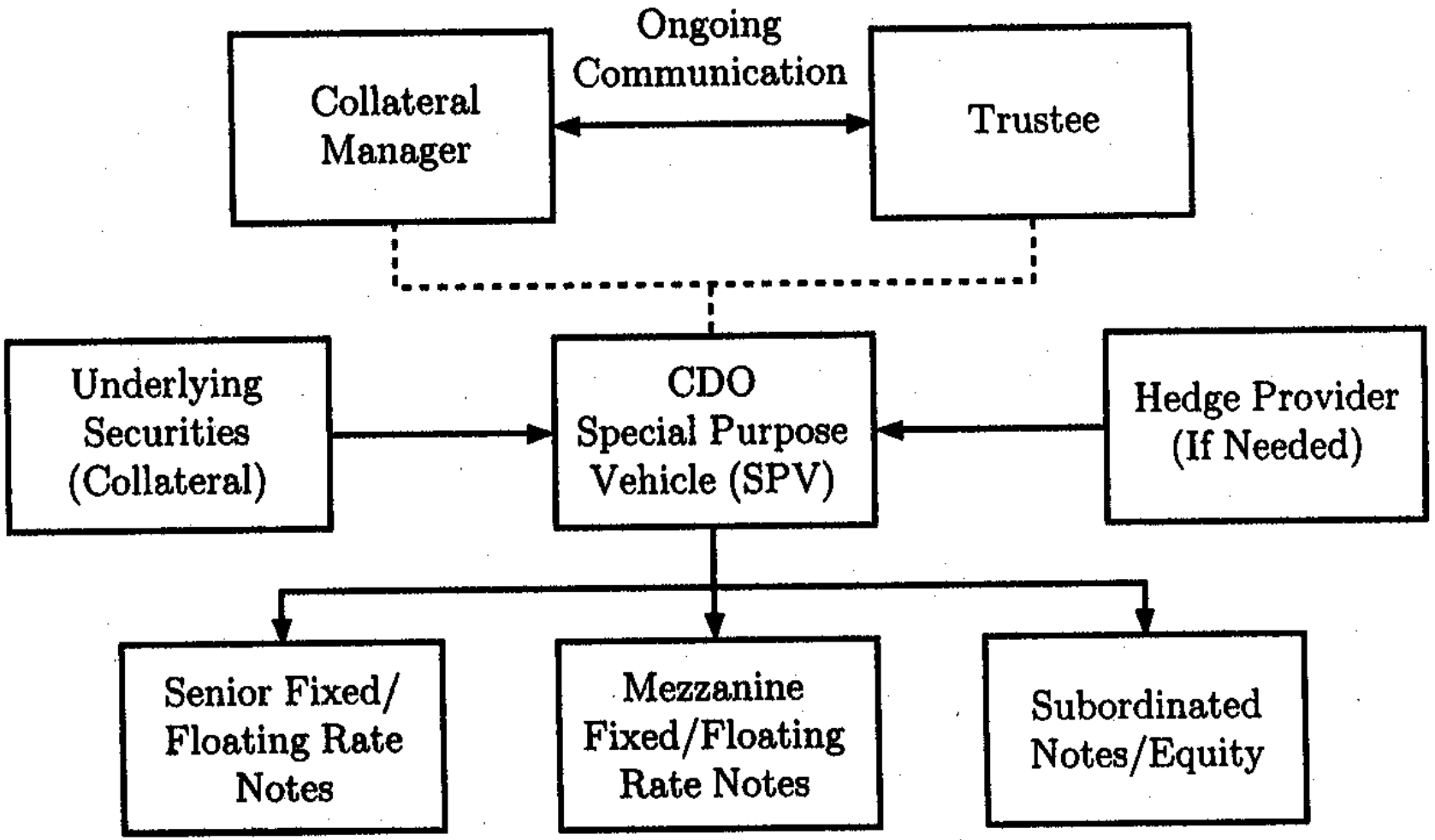
Collateralized Debt Obligations

A *collateralized debt obligation* (CDO) is a security backed by a pool of assets security (e.g. corporate bonds, bank loans).



A CDO cash flow structure allocates interest income and principal repayments from a collateral pool of different debt instruments to a prioritized collection of CDO securities, called *tranches*.

A CDO is a way of creating securities with widely different risk characteristics from a portfolio of debt instruments.



Typical CDO contractual relationships. (Source: Morgan Stanley.)

Contractual relationships involved in a CDO

- Collateral manager is charged with the selection and purchase of collateral assets for the special purpose vehicle (SPV), which is bankruptcy remote from its originator.

- The trustee is responsible for monitoring the contractual provisions.

- The hedge provider may provide various forms of swaps to hedge against interest rate risk and/or currency risk.

Prioritization scheme

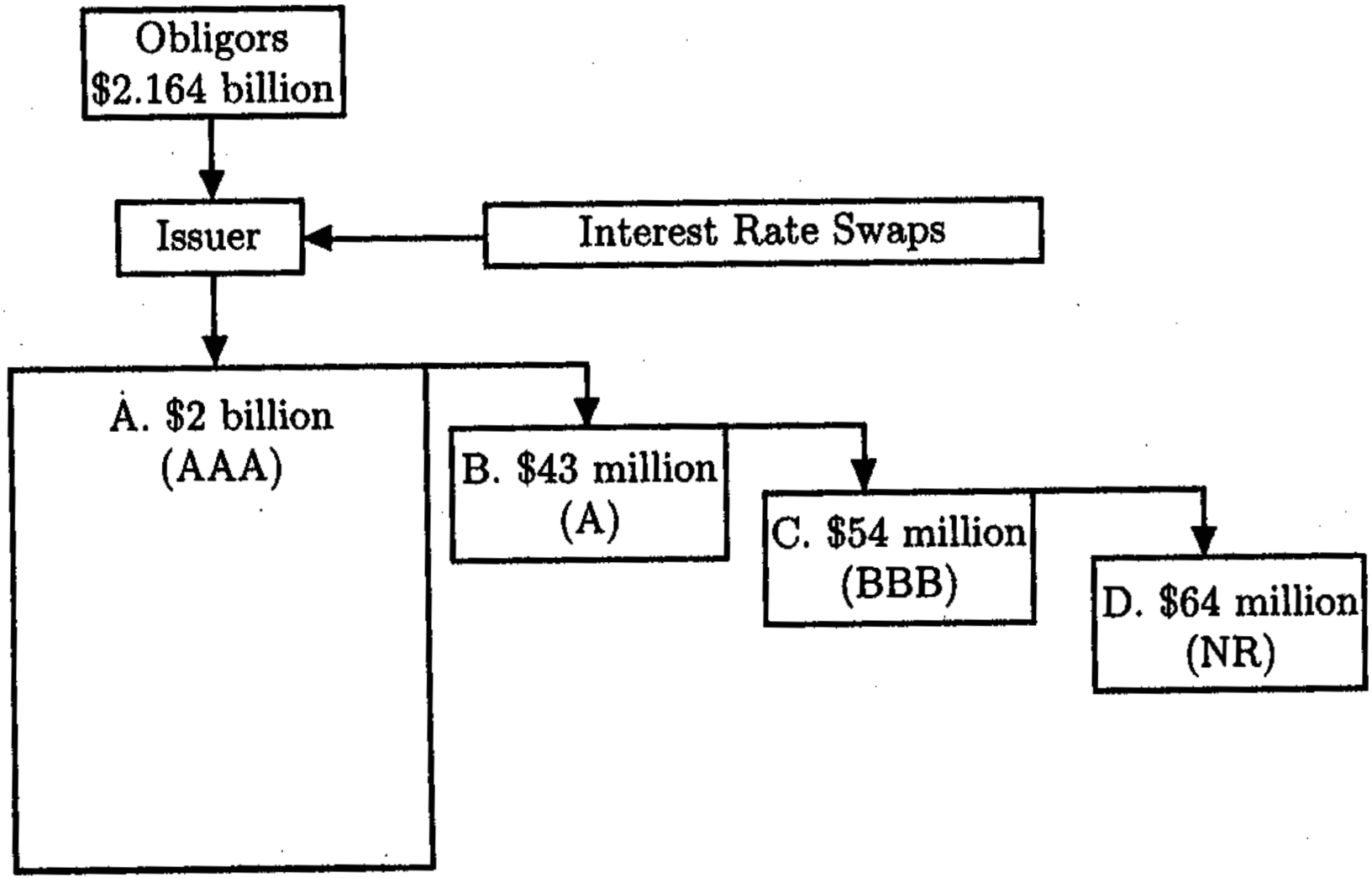
Senior CDO notes are paid before mezzanine and lower-subordinated notes are paid, with any residual cash flow paid to an equity piece.

The uncertainty regarding interest and principal payments to the CDO tranches is determined mainly by the number and timing of defaults of the collateral securities.

Cash flow CDOs – not subject to active trading by the CDO manager.

Market value CDOs – CDO tranches receive payments based on the mark-to-market returns of the collateral pool, hence trading performance.

- ramp-up period to purchase the collateral pool of assets.



NationsBank 1997-1 CLO tranches. (Source: Fitch.)

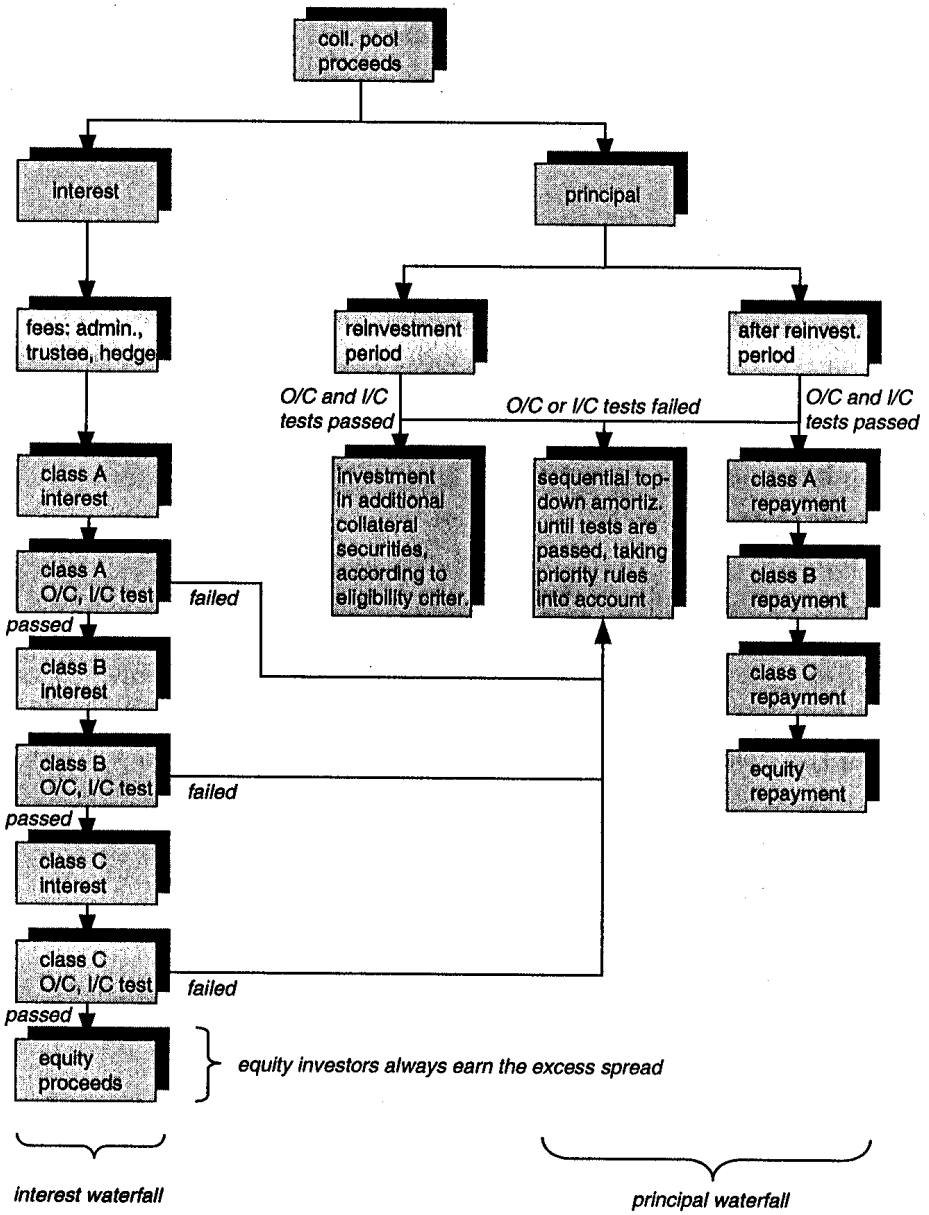
- The bulk of the underlying assets are floating-rate NationsBank loans rated BBB or BB. Any fixed-rate loans were significantly hedged against interest rate risk by interest rate swaps.

- The equity piece (not rated) is the first loss piece, receiving interest and principal payments only if all other note investors received their contractually promised payments.

Coverage tests

- Intended as an early warning (automatically redirecting cash flows) that interest or principal proceeds are running short for covering the notes coupons and / or repayments.

- In case of a failed coverage test, principal and interest proceeds are used for paying back the outstandings on notes sequentially until all tests are passed again.



Example of cash flow waterfalls in a cash flow CDO.

Regulatory capital

Regulatory wedge – what market requires (economic capital) and what regulators require (regulatory capital)?

Loans are 100% risk weight items and capital charges of 8% are levied on them.

- Forcing banks to allocate the same quantity of capital to support a loan to an AA-rated company as to a B-rated company. This would bias the investment decision in favor of the B-rated loans.
- This results in "pricing distortions", since capital costs of a loan are independent of the credit quality of the borrower.

Exploitation of regulatory arbitrage in securitization transactions

Risk-weighted assets are assumed to be equal to the total volume V of the pool.

$$RC_{pool} = \text{ regulatory capital of pool } = RWA \times 8\% = V \times 8\%.$$

Portfolios expected loss $EL = 50$bps and portfolio's weighted average net margin $NM = 130$bps.

$$RoE_{pool} = \text{ return on equity of pool } = \frac{NM - EL}{RC_{pool}} = \frac{130 - 50}{800} = 10\%.$$

The RoE measures the return, net of expected losses, in units of required equity.

Consider a synthetic CLO, the top 85% of volume of pool is protected by a credit default swap, and the first loss piece is 1.5%.

$$RC_{sec} = V \times 1.5\% \times 100\% + V \times 85\% \times \underset{\substack{\uparrow \\ \text{risk-adjusted factor}}}{20\%} \times 8\% = V \times 2.86\%$$

$$\begin{aligned} RoE_{sec} &= \frac{NM - EL - \text{COSTS}}{1.5\% \times 100\% + 85\% \times 20\% \times 8\%} \\ &= \frac{130 - 50 - 30}{286} = 17.48\%. \end{aligned}$$

Economics of CDOs

CDOs address some important market imperfections.

- Regulatory arbitrage and capital relief
- Illiquidity leading to a reduction in their market values

Balance sheet CDS is designed to remove loans from the balance sheets of banks, achieving capital relief and increasing the valuation of the assets through an increase in liquidity. It is normally cash flow type.

Arbitrage CDO is designed to capture some fraction of the likely difference between the total cost of acquiring collateral assets in the secondary market and the value received from management fees and the sale of the associated CDO structure. It can have either cash flow or market value structures.

Arbitrage spread opportunities

- The assets in the collateral pool are priced on a single asset basis.
- The tranching of notes really is a tranching of the loss distribution of the collateral pool. Since diversification decreases the risk of a portfolio, so that the price of the portfolio risk must be lower than the price obtained by just summing up exposure-weighted single risk.
- The spreads paid to notes investors are lower than the spreads earned on the bonds in the collateral pool.

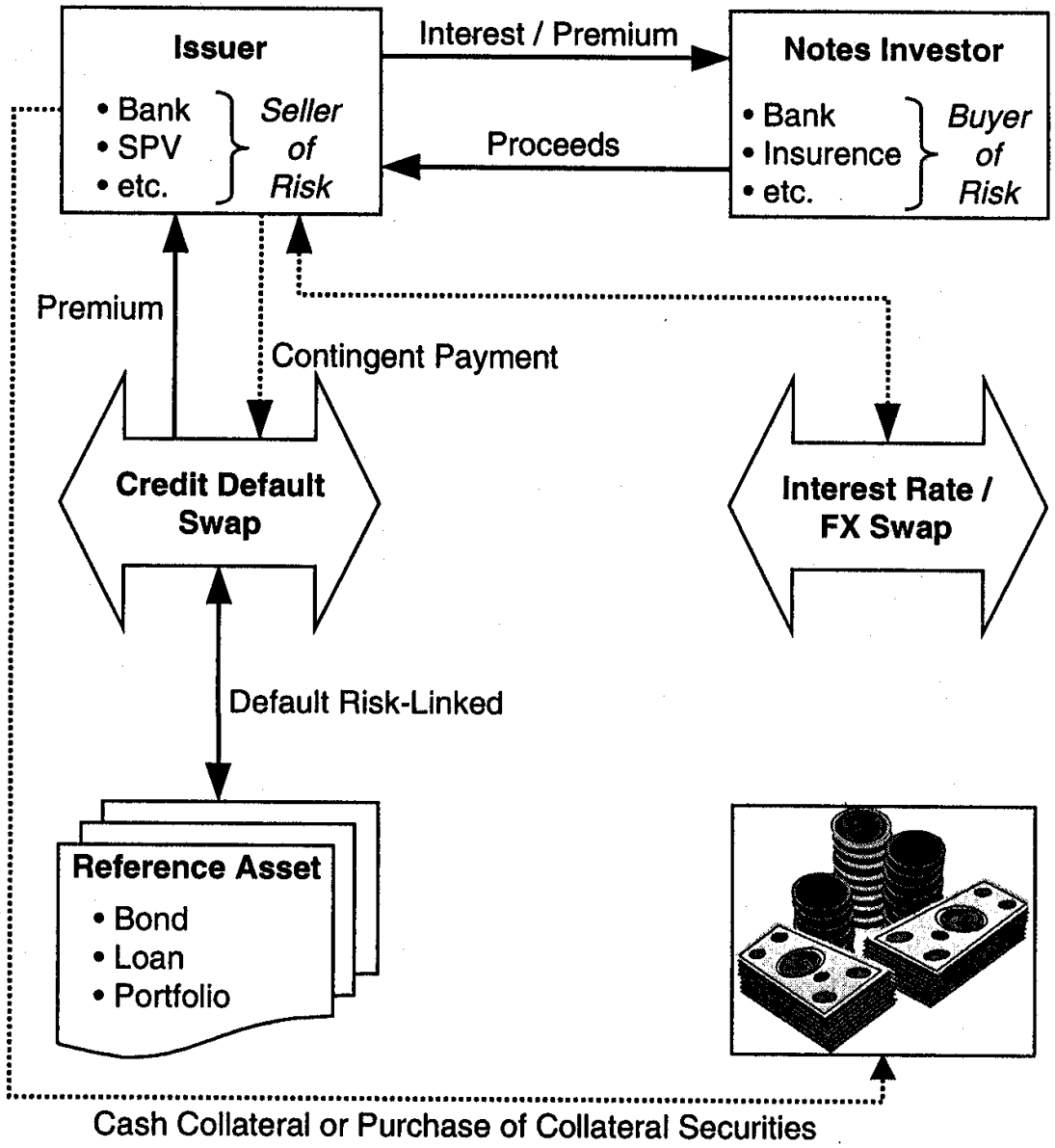
Risk distillation in synthetic CLO's

Credit risk is distilled from a reference portfolio of loans, then channeled to the credit markets.

- Create a special purpose vehicle (bankruptcy-remote from the originating bank) that issues the credit-linked notes.
- Credit-linked notes will be collateralized by AAA-rated securities, that is, they are the obligations of a fully collateralized SPV.

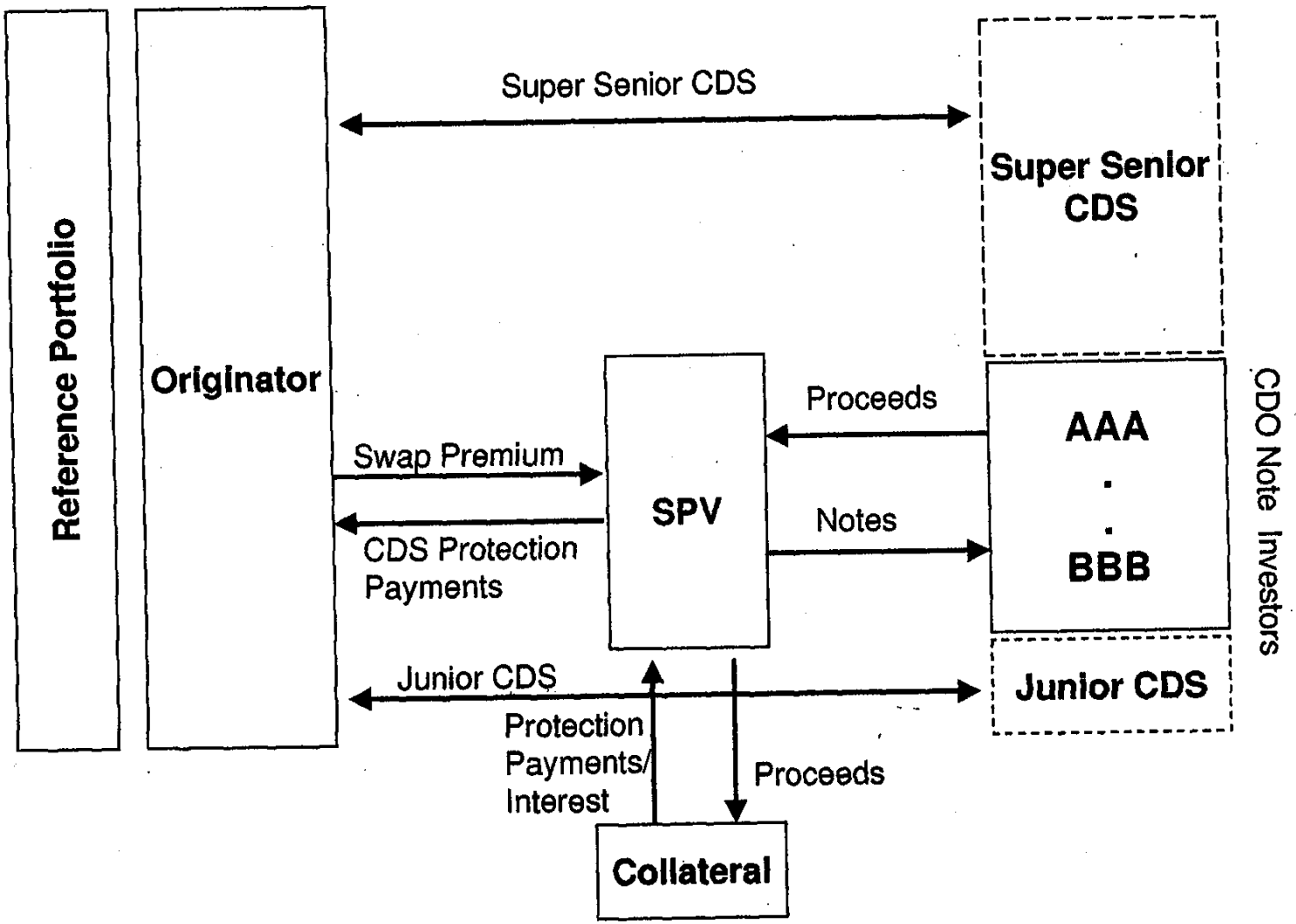
Credit-linked Notes

- The interest from the investment grade security and the periodic swap payments received from the default swap payments received from the default swap buyer are passed on to the CLN investors in the form of a yield on the notes.
- The CLN issuer is protected from default risk of the reference asset.
- Higher return for investors without directly getting into credit derivatives market. This is the same as buying a riskless FRN and selling a credit protection through a CDS.
- Conventional stream of cash flows with periodic fix/float coupons and principal at redemption, if no credit events occurs.
- The cash flows are altered upon the occurrence of a credit event experienced by a reference credit.



Example of a Credit-linked Note.

Example of a synthetic CDO transaction.



Moral hazard – asymmetric information

- Cherry picking – sorting assets into the portfolio pool based on the issuer's private information.
- In virtually every synthetic CDO and CLN, the 'buyer' of protection determines whether a credit event has occurred in the reference portfolio. Also the 'buyer' calculates the severity of its losses following a credit event, and how much the SPV will be required to pay under the swap.

Moody's advice – Good faith of the sponsor

No matter how carefully the transaction is structured, an aggressive protection buyer can interpret credit events more broadly than the seller intended, or obtain pricing for defaulted obligations that is unrealistic or not meaningful.

Example of a synthetic CLO – Olan Enterprises

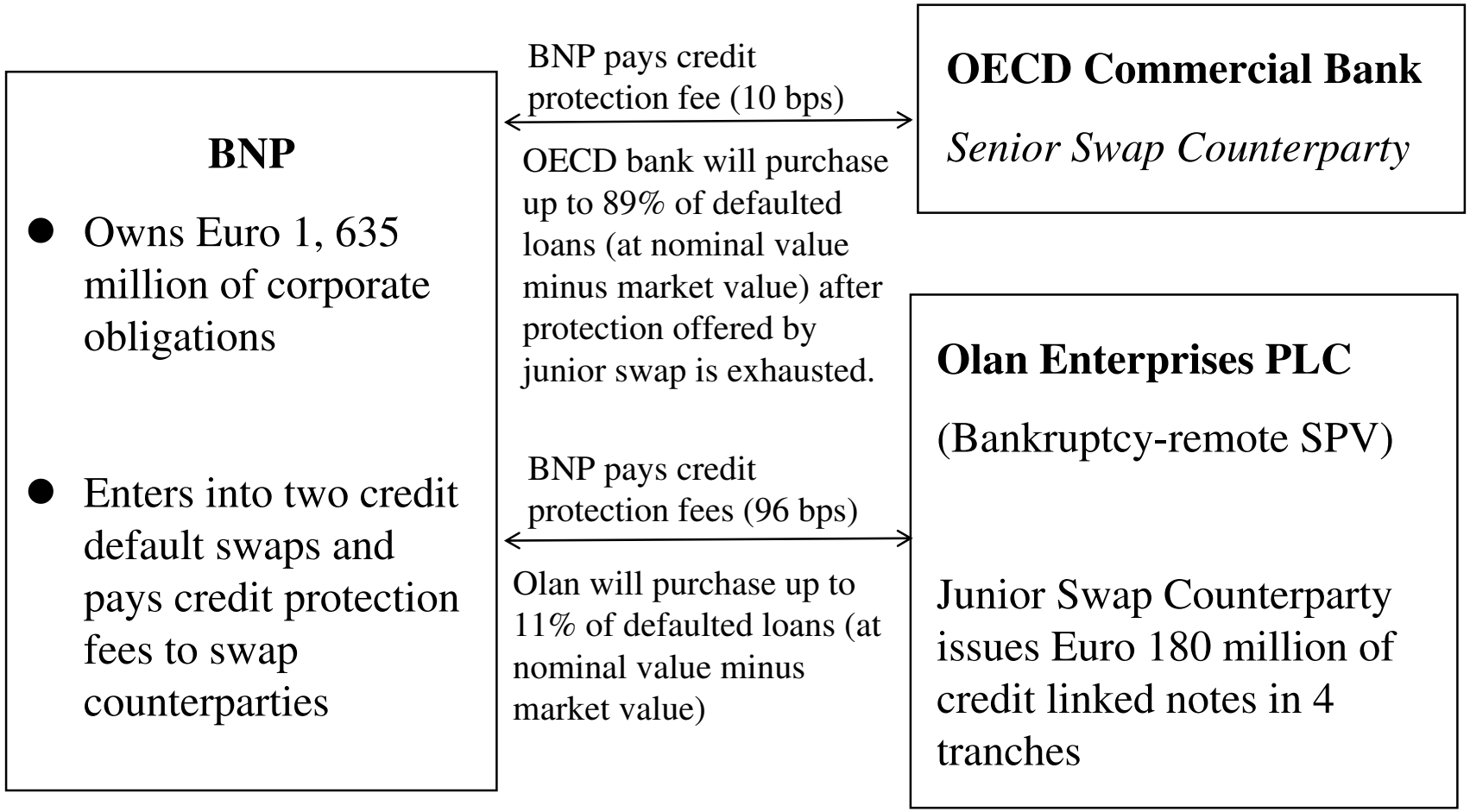
1. Olan financed its commitment under the junior credit default swap by issuing Eur 180 million of credit linked notes in 4 classes (11% of the reference portfolio). This is a *partially funded synthetic CDO*. The goal of partial funding is to deliver favorable capital requirement without the funding cost disadvantage problem.

2. Olan used the proceeds from the notes to purchase 5-year French Treasury bonds (OATs) as collateral. Should a reference credit be affected by a credit event, Olan must sell OATs to pay BNP's loss.

3. Olan receives the premium from the junior credit default swap. The fee, plus the coupon the AAA-collateral, funds Olan's interest obligations on the credit linked notes.

Olan 1 Transaction structure

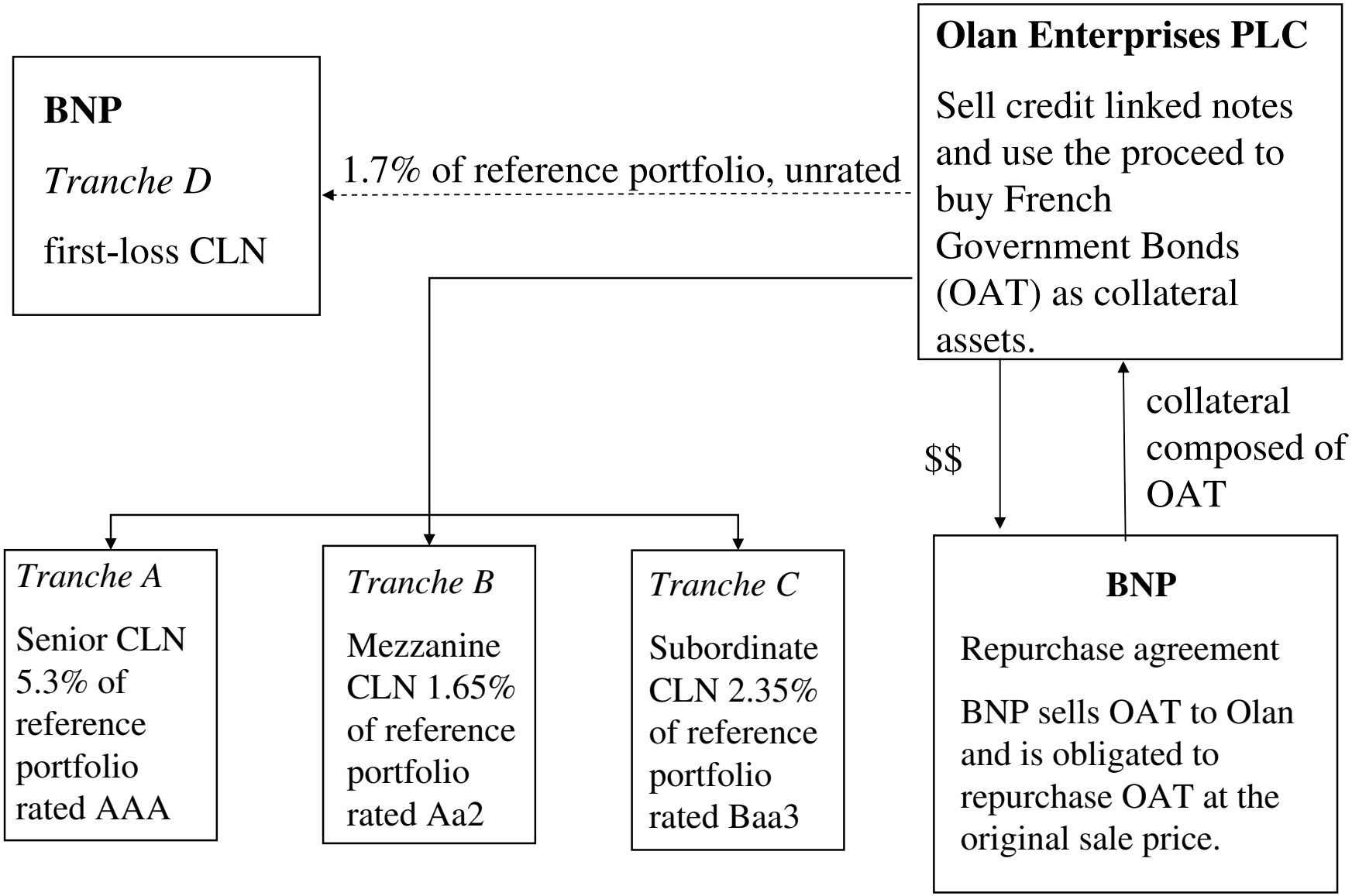
Launched by Banque Nationale de Paris (BNP) in 1999.



Total premium (as % of reference portfolio) = 0.96% × 11% + 0.10% × 89% = 0.1946%.

Note that 180 million = 1,635 million x 11%.

Olan 1 Transaction structure



Credit linked notes – public issues

	Class A	Class B	Class C	Class D
Amount (Euro)	86.65m	26.97m	38.42m	27.96m
Rating	AAA	Aa2	Baa3	unrated
bp over 3-month Euro-bor	30	40	150	NA
% of corporate credit exposure	5.3%	1.65%	2.35%	1.7%

Class D is absorb the first loss experienced by the reference portfolio.
This first-loss CLN was retained by BNP.

Transfer of credit risks

The 4 credit-linked notes have different exposures to credit risk.

- Class D funds the first level of losses (retained by BNP)

The credit risk beyond that funded by the SPV is shifted to an Organization for Economic Cooperation & Development (OECD) bank via a *Senior* default swap.

The embedded risks in the reference portfolio of loans are shifted without having to sell the underlying loans – *synthetic CLO*.

OATs as collateral

OATs are used as collateral, first for the credit protection of BNP, then for the repayment of classes A, B, C & D.

Repurchase agreement (mitigate the market risk associated with liquidation)

BNP is committed to repurchase the OATs sold to Olan at the original price paid by Olan.

Reference

1. L.S. Goodman, "Synthetic CDOs: an introduction," *Journal of Derivatives*, (Spring 2000) p. 60 – 72.

2. C.A. Stone and A. Zissu, "Synthetic collateralized loan obligations: Olan Enterprises, PLC," *Journal of Derivatives*, (Spring 2000) p.73 – 80.

Regulatory capital for synthetic CLOs

D = sponsoring bank's first loss (class D retained by sponsoring bank)

20% = risk-weight assigned to the notional amount of the senior credit swap

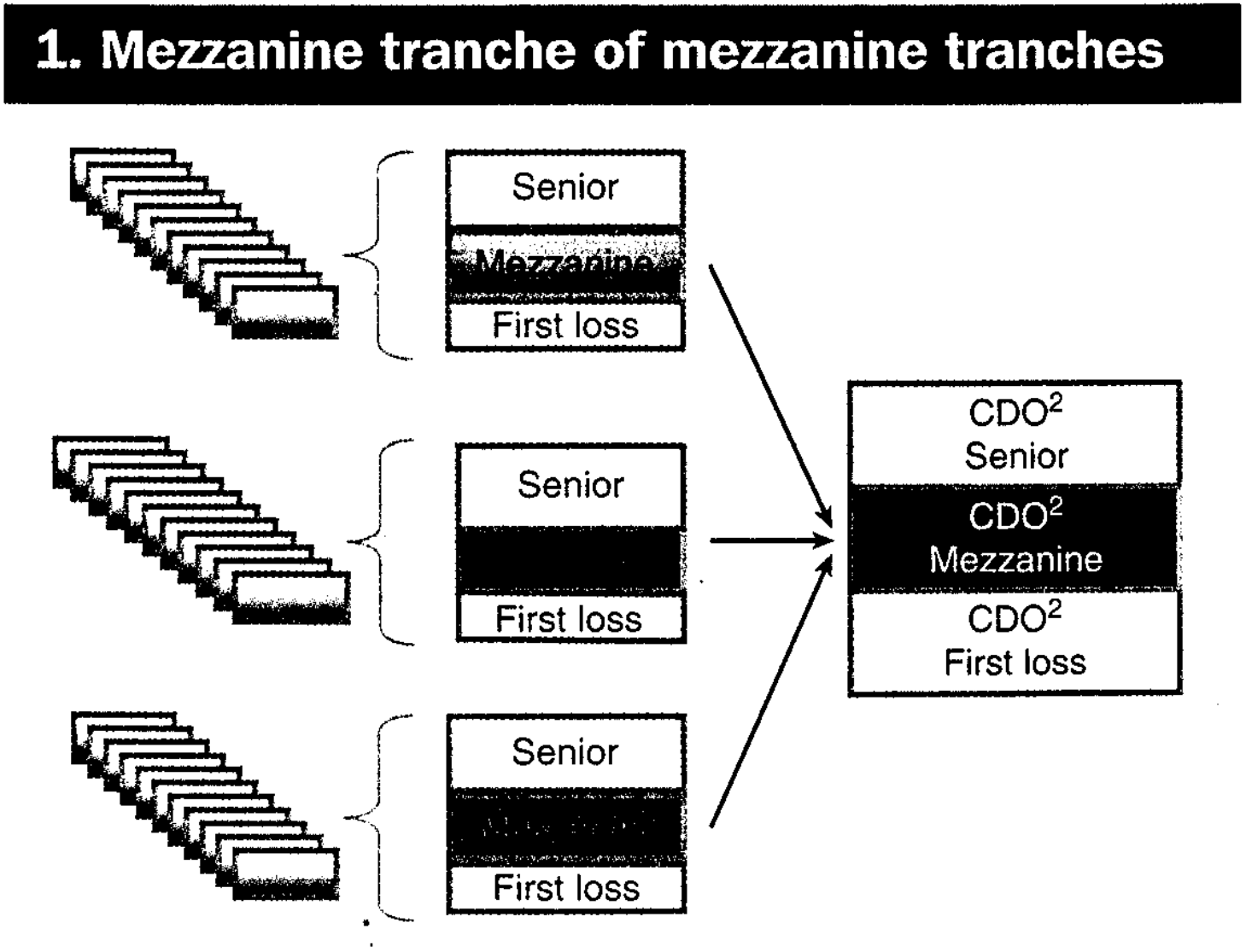
Senior = notional amount of senior credit swap

K_{fed} = capital requirement for the sponsoring US bank

= $\max(D, 0.08 \times (D + 0.2 \times \text{senior}) + 0 \times \text{junior}) = 1.7\%$

K_{CB} = capital requirement for the sponsoring bank under Commission Bancaire(French banking regulator) regulations

= $D + 8\% \times 20\% \times \text{senior} = 3.125\%$.



CDO^2 – a portfolio of synthetic CDO tranches is itself tranched into so-called super tranches.

Index tranches

- Participants in credit derivatives markets have developed indexes to track CDS spreads. For example, the Dow Jones iTraxx EUR 5 year index is the average credit default swap spread for a portfolio of 125 investment grade European companies. Successive tranches are responsible for 0% to 3%, 3% to 6%, 6% to 9%, 9% to 12% and 12% to 22% of the losses.

- An index tranche is not funded by the sale of a portfolio of credit default swaps, but the rules for determining payoff ensure that an index tranche is economically equivalent to the corresponding synthetic CDO tranche.

Implied correlations

- Calculate implied correlations from the spreads at which tranches trade using the standard market model.
- The implied correlation for a trade is the correlation that causes the value of the tranche to be zero.

Base correlations

Correlations that cause the total value of all tranches up to a certain point to have a value of zero.

For example, for DJ CDX IGNA 5 year index, the 0% to 10% base correlation is the correlation that causes the sum of values of the 0% to 3%, 3% to 7%, 7% to 10% tranches to be zero.

Impact of correlation on the values of tranches

Equity tranche

- The equity tranche bears the first loss exposure.
- The probability of smaller portfolio losses increases as correlation increases.

Hence, the tranche becomes valuable to the Protection Seller as it becomes relatively more likely that there will be no defaults and the protection sold will not be triggered.

Mezzanine tranche

- Not particularly sensitive to correlation

Other tranches

- Higher implied correlation means higher value to Protection Buyer.

Factors required in modeling CDOs credit risk

Rating of each CDO class is determined by credit enhancement, ongoing collateral credit performance, and the priority of interest in the cash flows generated by the pool of assets.

Risk analysis modeling

1. Number of defaults up to maturity.
2. The recovery rates of these defaults.
3. Default timing.

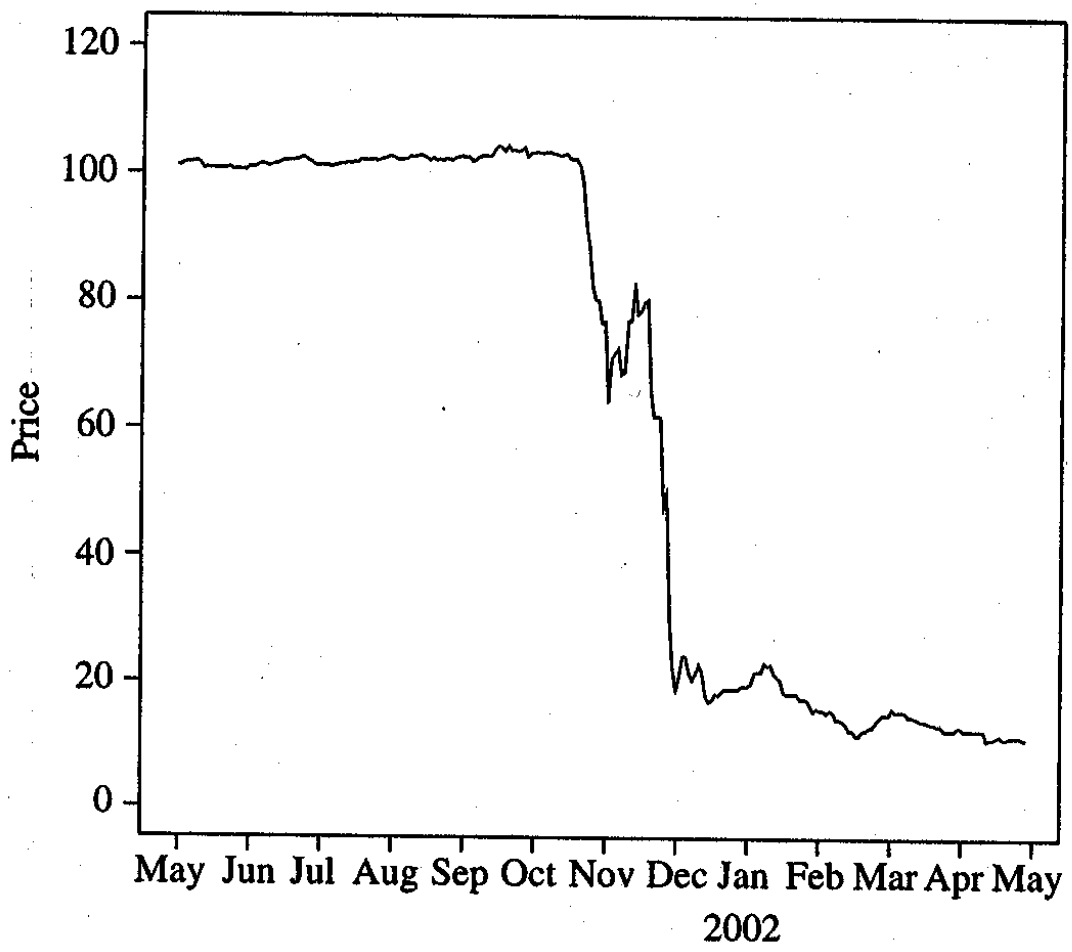
Default correlation

- Involves default timing.
- Correlation of assets.
- Use of diversity score.

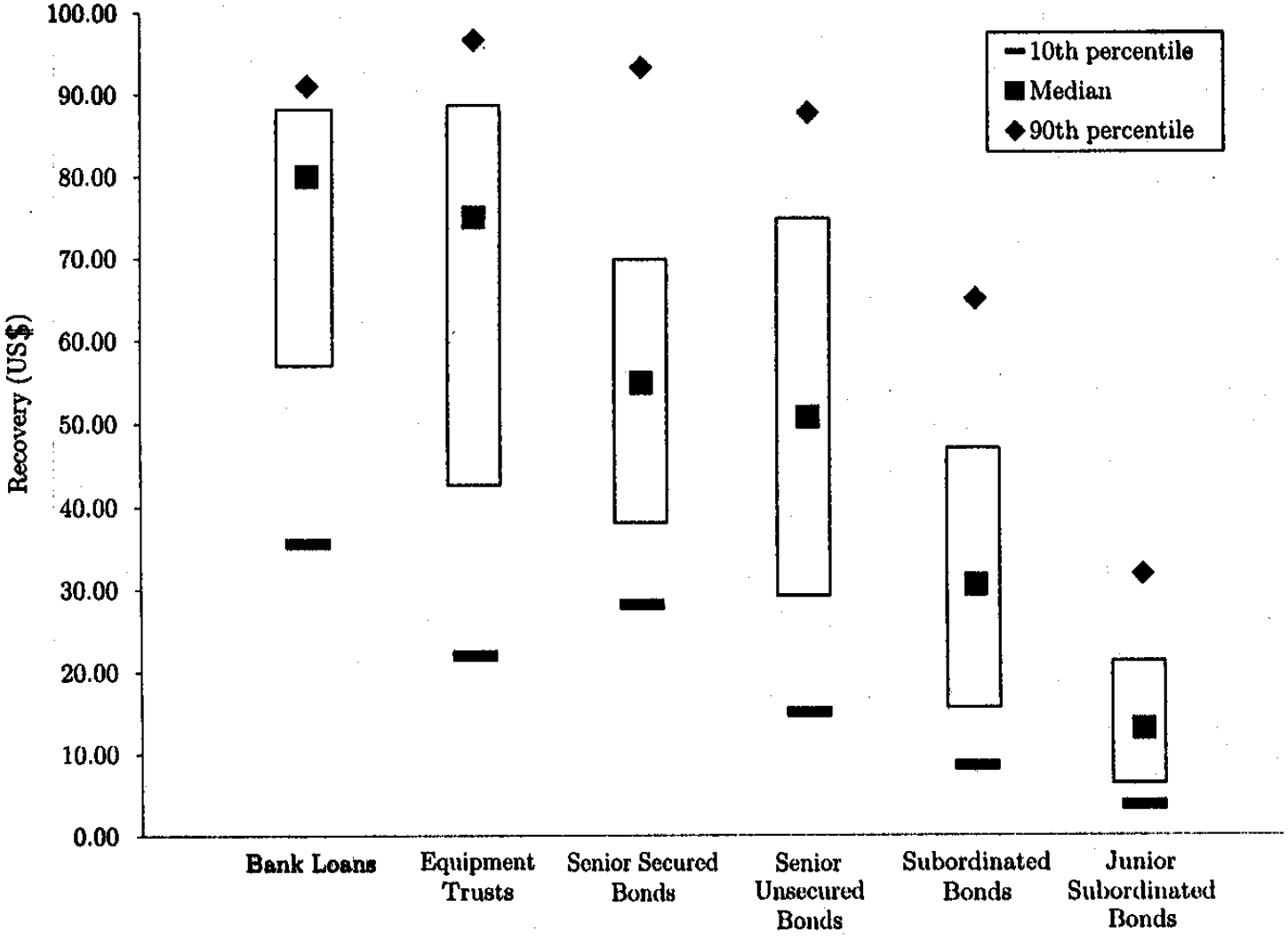
Recovery rates

- Averages across a large sample of different categories and industries.
- Estimated from other variables.

Timing of default is difficult to be predicted



The price of an Enron Corporation bond issued in 1997 and with maturity data 15 November 2005. The coupon rate was 6.625%. The Chapter 11 filing was recorded on 2 December 2001. (Source: Datastream.)



Distributions of recovery by seniority. (Source: Moody's.)

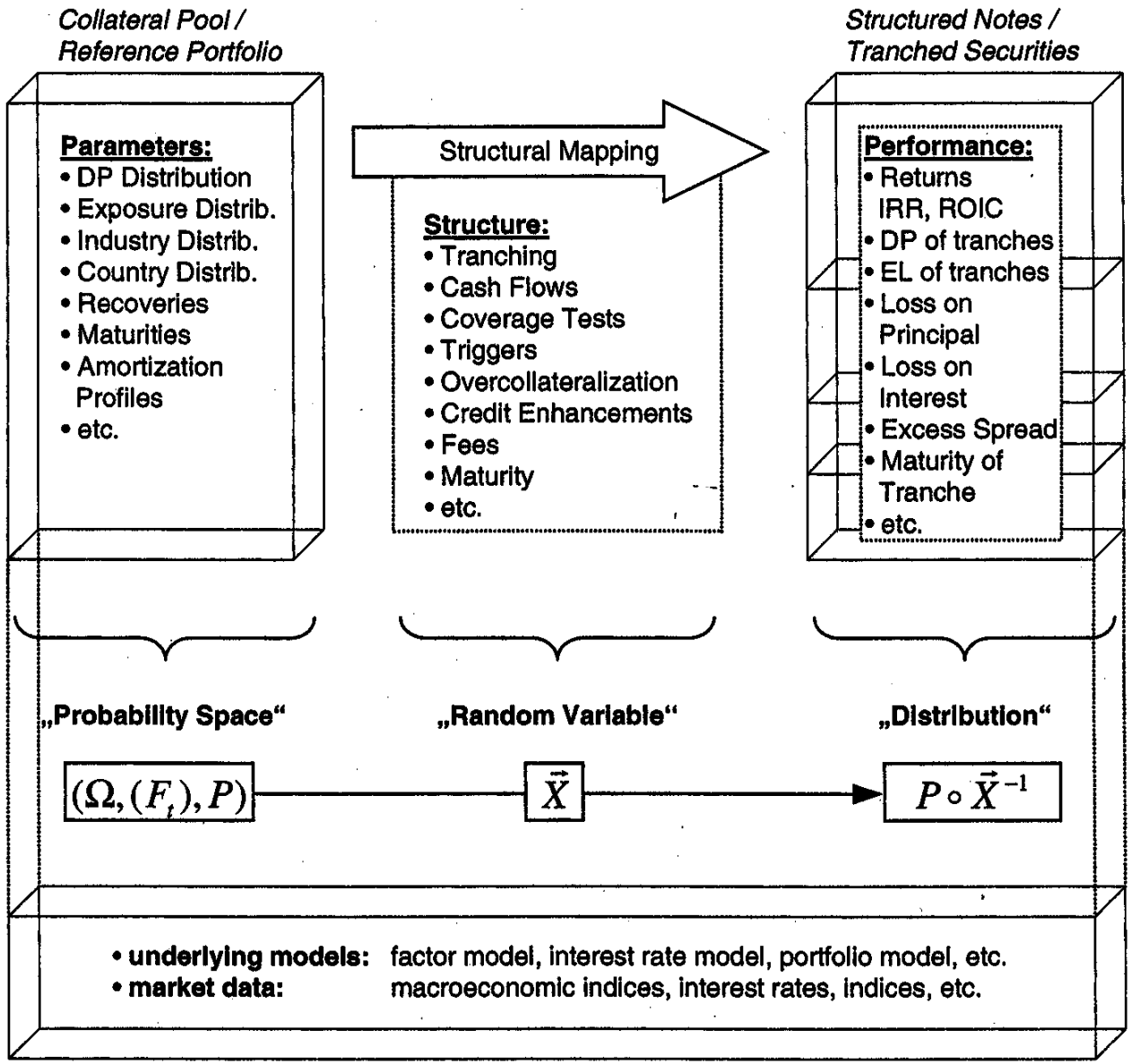
Measuring recovery on defaulted debt is difficult

- For traded debts, as a proxy for recovery, Moody's uses the trading price of the defaulted debt, expressed as a percentage of par and measured approximately one month after default.
- In principle, one would like to discount the eventual actual pay-outs to bondholders, net of procedural costs, back to the date of default.

Calibration of the default correlation structure of the risky assets is even more challenging

Scenario transformation approach

- Simulation of a default times vector for the portfolio underlying the transaction yields a so-called "asset-side scenario".
- Every single asset-side scenario is transformed into a so-called "liability-side scenario" by application of the structural definition of the transaction.
- All liability-side scenarios are aggregated to certain cash flow, return or loss distributions.



CDO modeling scheme.

CDOs from the Modeling Point of View

1. Constructing a model for the underlying portfolio. Such a model should include

   - multi-year horizons due to maturities longer than one year,
   - a sound factor model for measuring industry and country diversification in an appropriate manner, and
   - a model for short term interest rates for capturing the interest rate risk of floating rate securities and notes.

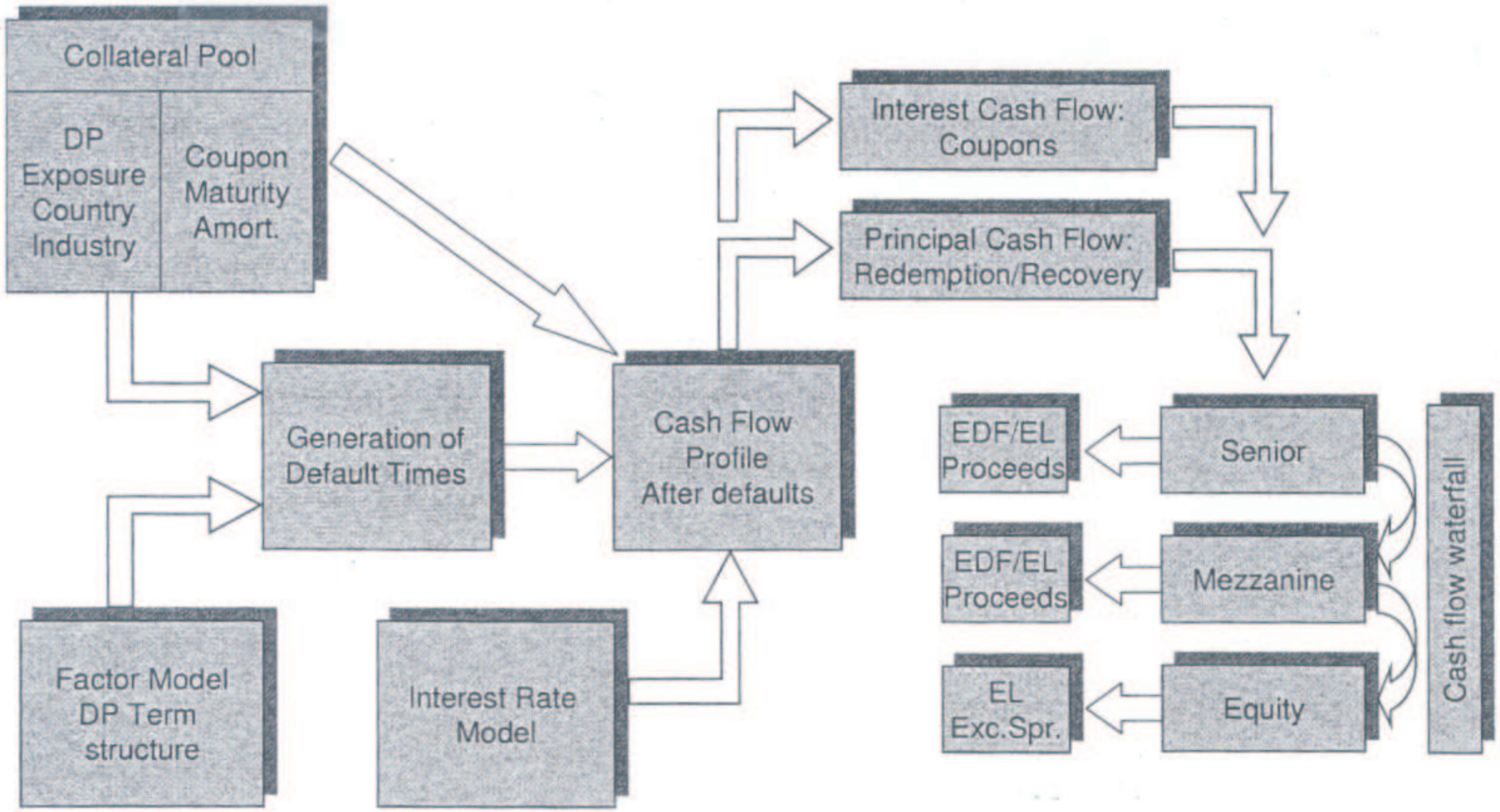
The difficulties lie on the generation of correlated default times of the collateral securities.

2 Modeling the cash flows of the structure

The cash flows of the structure *conditioned* on the simulated scenario from the portfolio model representing the performance of the collateral securities should be modeled by taking all cash flow elements of the structure, including

- subordination structure,
- fees and hedge premiums,
- principal and interest waterfalls,
- coverage tests,
- credit enhancement (e.g. overcollateralization),
- triggers (e.g. early amortization, call options), etc.

CDO modeling workflow based on default times.



Stylized CDOs

Consider a portfolio with m firms and define

$$\begin{aligned} Y_{t,i} &= \text{default indicator process of firm } i \\ &= \begin{cases} 1 & \text{if firm } i \text{ has defaulted up to time } t \\ 0 & \text{otherwise} \end{cases} . \end{aligned}$$

The cumulative loss

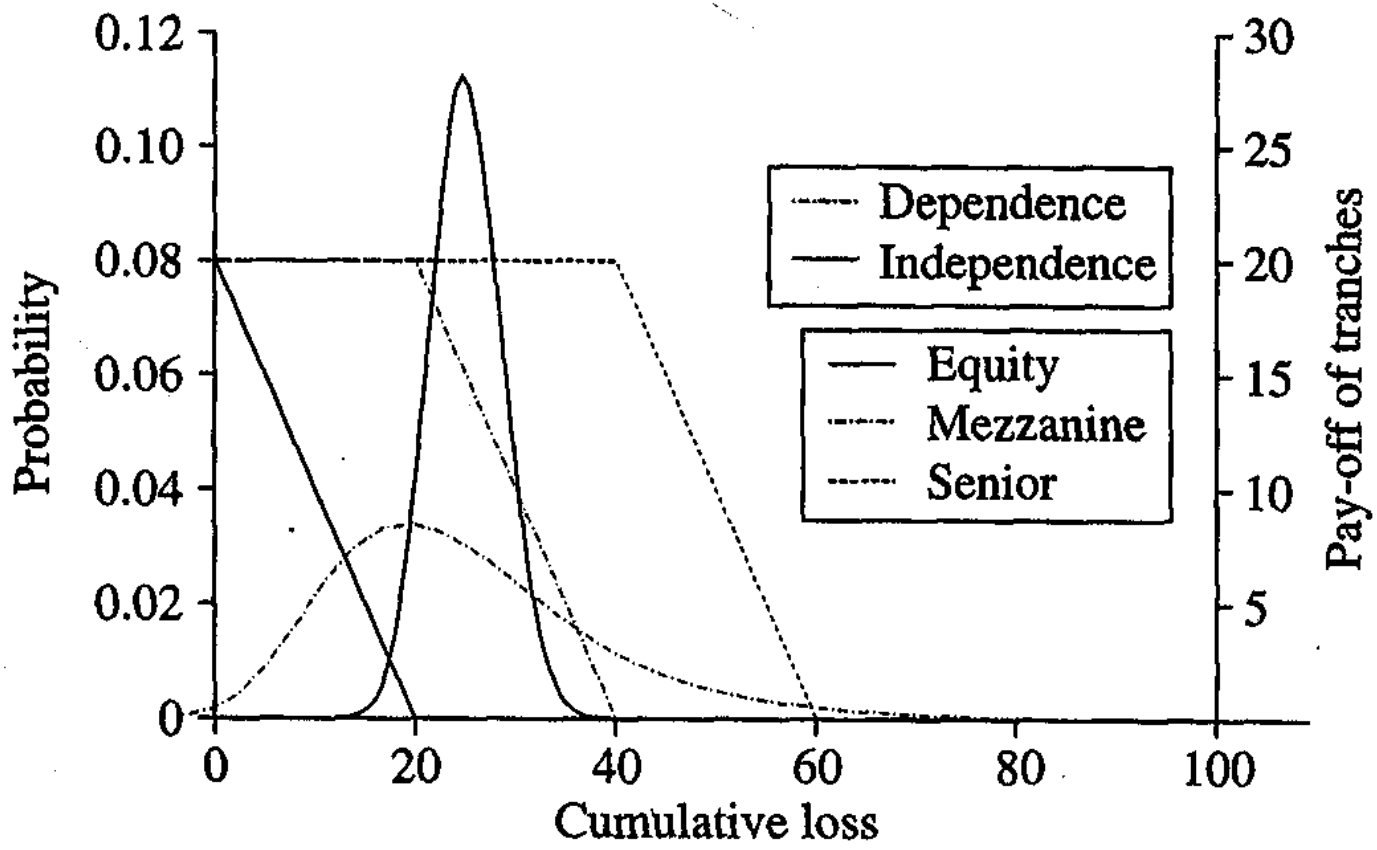
$$L_t = \sum_{i=1}^{m} \delta_i e_i Y_{t,i}$$

where e_i is the exposure of firm i and δ_i is the corresponding loss fraction.

We consider a CDO with k tranches, indexed by $k \in \{1, \cdots, k\}$, and characterized by *attachment points* $0 = K_0 < K_1 < \cdots < K_k \leq \sum_{i=1}^{m} e_i$. Initially, the notional is equal to $K_k - K_{k-1}$; it is reduced whenever there is a default event such that the cumulative loss falls in the layer $[K_{k-1}, K_k]$. The notional of tranche k at time t, $N_k(t)$, is given by

$$N_k(t) = f_k(L_t) \quad \text{with } f_t(\ell) = \begin{cases} K_k - K_{k-1}, & \text{for } \ell < K_{k-1}, \\ K_k - \ell & \text{for } \ell \in [K_{k-1}, K_k], \\ 0, & \text{for } \ell > K_k. \end{cases}$$

Note that f_k can be written more succinctly as $f_k(\ell) = (K_k - \ell)^+ - (K_{k-1} - \ell)^+$, i.e. the notional is equal to the sum of a long position in a put option on L_t with strike price K_k and a short position in a put with strike price K_{k-1}. Such positions are sometimes called a *put spread*.



Pay-off of a stylized CDO contract and distribution of the one-year loss L_1 for a default probability of 0.5% and different default correlations.

$m = 1,000$ firms, each with exposure one unit.

$K_1 = 20, K_2 = 40, K_3 = 60$, corresponding to 2%, 4% and 6% of the overall exposure.

Independent defaults

- L_1 is typically close to its mean due to diversification effects within the portfolio. Hence it is quite unlikely that a tranche k with lower attachment point K_{k-1} substantially larger than $E(L_1)$ the senior tranche in the Figure) suffers a loss, so the value of such a tranche is quite high.

- Since the attachment point K_1 on the equity tranche is typically lower than $E(L_1)$, it is quite unlikely that L_1 is substantially smaller than K_1, and the value of the equity tranche is low.

- If defaults are (strongly) dependent, diversification effects in the portfolio are less pronounced.

- Realizations with L_1 bigger than the lower attachment point K_2 of the senior tranche are more likely, as are realizations with L_1 smaller than the upper attachment point K_1 of the equity tranche.

- The reduces the value of tranches with high seniority and increases the value of the equity tranche compared with the case with (almost) independent defaults.