Forward contracts and futures

A forward is an agreement between two parties to buy or sell an asset at a pre-determined future time for a certain price.

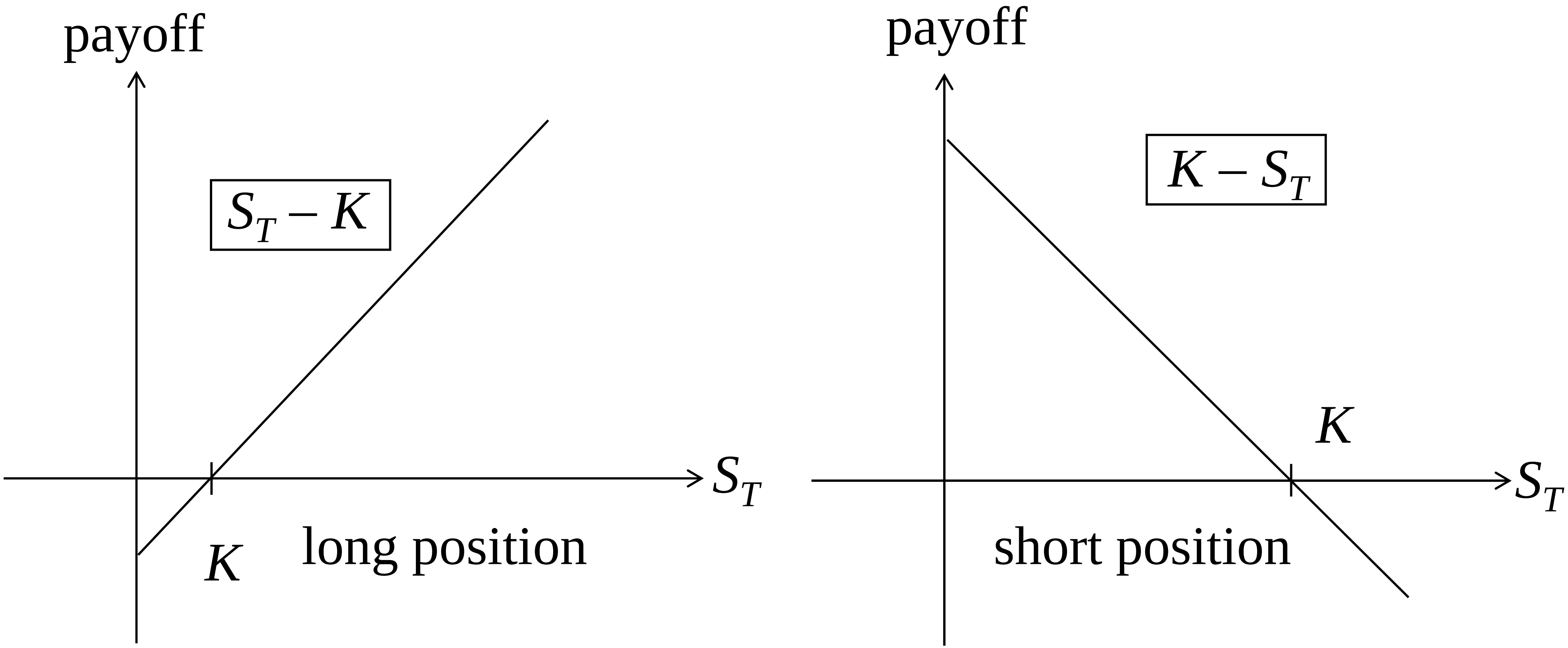
Goal To hedge against the price fluctuation of commodity.

- Intension of purchase decided earlier, actual transaction done later.

- The forward contract needs to specify the delivery price, amount, quality, delivery date, means of delivery, etc.

Potential default of either party: writer or holder.

Terminal payoff from forward contract



K = delivery price, S_T = asset price at maturity

Zero-sum game between the writer (short position) and owner (long position).

Since it costs nothing to enter into a forward contract, the terminal payoff is the investor's total gain or loss from the contract.

Forward price for a forward contract is defined as the delivery price which make the value of the contract at initiation be zero.

Question

Does it relate to the expected value of the commodity on the delivery date?

$$\begin{aligned} &\text{Forward price} \\ = \ &\text{spot price} + \underbrace{\text{cost of fund} + \text{storage cost}}_{\text{cost of carry}} \end{aligned}$$

Example

- Spot price of one ton of wood is $10,000
- 6-month interest income from $10,000 is $400
- storage cost of one ton of wood is $300

6-month forward price of one ton of wood
= $10,000 + 400 + $300 = $10,700.

Explanation

Suppose the forward price deviates too much from $10,700, the construction firm would prefer to buy the wood now and store that for 6 months (though the cost of storage may be higher).

The writer of the contract should be compensated by the cost of carry of the commodity for the 6-month period.

No arbitrage principle

Arbitrage means the realization of a guaranteed risk free profit with a trade or series of trades in the market.

• Take a position in the market, which has zero net cost; but guarantee
  (i) no losses in the future
  (ii) some chance of making profit.

Arbitrage opportunities should disappear quickly in an efficient and frictionless market.

No arbitrage principle enforces price.

Example of an arbitrage opportunity

- spot price of oil is US$19
- quoted 1-year forward price of oil is US$25
- 1-year US dollar interest rate is 5% pa
- storage cost of oil is 2% pa

Any arbitrage opportunity? *Yes*

Sell the forward and expect to receive US$25 one year later.
Borrow $19 now to acquire oil, pay back $19 (1 + 0.05) = $19.95 a year later. Also, need to spend $0.38 as storage cost.

Total cost = $20.33 < $25 to be received.

Close out all positions by delivering the oil to honor the forward.
At maturity of the forward contract, guaranteed riskless profit = $4.67.

Price and value of a forward

July 1: October forward price of silver is quoted at $30 (delivery price)

July 8: new quoted price of same October forward becomes $35

Investor needs to pay nothing to enter into a forward contract. The October forward entered on July 1 earlier now has *positive value* since the new forward price has increased to $35.

Imagine while holding July 1 forward, he can short another forward of October delivery on July 8. He is secured to receive $35 – $30 = $5 on the delivery date.

* Forward price and delivery price are the same initially, but forward price is liable to change due to price fluctuations of underlying asset.

* Forward contracts are traded over-the-counter, no money changes hand initially and during the life time of the contract.

Value and price of forward contract

No intermediate settlement is required. We write

$f(S, \tau)$ = value of forward, $F(S, \tau)$ = forward price,

τ = time to expiration,

S = spot price of the underlying asset.

Further, we let

$D(\tau)$ = cash discount factor over the remaining life of the forward

$G(\tau)$ = cash growth factor over the remaining life of the forward = $1 / D(\tau)$.

Remark

If the interest rate r is constant and interests are compounded continuously, then $G(\tau) = e^{r\tau}$.

Portfolio A: long one forward and cash amount $KD(\tau)$
Portfolio B: one unit of underlying asset

Both portfolios grow to become one unit of asset at maturity. Forward value is given by

$$\boxed{f = S - KD(\tau)}$$

The forward price is defined to be the delivery price which makes $f = 0$, so $K = SG(\tau)$. Hence, forward price is given by

$$\boxed{F(S, \tau) = SG(\tau)}$$

Suppose $F > SG(\tau)$, an arbitrageur borrows S dollars, buy the asset and short one forward; will receive F dollars under the forward contract and repay the loan of $SG(\tau)$. The arbitrage profit is

$$F - SG(\tau).$$

The presence of replicating strategy dictates the forward price. The forward price is not given by the expected value of the asset price at maturity.

Discrete dividend paying asset

Suppose the underlying asset pays dividends.

$D =$ present value of all dividends received from holding the asset during the life of forward

then $$f = S - [D + KD(\tau)]$$

so that

$$F = (S - D)G(\tau).$$

Cost of carry

Additional costs to hold the commodities, like storage, insurance, deterioration, etc. These can be considered as negative dividends. Treating U as $-D$, we obtain

$$F = (S + U)e^{r\tau},$$

U = present value of total cost during remaining life of the forward.

Suppose the cost are paid continuously, we have

$$F = Se^{(r+u)\tau},$$

u = cost per annum as a proportion of spot price.

In general, $F = Se^{b\tau}$, where b is the cost of carry.

Futures contracts

A futures contract is a legal agreement between a buyer (seller) and an **established exchange** or its clearing house in which the buyer (seller) agrees to take (make) delivery of a financial entity at a specified price at the end of a designated period of time. Usually the exchange specifies certain standardized features.

Futures price – price at which the parties agree to transact in the future.

Settlement or delivery date – designated date at which the parties must transact

Marking to market the account

Pay or receive from the writer the change in the future price through the *margin account* so that payment required on the maturity date is simply the spot price on that date.

Role of the clearinghouse

Guarantee function

To guarantee that the two parties of the futures must honor the transaction – eliminate the **counterparty risk** through the margin account.

Trading platform

Provide the **platform** for parties of a futures contract to unwind their position prior to the settlement date.

Margin requirements

Initial margin – paid at inception as deposit for the contract.

Maintenance margin – minimum level before the investor is required to deposit additional margin.

Variation margin – amount required to bring the equity back to its initial margin level.

For example, today's futures price for gold (Dec. contract) is $400 per ounce, and the contract size is 200 ounces. At the end of this trading day, the futures price has dropped from $400 to $397. The investor has a loss of $\$3 \times 200 = \$600$. This amount will be taken away from investor's margin account.

Credit risk is limited to one-day performance period

Difference in payment schedules may lead to difference in futures and forward prices since different interest rates are applied on intermediate payments.

Hang-Seng index

market capitalization weighted index

$$\text{index}_{\text{current}} = \text{index}_{\text{previous}} \times \frac{\text{current market cap of the stock basket}}{\text{previous market cap of the stock basket}}$$

market capitalization of stock i is $M_i = w_i \times P_i$,
w_i = number of shares outstanding for the stock
P_i = current stock price

33 blue chip stocks are included in the basket:
4 stocks in banking & finance;
4 stocks in public utilities;
11 stocks in real estate;
14 stocks in commerce & industry.

Hang Seng index futures

- The underlying is the Hang Seng index.
- One index point corresponds to $50.
- Contracts are available for the present month, next month, and the two closest quarter months (March, June, September, December).

Settlement

- On the last but one trading day at the end of the month.
- Take the average of the index value at every 5-minute interval as the settlement value.

Index futures arbitrage

If $F > SG(\tau)$, profits can be made by buying the basket of stocks that underlying the index and shorting the futures contract.

Difficulties in actual implementation

1. Require significant amount of capital
   e.g. Shorting 2,000 Hang Seng index futures requires the purchase of one *billion* worth of stock.

2. Timing risk
   Stock prices move quickly, there exist lags in the buy in and buy out processes.

3. Settlement
   The unwinding of positions must be done in 47 steps on the settlement date.

4. Stocks must be bought or sold in board lot. One can only approximate the proportion of the stocks in the index calculation formula.

5. Dividend dates and dividend amounts are uncertain. Note that dividends cause the stock price to drop and affect the index value.

6. Transaction costs in the buy in and buy out; and interest losses in the margin account.

Currency forward

- S be the domestic currency price of one unit of foreign currency
- K be the delivery price (in domestic currency) of the currency forward
- f be the value of the currency forward (in domestic currency)
- r_d and r_f are the domestic and foreign riskless interest rates (assumed to be constant)
- τ is the time to expiry of the currency forward

Portfolio A: long one currency forward and domestic currency of amount $Ke^{-r_d \tau}$

Portfolio B: $Ke^{-r_f \tau}$ units of foreign currency

Interest Rate Parity Relation

On the maturity date of the forward, both portfolios worth one unit of foreign currency. Their present values should be the same so

$$f + Ke^{-rd\tau} = Se^{-rf\tau}.$$

The delivery price *K* is set so that the value of the currency forward at initiation is zero. Hence, the forward price is

$$K = Se^{(r_d - r_f)\tau}.$$

Price of a currency forward

Here, $r_d - r_f$ is the cost of carry of holding the foreign currency.

Let $B_d(\tau)$ and $B_f(\tau)$ denote the domestic currency and foreign currency price of a riskless domestic and foreign bond with unit par, respectively. Then, the price of currency forward is

$$F = S\,\frac{B_f(\tau)}{B_d(\tau)}.$$

Non-deliverable forwards

Synthetic foreign currency forward contracts on non-convertible currencies or thinly traded currencies.

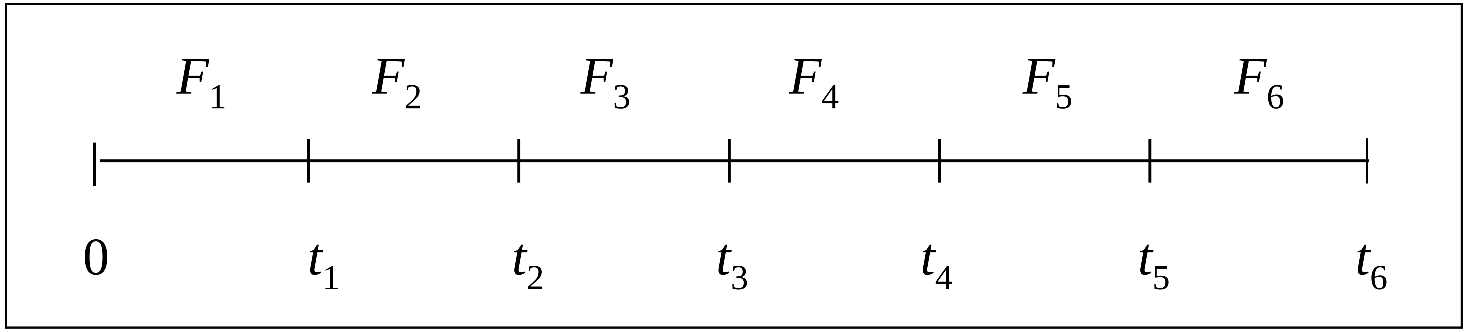
To hedge or take exposure to movements of currency markets without actually dealing in the underlying.

Cash settlement in domestic dollars at maturity.

- The demand arises principally out of regulatory and liquidity issues in the underlying currency.
- For example, overseas players are essentially barred from access to the domestic New Taiwan dollar spot and forward markets. The NDF does not involve any buying, selling, borrowing or lending of the New Taiwan dollars.

American currency forward (HSBC product)

Consider a 6-month forward contract. The exchange rate over each one-month period is preset to assume some constant value.



The holder can exercise parts of the notional at any time during the life of the forward, but she has to exercise all by the maturity date of the currency forward.

Questions

1. What should be the optimal exercise policies adopted by the holder?

2. How to set the predetermined exchange rates so that the value of the American currency forward is zero at initiation?

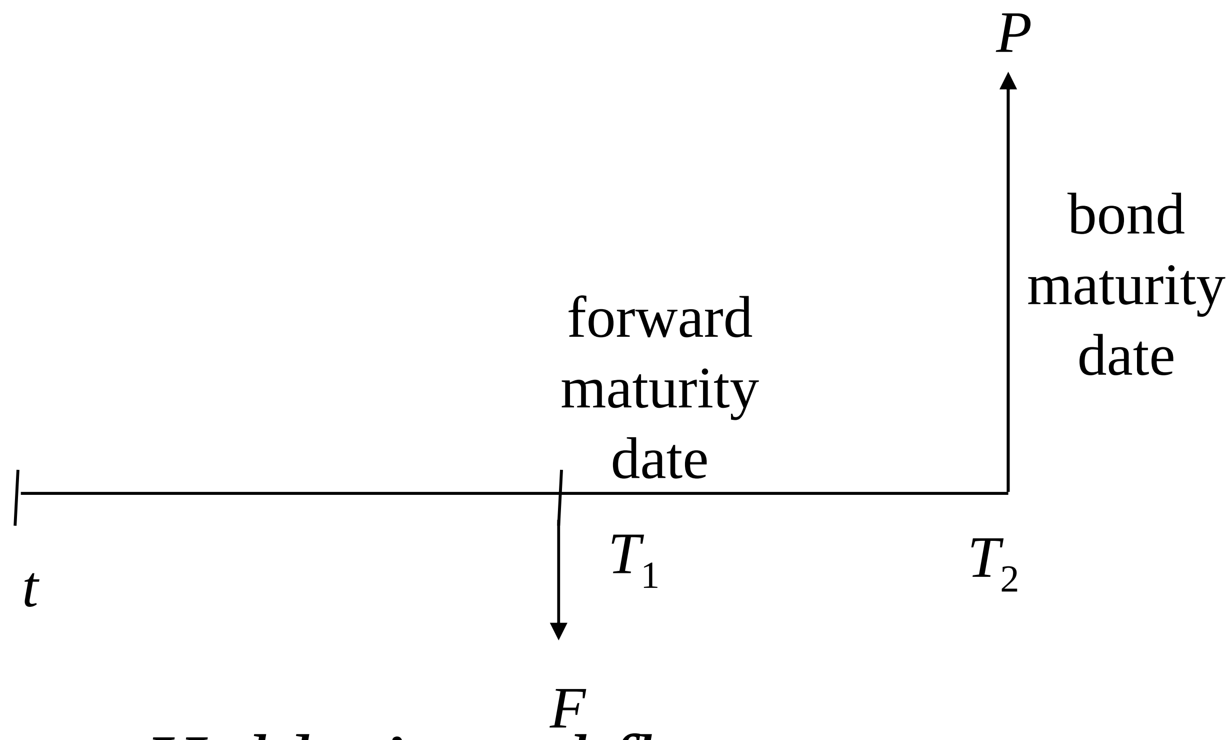
Pricing considerations

- The critical exchange rate $S^*(\tau)$ is independent of the amount exercised. Hence, when S reaches $S^*(\tau)$, the whole should be exercised (though the holder may not have the whole notional amount of foreign currency available).
- Set $F_j = F_i e^{(r_d - r_f)(j-1)\Delta t}, j = 2, 3, \cdots, 6;$ this is because the forward price grows by the factor $e^{(r_d - r_f)\Delta t}$ over each Δt time interval.

Determine F_1 such that the value of the American currency forward at initiation is zero.

Bond forward

The underlying asset is a zero-coupon bond of maturity T_2 with a settlement date T_1, where $t < T_1 < T_2$.



Holder's cashflows

The holder pays the delivery price F of the bond forward on forward maturity date T_1 to receive a bond with par value P on maturity date T_2.

Bond forward price in terms of traded bond prices

Let $B_t(T)$ denote the traded bond price at current time t with maturity date T.

Present value of the net cashflows

$$= -F\,B_t(T_1) + P\,B_t(T_2).$$

To determine the forward price F, we set the above value zero and obtain

$$\mathrm{F} = P\,B_t(T_2) \,/\, B_t(T_1).$$

Forward interest rate

The forward price should be related to the forward interest rate $R(t;\ T_1,\ T_2)$. The forward rate is the interest rate determined at the current time t which is applied over the future period $[T_1, T_2]$.

$$F[1 + R(t; T_1, T_2)(T_2 - T_1)] = P$$

so that

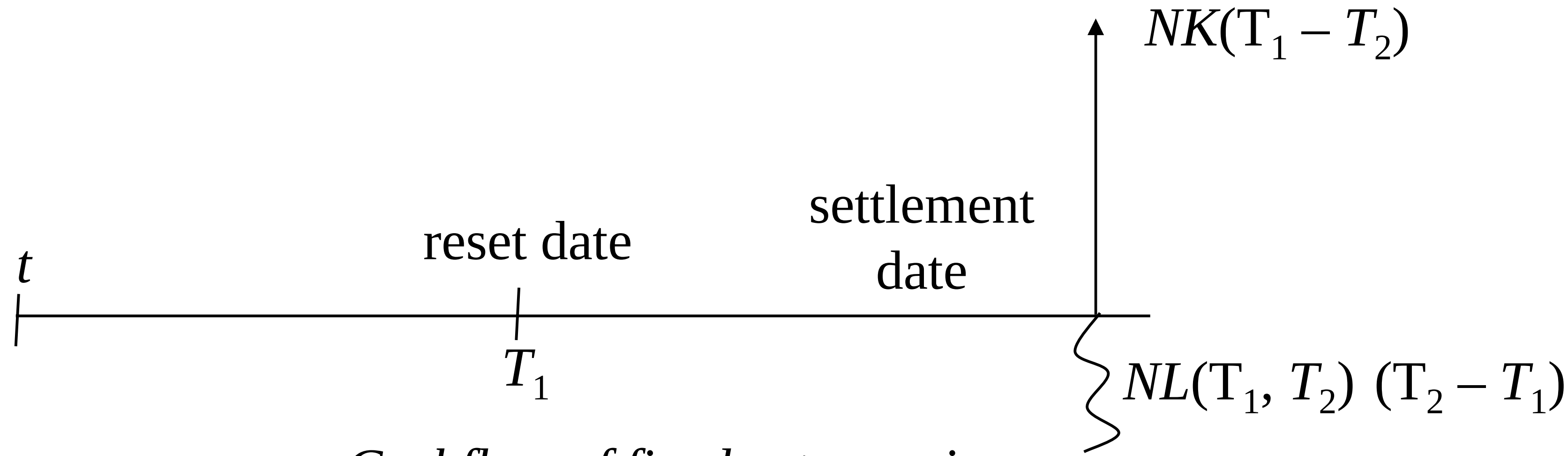
$$R(t; T_1, T_2) = \frac{1}{T_2 - T_1}\left[\frac{B_t(T_1)}{B_t(T_2)} - 1\right].$$

Forward rate agreement

$L[T_1, T_2]$ = LIBOR rate observed at T_1

for accrual period $[T_1, T_2]$

K = fixed rate



Cashflow of fixed rate receiver

FRA is an agreement between two counterparties to exchange floating and fixed interest payments on future settlement date T_2. The floating rate will be LIBOR rate $L[T_1, T_2]$ as observed on the future reset date T_1.

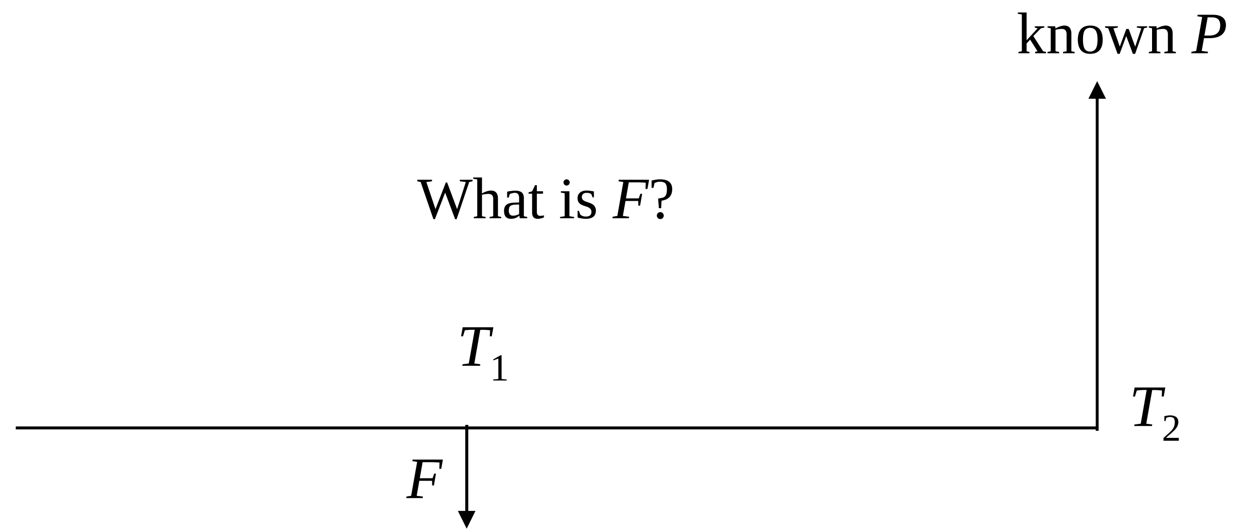
To the fixed rate receiver, she pays N at T_1, which becomes $NL[T_1, T_2](T_2 - T_1)$ at time T_2, and receives fixed interest of amount $NK(T_2 - T_1)$ at time T_2.

The cashflows of the fixed rate payer can be replicated by

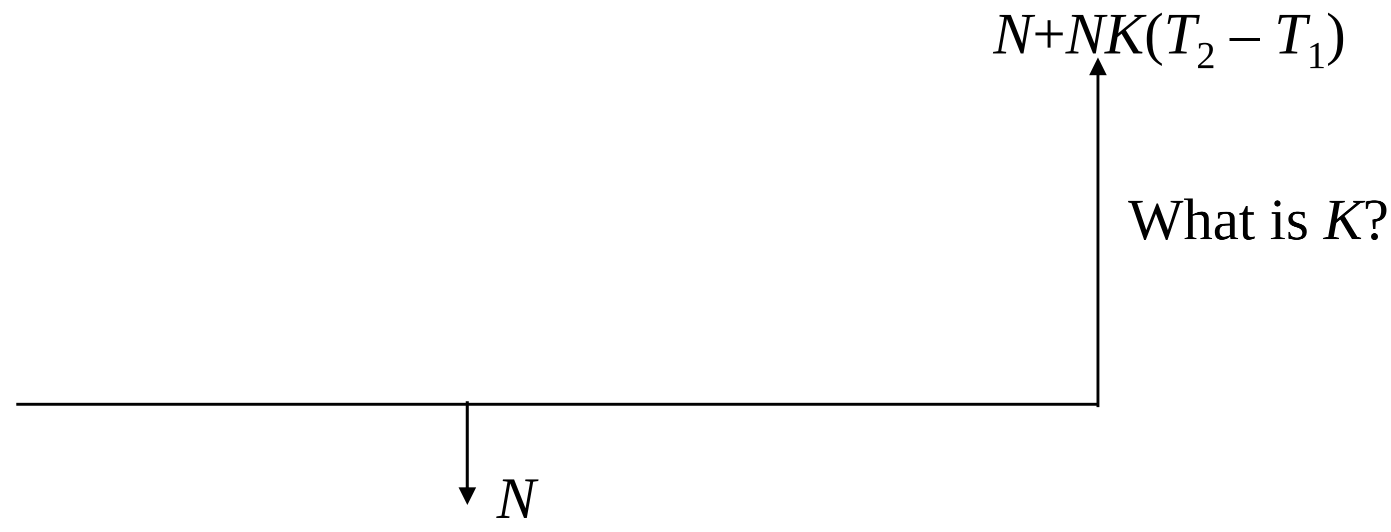
(i) long holding of $N[1 + K(T_2 - T_1)]$ units of T_2-maturity zero coupon bond with unit par

(ii) short holding of N units of T_1-maturity zero coupon bond with unit par.

Comparison between forward contract and FRA



forward contract – determination of F



forward rate agreement – determination of K

Value of the replicating portfolio at the current time

$= N[1 + K(T_2 - T_1) B_t(T_2) - B_t(T_1)].$

We find K such that the above value is zero.

$$K = \underbrace{\frac{1}{T_2 - T_1}\left[\frac{B_t(T_1)}{B_t(T_2)} - 1\right]}_{\text{forward rate over } [T_1,\, T_2]}.$$

K is the expectation of the LIBOR rate $L[T_1, T_2]$ over the time period $[T_1, T_2]$.

Equality of futures and forward price under constant interest rate

Let F_i and G_i denote the forward price and futures price at the end of i^{th} day respectively.

δ = constant interest rate per day

gain/loss of futures on i^{th} day = $G_i - G_{i-1}$ and will grow to $(G_i - G_{i-1})\, e^{\delta(n-i)}$ at maturity.

Suppose investor keeps changing the amount of futures held, say, α_i units at the end of $(i - 1)$th day. The portfolio value is given by

$$\Pi = \sum_{i=1}^{n} \alpha_i (G_i - G_{i-1}) e^{\delta(n-i)}.$$

Portfolio A: long $F_0\, e^{-\delta n}$ bond and

long unit of forward contract

Portfolio B: long $G_0 e^{-\delta n}$ bonds and

long $e^{-\delta(n-i)}$ units of futures on the i^{th} day

(decrease the long position day by day)

value of portfolio $A = F_0 + S_n - F_0 = S_n$ = asset price on n^{th} day

value of portfolio B $= G_0 + \sum_{i=1}^{n} e^{-\delta(n-i)}(G_i - G_{i-1})e^{\delta(n-i)}$

$$= G_0 + \sum_{i=1}^{n} G_i - G_{i-1} = G_0 G_n - G_0 = G_n$$

but $G_n = S_n$ since futures price = asset price at maturity.

Both portfolios have the same value at maturity so same value at initiation. This gives $F_0 = G_0$.