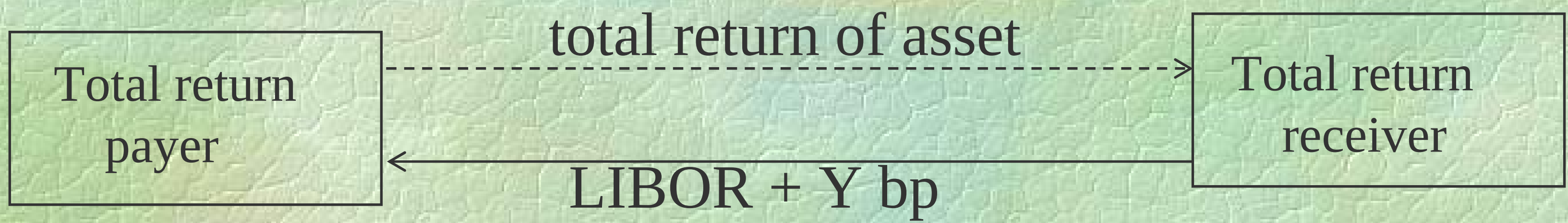
Total return swap

- Exchange the total economic performance of a specific asset for another cash flow.



Total return comprises the sum of interests, fees and any change-in-value payments with respect to the reference asset.

A commercial bank can hedge all credit risk on a loan it has originated. The counterparty can gain access to the loan on an off-balance sheet basis, without bearing the cost of originating, buying and administering the loan.

The payments received by the total return receiver are:

1. The coupon $\bar{c}$ of the bond (if there were one since the last payment date T_{i-1})
2. The price appreciation $\left(\overline{C}(T_i) - \overline{C}(T_{i-1})\right)^+$ of the underlying bond C since the last payment (if there were only).
3. The recovery value of the bond (if there were default).

The payments made by the total return receiver are:

1. A regular fee of LIBOR + s^{TRS}
2. The price depreciation $\left(\overline{C}(T_{i-1}) - \overline{C}(T_i)\right)^+$ of bond C since the last payment (if there were only).
3. The par value of the bond C if there were a default in the meantime).

The coupon payments are netted and swap's termination date is earlier than bond's maturity.

Some essential features

1. The receiver is synthetically long the reference asset without having to fund the investment up front. He has almost the same payoff stream as if he had invested in risky bond directly and funded this investment at LIBOR + s^{TRS}.
2. The TRS is marked to market at regular intervals, similar to a futures contract on the risky bond. The reference asset should be liquidly traded to ensure objective market prices for making to market (determined using a dealer poll mechanism).
3. The TRS allows the receiver to leverage his position much higher than he would otherwise be able to (may require collateral). The TRS spread should not be driven by the default risk of the underlying asset but also by the credit quality of the receiver.

Alternative financing tool

- The receiver wants financing to invest $100 million in the reference bond. It approaches the payer (a financial institution) and agrees to the swap.
- The payer invests $100 million in the bond. The payer retains ownership of the bond for the life of the swap and has much less exposure to the risk of the receiver defaulting.
- The receiver is in the same position as it would have been if it had borrowed money at LIBOR + s^{TRS} to buy the bond. He bears the market risk and default risk of the underlying bond.

Motivation of the receiver

1. Investors can create *new assets* with a *specific maturity* not currently available in the market.
2. Investors gain *efficient off-balance sheet exposure* to a desired asset class to which they otherwise would not have access.
3. Investors may achieve a *higher leverage on capital* – ideal for hedge funds. Otherwise, *direct asset ownership* is on *on-balance sheet* funded investment.
4. Investors can *reduce administrative costs* via an off-balance sheet purchase.
5. Investors can *access entire asset classes* by receiving the total *return on an index*.

Motivation of the payer

The payer creates a hedge for both the *price risk* and *default risk* of the reference asset.

* A long-term investor, who feels that a reference asset in the portfolio may *widen in spread* in the *short term* but will recover later, may enter into a total return swap that is shorter than the maturity of the asset. This structure is *flexible* and *does not require a sale of the asset* (thus accommodates a temporary *short-term negative view* on an asset).

What would be the difference on the cost to the TRS receiver comparing with an outright purchase?

- The funding cost above LIBOR for the receiver in an outright purchase will be somewhat reflected in the credit spread demanded in the fee stream LIBOR + Ybp.

- Another source of value difference lies in the marking-to-market of the TRS.

In an outright purchase, the adjustment in the price of the defaultable bound at T_N and T_O is

$$\overline{C}(T_N) - \overline{C}(T_O) = \sum_{i=1}^{N} \left[\overline{C}(T_i) - \overline{C}(T_{i-1}) \right].$$

Due to marking-to-market mechanism, $\left[\overline{C}(T_i) - \overline{C}(T_{i-1})\right]$ is paid at T_i instead of T_N. The extra cost due to difference in value of this adjustment at T_i is

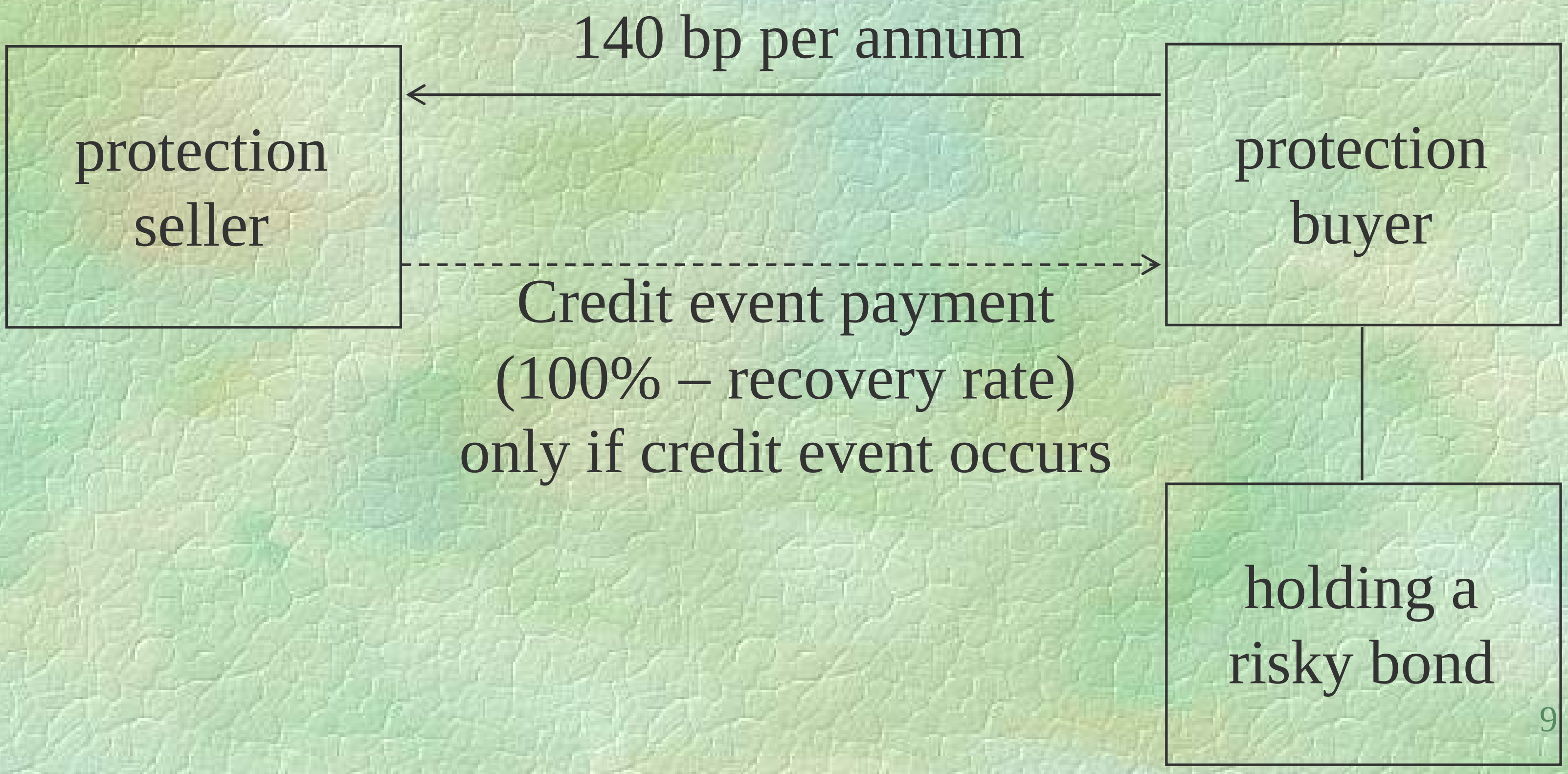
$$\overline{C}(T_i) - \overline{C}(T_{i-1}) = \left[1 - B(T_i, T_N)\right].$$

Rule of thumb

Bonds that are initially trade at a discount at par should command a positive TRS spread since $\left[\overline{C}(T_i) - \overline{C}(T_{i-1})\right]$ has a higher chance to be positive.

Credit default swaps

The protection seller receives fixed periodic payments from the protection buyer in return for making a single contingent payment covering losses on a reference asset following a default.



Protection seller

- earns investment income with no funding cost
- gains customized, synthetic access to the risky bond

Protection buyer

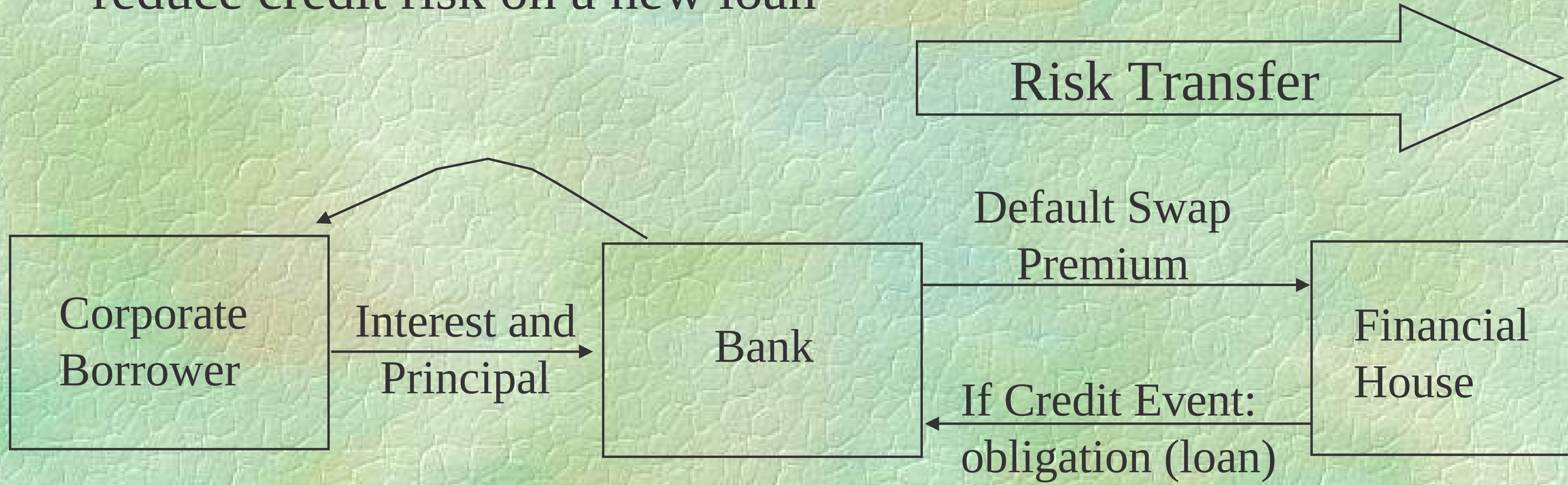
- hedges the default risk on the reference asset

1. Very often, the bond tenor is longer than the swap tenor. In this way, the protection seller does not have exposure to the full market risk of the bond.
2. *Basket default swap* – gain additional yield by selling default protection on several assets.

A bank lends 10mm to a corporate client at L + 65bps. The bank also buys 10mm default protection on the corporate loan for 50bps.

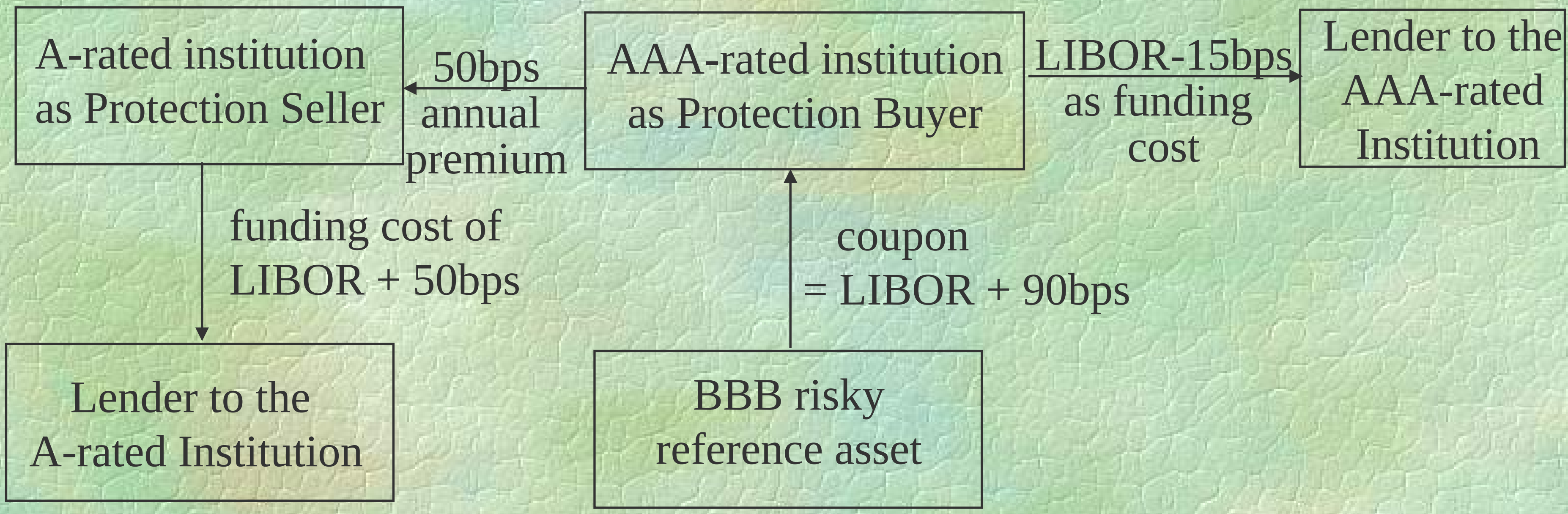
Objective achieved

- maintain relationship
- reduce credit risk on a new loan



Default Swap Settlement following Credit Event of Corporate Borrower

Funding cost arbitrage – Credit default swap



The combined risk faced by the Protection Buyer:

• default of the BBB-rated bond

• default of the Protection Seller on the contingent payment

The AAA-rated Protection Buyer creates a *synthetic AA-asset* with a coupon rate of LIBOR + 90bps – 50bps = LIBOR + 40bps.

This is better than LIBOR + 30bps, which is the coupon rate of a AA-asset (net gains of 10bps).

For the A-rated Protection Seller, it gains synthetic access to a BBB-rated asset with earning of net spread of

50bps – [(LIBOR + 90bps) – (LIBOR + 50bps)] = 10bps

the A-rated Protection Seller earns
40bps if it owns the BBB asset directly

In order that the credit arbitrage works, the funding cost of the default protection seller must be *higher* than that of the default protection buyer.

Example

Suppose the A-rated institution is the Protection buyer, and assume that it has to pay 60bps for the credit default swap premium (higher premium since the AAA-rated institution has lower counterparty risk).

The net loss of spread = (60 – 40) = 20bps.

Valuation of a credit default swap

- Notional principal is $1.
- We assume that default events, interest rates, and recovery rates are mutually independent.
- The claim in the event of default is the face value plus accrued interest.
- Suppose first that default can occur only at times t_1, t_2, …, t_n.

T: Life of credit default swap in years

P_i: Risk-neutral probability of default at time t_i

$\hat{R}$: Expected recovery rate on the reference obligation in a risk-neutral world (this is assumed to be independent of the time of the default)

$u(t)$: Present value of payments at the rate of $1 per year on payment dates between time zero and time t

$e(t)$: Present value of a payment at time t equal to $t - t^*$ dollars, where t^* is the payment date immediately preceding time t (both t and t^* are measured in years

$v(t)$: Present value of \$1 received at time t

w: Payment per year made by credit default swap buyer per dollar

s: Value of w that causes the credit default swap to have a value of zero

π The risk-neutral probability of no credit event during the life of the swap

$A(t)$: Accrued interest on the reference obligation at time t as a percent face value

The value of π is one minus the probability that a credit event will occur.

$$\pi = 1 - \sum_{i=1}^{n} p_i.$$

- The payments last until a credit event or until time T, whichever is sooner. The present value of the payments is therefore

$$w\sum_{i=1}^{n}[u(t_i)+e(t_i)]p_i + w\pi u(T).$$

- If a credit event occurs at time t_i, the risk-neutral expected value of the reference obligation, as a percent of its face value, is $[1+A(t_i)]\hat{R}$. The risk-neutral expected payoff from the CDS is therefore

$$1-[1+A(t_i)]\hat{R} = 1-\hat{R}-A(t_i)\hat{R}.$$

The present value of the expected payoff from the CDS is

$$\sum_{i=1}^{n}[1-\hat{R}-A(t_i)\hat{R}]p_i v(t_i).$$

The value of the credit default swap to the buyer is the present value of the expected payoff minus the present value of the payments made by the buyer:

$$\sum_{i=1}^{n}[1-\hat{R}-A(t_i)\hat{R}]p_i v(t_i)-w\sum_{i=1}^{n}[u(t_i)+e(t_i)]p_i+w\pi u(T).$$

The CDS spread, *s*, is the value of *w* that makes this expression zero:

$$s = \frac{\sum_{i=1}^{n}[1-\hat{R}-A(t_i)\hat{R}]p_i v(t_i)}{\sum_{i=1}^{n}[u(t_i)+e(t_i)]p_i+\pi u(T)}.$$

The variable *s* is referred to as the *credit default swap spread*, or *CDS spread*. It is the payment per year, as a percent of the notional principal, for a newly issued credit default swap.

Numerical example

Suppose that the risk-free rate is 5% per annum with semiannual compounding and that, in a five-year credit default swap where payments are made semiannually, defaults can take place at the end of years 1, 2, 3, 4, and 5. The reference obligation is a five-year bond that pays a coupon semiannually of 10% per year. Default times are immediately before coupon payment dates on this bond.

Assume that the probabilities of default are

$p_1 = 0.0224$, $p_2 = 0.0247$, $p_3 = 0.0269$, $\mathrm{p}_4 = 0.0291$,

$\mathrm{p}_5 = 0.0312$, and $\pi = 0.8657$,

and the expected recovery rate is 0.3. In this case,

$A(t_i) = 0.05$ and $e(t_i) = 0$ for all i.

Also, $v(t_i) = 0.9518$, $v(t_2) = 0.9060$, $v(t_3) = 0.8623$,

$v(t_4) = 0.8207$ and $v(t_5) = 0.7812$, while

$u(t_1) = 0.9637$, $u(t_2) = 1.8810$, $u(t_3) = 2.7541$,

$u(t_4) = 3.5851$, and $u(t_5) = 4.3760$.

The numerator is

$(1 - 0.3 - 0.05 \times 0.03)$

$\times (0.0224 \times 0.9518 + 0.0247 \times 0.9060 + 0.0269 \times 0.8623$
$+ 0.09291 \times 0.8207 + 0.0312 \times 0.7812)$

or 0.0788. The denominator is

$0.0224 \times 0.9637 + 0.0247 \times 1.8810 + 0.0269 \times 2.7541$

$+ 0.0291 \times 3.5851 + 0.0312 \times 4.3760 + 0.8657 \times 4.3760$

or 4.1712. The CDS spread, *s*, is therefore 0.7888/4.1712 = 0.1891, or 189.1 basis points. This means that payments equal to 0.5 × 1.891 = 0.09455% are made every six months.

Supply and demand drive the price

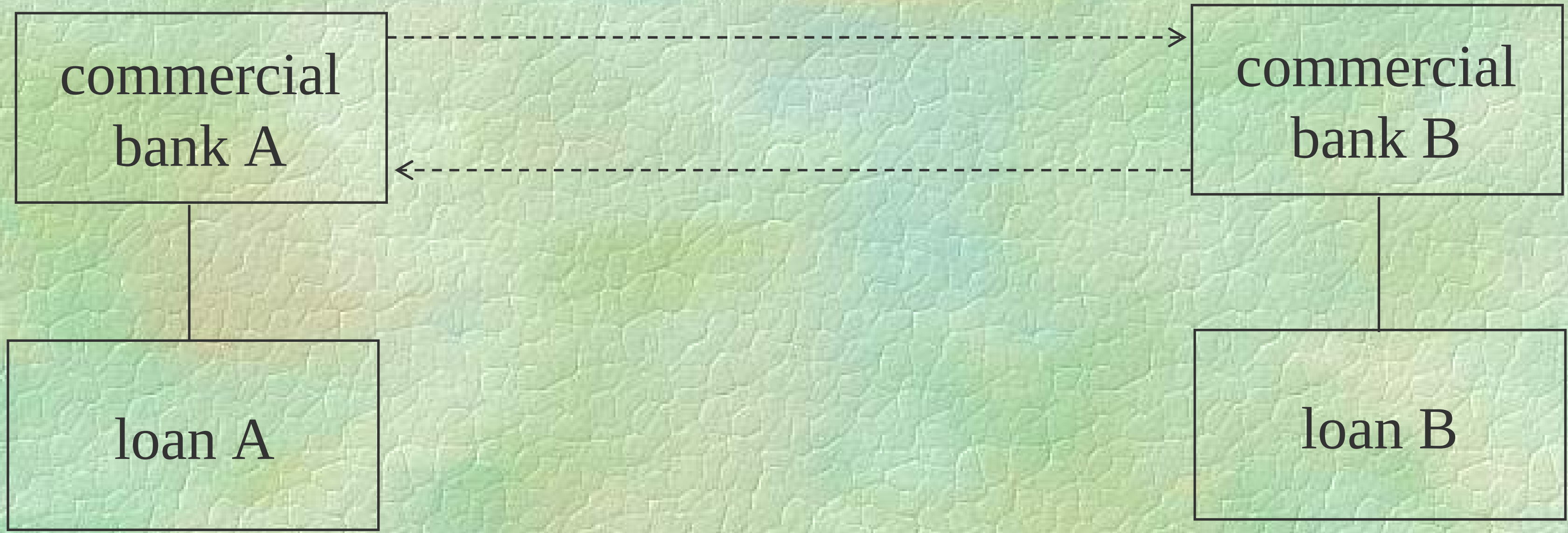
Credit Default Protection Referencing a 5-year Brazilian Eurobond (May 1997)

Chase Manhattan Bank	240bps
Broker Market	285bps
JP Morgan	325bps

* It is very difficult to estimate the recovery rate upon default.

Credit default exchange swaps

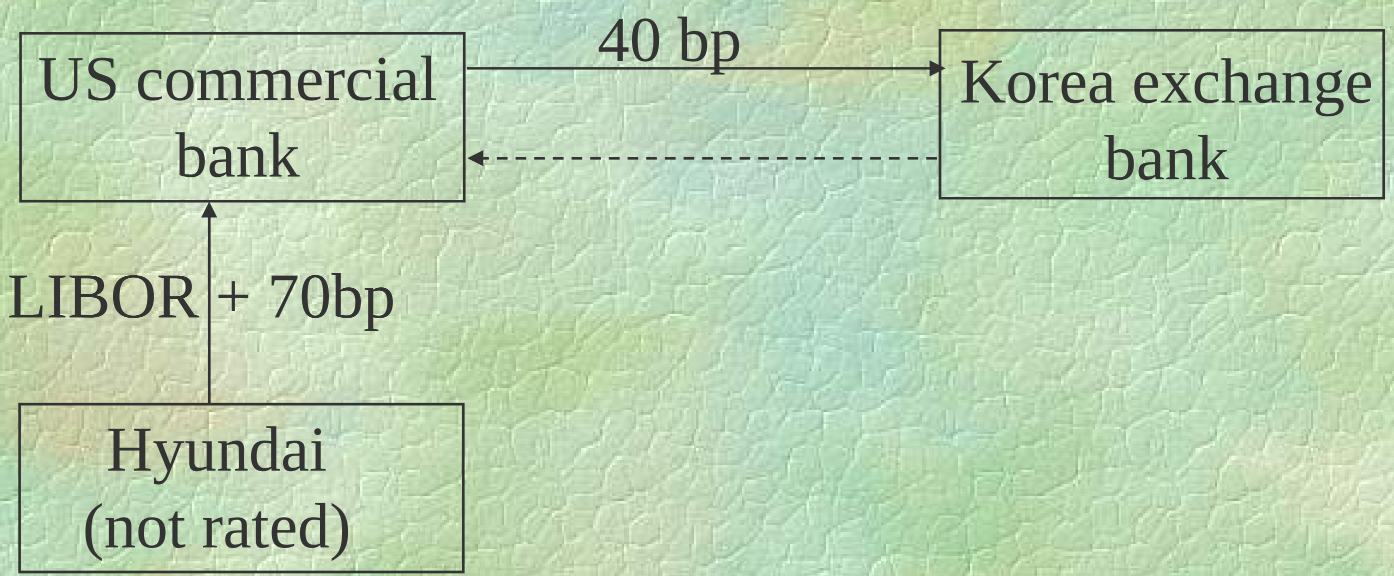
Two institutions that lend to different regions or industries can diversify their loan portfolios in a single non-funded transaction – hedging the concentration risk on the loan portfolios.



* contingent payments are made only if credit event occurs on a reference asset
* periodic payments may be made that reflect the different risks between the two reference loans

Counterparty risk

Before the Fall 1997 crisis, several Korean banks were willing to offer credit default protection on other Korean firms.



* Political risk, restructuring risk and the risk of possible future war lead to potential high correlation of defaults.

Advice: Go for a *European bank* to buy the protection.

- In order that funding cost arbitrage works, the Protection Buyer should have a higher credit rating than the Protection Seller. It is advantageous for the Protection Buyer to hold the risky asset to take advantage of the lower funding cost.
- Before the 1997 crisis in Korea, Korean financial institutions are willing to order protection on Korean bonds. The financial melt down caused failure of compensation payment on defaulting Korean bonds by the Korean Protection Sellers.

Risks inherent in credit derivatives

- *counterparty risk* – counterparty could renege or default
- *legal risk* – arises from ambiguity regarding the definition of default
- *liquidity risk* – thin markets (declines when markets become more active)
- *model risk* – probabilities of default are hard to estimate

Market efficiencies provided by credit derivatives

1. Absence of the reference asset in the negotiation process - flexibility in setting terms that meet the needs of both counterparties.

2. *Short sales* of credit instruments can be executed with reasonable *liquidity* - hedging existing exposure or simply *profiting from a negative credit view*. Short sales would open up a wealth of arbitrage opportunities.

3. Offer considerable flexibilities in terms of *leverage*. For example, a hedge fund can both *synthetically finance the position* of a portfolio of bank loans but avoid the administrative costs of direct ownership of the asset.

Spread-lock interest rate swaps

Enables an investor to lock in a swap spread and apply it to

an interest rate swap executed at some point in the future.

- The investor makes an agreement with the bank on

(i) swap spread, (ii) a Treasury rate.

- The sum of the rate and swap spread equals the fixed rate paid by the investor for the life of the swap, which begins at the end of the three month (say) spread-lock.

- The bank pays the investor a floating rate. Say, 3-month LIBOR.

Example

- The current 5yr swap rate is 8% while the 5yr benchmark government bond rate is 7.70%, so the current spread is 30bp an historically low level.
- A company is looking to pay fixed using an Interest Rate Swap at some point in the year. The company believes however, that the bond rate will continue to fall over the next 6 months. They have therefore decided not to do anything in the short term and look to pay fixed later.
- It is now six months later and as they predicted, rates did fall. The current 5 yr bond rate is now 7.40% so the company asks for a 5 yr swap rate and is surprised to learn that the swap rate is 7.90%. While the bond rate fell 30bp, the swap rate only fell 10bp. Why?

Explanations

- The swap spread is largely determined by demand to pay or receive fixed rate.
- As more parties wish to pay fixed rate, the "price" increases, and therefore the spread over bond rates increases.
- It would appear that as the bond rate fell, more and more companies elected to pay fixed, driving the swap spread from 30bp to 50bp.
- While the company has saved 10bp, it could have used a Spread-lock to do better.

- When the swap rate was 8% and the bond yield 7.70%, the company could have asked for a six month Spread-lock on the 5yr Swap spread.
- While the spot spread was 30bp, the 6mth forward Spread was say 35bp.
- The company could "buy" the Spread-lock for six months at 35bp. At the end of the six months, they can then enter a swap at the then 5yr bond yield plus 35bp, in this example a total of 7.75%. The Spread-lock therefore increases the saving from 10bp to 25bp.

- A Spread-lock allows the Interest Rate Swap user to lock in the forward differential between the Interest Rate Swap rate and the underlying Government Bond Yield (usually of the same or similar tenor).
- The Spread-lock is not an option, so the buyer is obliged to enter the swap at the maturity of the Spread-lock.

Price of a currency forward

Here, $r_d - r_f$ is the cost of carry of holding the foreign currency.

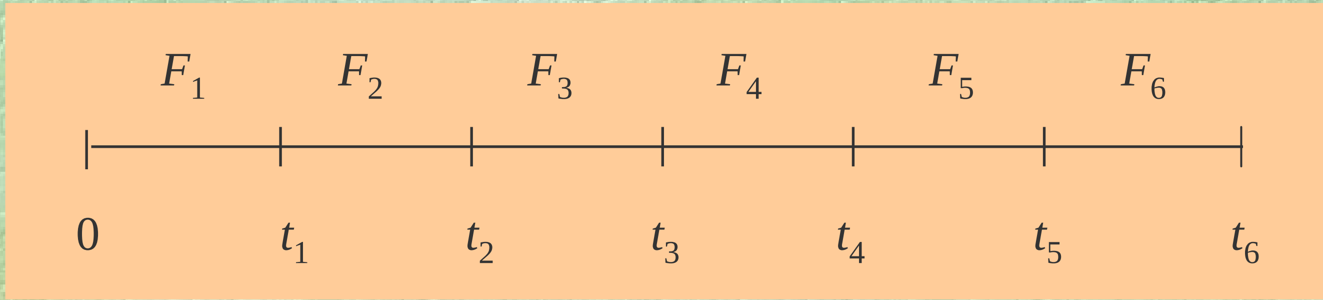
Let $B_d(\tau)$ [$B_f(\tau)$] denote the price of domestic (foreign) discount bond with unit par in domestic (foreign) currency.

Then, the price of currency forward is

$$F = S\,\frac{B_f(\tau)}{B_d(\tau)}.$$

American currency forward (HSBC product)

Consider a 6-month forward contract. The exchange rate over each one-month period is preset to assume some constant value.



The holder can exercise parts of the notional at any time during the life of the forward, but she has to exercise all by the maturity date of the currency forward.

Questions

1. What should be the optimal exercise policies adopted by the holder?

2. How to set the predetermined exchange rates so that the value of the American currency forward is zero at initiation?

Pricing considerations

- The critical exchange rate $S^*(\tau)$ is independent of the amount exercised. Hence, when S reaches $S^*(\tau)$, the whole should be exercised (though the holder may not have the whole notional amount of foreign currency available).
- Set $F_j = F_1 e^{(r_d - r_f)(j-1)\Delta t}, j = 2, 3, \cdots, 6;$ this is because the forward price grows by the factor $e^{(r_d - r_f)\Delta t}$ over each Δt time interval.

Determine F_1 such that the value of the American currency forward at initiation is zero.

Auto-Cancellable Equity Linked Swap

Contract Date: June 13, 2003

Effective Date: June 18, 2003

Termination Date:
The earlier of (1) June 19, 2006 and (2) the Settlement Date relating to the Observation Date on which the Trigger Event takes place (maturity uncertainty).

Trigger Event:
The Trigger Event is deemed to be occurred when the closing price of the Underlying Stock is at or above the Trigger Price on an Observation Date.

Observation Dates:
1. Jun 16, 2004, 2. Jun 16, 2005, 3. Jun 15, 2006

Settlement Dates:
With respect to an Observation Date, the 2nd business day after such Observation Date.

- In order that funding cost arbitrage works, the Protection Buyer should have a higher credit rating than the Protection Seller. It is advantageous for the Protection Buyer to hold the risky asset to take advantage of the lower funding cost.
- Before the 1997 crisis in Korea, Korean financial institutions are willing to order protection on Korean bonds. The financial melt down caused failure of compensation payment on defaulting Korean bonds by the Korean Protection Sellers.

Underlying Stock: HSBC (0005.HK)
Notional: HKD 83,000,000.00
Trigger Price: HK$95.25

Party A pays:
For Calculation Period 1 – 4: 3-month HIBOR + 0.13%,
For Calculation Period 5 – 12: 3-month HIBOR - 0.17%

Party B pays:
On Termination Date,
8% if the Trigger Event occurred on Jun 16, 2004;
16% if the Trigger Event occurred on Jun 16, 2005;
24% if the Trigger Event occurred on Jun 15, 2006; or
24% if the Trigger Event occurred on Jun 15, 2006; or
0% if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred
Party A pays: Notional Amount
Party B delivers: 1,080,528 shares of the Underlying Stock

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year

Party B pays Party A an upfront fee of HKD1,369,500.00 (i.e. 1.65% on Notional) on Jun 18, 2003.

Model Formulation

- This swap may be visualized as an auto knock-out equity forward with terminal payoff

  1,080,528 x terminal stock price - Notional.

- Modeling of the equity risk: The stock price follows the trinomial random walk. The "clock" of the stock price trinomial tree is based on trading days. When we compute the drift rate of stock and "equity" discount factor, "one year" is taken as the number of trading days in a year.
- The net interest payment upon early termination is considered as knock-out rebate. The contribution of the potential rebate to the swap value is given by the Net Interest Payment times the probability of knock-out.
- The Expected Net Interest Payment is calculated based on today's yield curve. Linear interpolation on today's yield curve is used to find the HIBOR at any specific date. The dynamics of interest rate movement has been neglected for simplicity since only Expected Net Interest Payment (without cap or floor feature) appears as rebate payment.

Quanto version

Underlying Stock: HSBC (0005.HK)
Notional: USD 10,000,000.00
Trigger Price: HK$95.25

Party A pays:
For Calculation Period 1 – 4: 3-month LIBOR
For Calculation Period 5 – 12: 3-month LIBOR - 0.23%,

Party B pays:
On Termination Date,
7% if the Trigger Event occurred on Jun 16, 2004;
14% if the Trigger Event occurred on Jun 16, 2005;
21% if the Trigger Event occurred on Jun 15, 2006; or
0% if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred
Party A pays: Notional Amount
Party B delivers: Number of Shares of the Underlying Stock

Number of Shares: Notional x USD-HKD Spot Exchange Rate on Valuation Date / Trigger Price

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year

Party B pays Party A an upfront fee of USD150,000.00 (i.e. 1.5% on Notional) on Jun 18, 2003.

Model Formulation

- By the standard quanto prewashing technique, the drift rate of the HSBC stock in US currency = $r_{HK} - q_S - \rho\,\sigma_S\,\sigma_F$,

  where r_{HK} = riskfree interest rate of HKD
  q_S = dividend yield of stock
  ρ = correlation coefficient between stock price and exchange rate
  σ_S = annualized volatility of stock price
  σ_F = annualized volatility of exchange rate

- Terminal payoff (in US dollars)

  = Notional / Trigger Price (HKD) x terminal stock price (HKD) - Notional.

- The exchange rate F does not enter into the model since the payoff in US dollars does not contain the exchange rate. The volatility of F appears only in the quanto-prewashing formula.

Worst of two stocks

Contract Date: June 13, 2003
Effective Date: June 18, 2003

Underlying Stock: The Potential Share with the lowest Price Ratio with respect to each of the Observation Dates.

Price Ratio: In respect of a Potential Share, the Final Share Price divided by its Initial Share Price.

Final Share Price: Closing Price of the Potential Share on the Observation Date

Potenital Share	Initial Share Price	Trigger Price	Number of Shares
HSBC (0005.HK)	95.25	95.25	1,080,528
HK Electric (0006.HK)	29.00	29.00	3,549,193

Party A pays:
For Calculation Period 1 – 4: 3-month HIBOR + 0.13%,
For Calculation Period 5 – 12: 3-month HIBOR - 0.17%,

Party B pays:
On Termination Date,
10% if the Trigger Event occurred on Jun 16, 2004;
20% if the Trigger Event occurred on Jun 16, 2005;
30% if the Trigger Event occurred on Jun 15, 2006; or
0% if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred
Party A pays: Notional Amount
Party B delivers: Number of Shares of the Underlying Stock as shown above

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year

Party B pays Party A an upfront fee of HKD1,369,500.00 (i.e. 1.65% on Notional) on Jun 18, 2003.