## Basic interest rate and currency swap products

Basic forward products

- Bond forward
- Forward rate agreement and forward interest rate
- American currency forward

Valuation of vanilla interest rate swap

- Pricing off the yield curve

Currency swaps

- Origin of currency swaps (IBM and Swiss bank)


## Bond forward

The underlying asset is a zero-coupon bond of maturity $T_{2}$ with a settlement date $T_{1}$, where $t<T_{1}<T_{2}$.


The holder pays the delivery price $F$ of the bond forward on the forward maturity date $T_{1}$ to receive a bond with par value $P$ on the maturity date $T_{2}$.

## Bond forward price in terms of traded bond prices

Let $B_{t}(T)$ denote the traded price of unit par discount bond at current time $t$ with maturity date $T$.

Present value of the net cashflows

$$
=-F B_{t}\left(T_{1}\right)+P B_{t}\left(T_{2}\right)
$$

To determine the forward price $F$, we set the above value zero and obtain

$$
F=P B_{t}\left(T_{2}\right) / B_{t}\left(T_{1}\right)
$$

The forward price is given in terms of known market bond prices observed at time $t$ with maturity dates $T_{1}$ and $T_{2}$.

## Forward interest rate

The forward price should be related to the forward interest rate $R\left(t ; T_{1}, T_{2}\right)$. The forward rate is the interest rate determined at the current time $t$ which is applied over the future period $\left[T_{1}, T_{2}\right]$. Recall the relations

$$
F\left[1+R\left(t ; T_{1}, T_{2}\right)\left(T_{2}-T_{1}\right)\right]=P
$$

and

$$
F=P B_{t}\left(T_{2}\right) / B_{t}\left(T_{1}\right)
$$

so that

$$
R\left(t ; T_{1}, T_{2}\right)=\frac{1}{T_{2}-T_{1}}\left[\frac{B_{t}\left(T_{1}\right)}{B_{t}\left(T_{2}\right)}-1\right]
$$

## Forward rate agreement

FRA is an agreement between two counterparties to exchange floating and fixed interest payments on future settlement date $T_{2}$.

- The floating rate will be the LIBOR rate $L\left[T_{1}, T_{2}\right]$ as observed on the future reset date $T_{1}$.

Question
Should the fixed rate be equal to the forward rate over the same period as observed today?


An amount $N$ paid out at $T_{1}$ would become

$$
N\left\{1+L\left[T_{1}, T_{2}\right]\left(T_{2}-T_{1}\right)\right\} \text { at time } T_{2} .
$$

The cash flows of the fixed rate receiver can be replicated by
(i) long holding of $N\left[1+K\left(T_{2}-T_{1}\right)\right]$ units of $T_{2}$-maturity zero coupon bond with unit par
(ii) short holding of $N$ units of $T_{1}$-maturity zero coupon bond with unit par.

Value of the replicating portfolio at the current time
$=N\left\{\left[1+K\left(T_{2}-T_{1}\right)\right] B_{t}\left(T_{2}\right)-B_{t}\left(T_{1}\right)\right\}$.
We find $K$ such that the above value is zero.

$$
K=\frac{1}{T_{2}-T_{1}}\left[\frac{B_{t}\left(T_{1}\right)}{B_{t}\left(T_{2}\right)}-1\right] .
$$

forward rate over [ $T_{1}, T_{2}$ ]
$K$ is the forward price of the LIBOR rate $L\left[T_{1}, T_{2}\right]$ over the time period $\left[T_{1}, T_{2}\right.$ ].

## Comparison between forward contract and FRA

What is $F$ ?
forward contract - determination of $F$$T_{1} T_{2}$


## Price of a currency forward

Here, $r_{d}-r_{f}$ is the cost of carry of holding the foreign currency.

Let $B_{d}(\tau)\left[B_{f}(\tau)\right]$ denote the price of domestic (foreign) discount bond with unit par in domestic (foreign) currency.

Then, the price of currency forward is

$$
F=S \frac{B_{f}(\tau)}{B_{d}(\tau)}
$$

## American currency forward (HSBC product)

Consider a 6-month forward contract. The exchange rate over each one-month period is preset to assume some constant value.


The holder can exercise parts of the notional at any time during the life of the forward, but she has to exercise all by the maturity date of the currency forward.

## Questions

1. What should be the optimal exercise policies adopted by the holder?
2. How to set the predetermined exchange rates so that the value of the American currency forward is zero at initiation?

## Pricing considerations

- The critical exchange rate $S^{*}(\tau)$ is independent of the amount exercised. Hence, when $S$ reaches $S^{*}(\tau)$, the whole should be exercised (though the holder may not have the whole notional amount of foreign currency available).
- Set $F_{j}=F_{1} e^{\left(r_{d}-r_{f}\right)(j-1) \Delta t}, j=2,3, \cdots, 6$; this is because the forward price grows by the factor $e^{\left(r_{d}-r_{f}\right) \Delta t}$ over each $\Delta t$ time interval.

Determine $F_{1}$ such that the value of the American currency forward at initiation is zero.


6-month LIBOR

## Direct swap agreement

In an interest swap, two parties agree to exchange periodic interest payments.

- One party is the fixed-rate payer, and the other party is the floatingrate payer, where the interest rate floats with some reference rate.


## Example of an interest rate swap

Notional amount $=\$ 50$ million
fixed-rate $=10 \%$
floating rate $=6-$ month LIBOR
Tenor $=3$ years, semi-annual payments

| 6-month | Cash flows |  |  |
| :---: | :---: | :---: | :---: |
| period | Net (float-fix) | floating rate bond |  |
| 0 | 0 | -50 | 50 |
| 1 | $\mathrm{LIBOR}_{1} / 2 \times 50-2.5$ | $\mathrm{LIBOR}_{1} / 2 \times 50$ | -2.5 |
| 2 | $\mathrm{LIBOR}_{2} / 2 \times 50-2.5$ | $\mathrm{LIBOR}_{2} / 2 \times 50$ | -2.5 |
| 3 | $\mathrm{LIBOR}_{3} / 2 \times 50-2.5$ | $\mathrm{LIBOR}_{3} / 2 \times 50$ | -2.5 |
| 4 | $\mathrm{LIBOR}_{4} / 2 \times 50-2.5$ | $\mathrm{LIBOR}_{4} / 2 \times 50$ | -2.5 |
| 5 | $\mathrm{LIBOR}_{5} / 2 \times 50-2.5$ | $\mathrm{LIBOR}_{5} / 2 \times 50$ | -2.5 |
| 6 | $\mathrm{LIBOR}_{6} / 2 \times 50-2.5$ | $\mathrm{LIBOR}_{6} / 2 \times 50$ | -52.5 |

A swap can be interpreted as a package of cash market instruments.

- Buy $\$ 50$ million par of a 3 -year floating rate bond that pays 6 -month LIBOR semi-annually.
- Finance the purchase by borrowing $\$ 50$ million for 3 years at 10\% interest rate paid semi-annually.

Fixed-rate payer

- long position in a floating-rate bond
- short position in a fixed rate bond


## Uses and characteristics

- One transaction can effectively establish a payoff equivalent to a package of forward contracts.
- Interest rate swaps now provide more liquidity than forward contracts, in particular for long-term forward contracts.
- Used to alter the cash flow characteristics of an institution's asset so as to provide a better match between assets and liabilities.


## Valuation of interest rate swap

- When a swap is entered into, it typically has zero value.
- Valuation involves finding the fixed coupon rate $K$ such that fixed and floating legs have equal value at inception.
- Consider a swap with payment dates $t_{1}, t_{2}, \ldots, t_{N}$ set in the terms of the swap.

$$
\left(t_{i}-t_{i-1}\right) \times K \times N
$$



## Valuation (cont'd)

- Fixed payment at $t_{i}$ is $\left(t_{i}-t_{i-1}\right) \times K \times N$ where $N$ is the notional principal, $t_{i}-t_{i-1}$ is the tenor period. The fixed payments are packages of bonds with par $K \times N$.
- To generate the floating rate payments, we invest a floating rate bond of par value $\$ N$ and use the floating rate interest earned to honor the floating leg payments. At maturity, $\$ N$ remains but all the intermediate floating rate interests are forgone.
"Assume forward rates will be realized" rule

1. Calculate the swap's net cash flows on the assumption that LIBOR rates in the future equal today's forward LIBOR rates.
2. Set the value of the swap equal to the present value of the net cash flows using today's LIBOR zero curve for discounting.

## Valuation (cont'd)

- Let $B(0, t)$ be the discount bond price with maturity $t$.
- Sum of percent value of floating leg payments $=N\left[1-B\left(0, t_{N}\right)\right]$; sum of present value of fixed leg payments =

$$
(N \times K) \sum_{i=1}^{N}\left(t_{i}-t_{i-1}\right) B\left(0, t_{i}\right) .
$$

- Hence, the swap rate is given by

$$
K=\frac{1-B\left(0, t_{N}\right)}{\sum_{i=1}^{N}\left(t_{i}-t_{i-1}\right) B\left(0, t_{i}\right)} .
$$

## Swap rate curves

- From traded discount bonds, we may construct the implied forward rates; then the equilibrium swap rates are determined from these forward rates.
- Turning around, with the high liquidity of the swap market, and available at so many maturities, it is the swap rates that drive the prices of bonds. That is, the fixed leg of a par swap (having zero value) is determined by the market.
- For swap-based interest rate derivatives, swap rates constitute the more natural set of state variables, rather than the forward rates.


## Numerical Example: Determining the Swap Rate

Three-year swap, notional amount $\$ 100$ thousand
Fixed-rate receiver
Actual/360 day count basis, quarterly payments

Floating-rate receiver
3-month LIBOR, actual/360 day count basis, quarterly payments and reset.

Swap rate is the rate that will produce fixed cash flows whose present value will equal the present value of the floating cash flows.

| $(1)$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(2)$ |  | $(3)$ | $(4)$ | $(5)$ | $(6)$ |  | $(7)$ |
| Period | Futures <br> Price | Forward <br> Rate | Discount <br> Factor | Floating <br> Cash Flow | PV of <br> Floating CF | PV of <br> Fixed CF |  |
| 1 |  | 4.05 | 1.00000 |  |  |  |  |
| 2 | 95.85 | 4.15 | 0.990 | 1,012 | 1,002 | 1,234 |  |
| 3 | 95.45 | 4.55 | 0.980 | 1,049 | 1,027 | 1,235 |  |
| 4 | 95.28 | 4.72 | 0.969 | 1,150 | 1,113 | 1,221 |  |
| 5 | 95.10 | 4.90 | 0.957 | 1,193 | 1,141 | 1,206 |  |
| 6 | 94.97 | 5.03 | 0.945 | 1,279 | 1,209 | 1,230 |  |
| 7 | 94.85 | 5.15 | 0.933 | 1,271 | 1,186 | 1,176 |  |
| 8 | 94.75 | 5.25 | 0.921 | 1,287 | 1,186 | 1,148 |  |
| 9 | 94.60 | 5.40 | 0.909 | 1,327 | 1,206 | 1,146 |  |
| 10 | 94.50 | 5.50 | 0.897 | 1,365 | 1,224 | 1,130 |  |
| 11 | 94.35 | 5.65 | 0.885 | 1,390 | 1,229 | 1,115 |  |
| 12 | 94.25 | 5.76 | 0.872 | 1,459 | 1,272 | 1,123 |  |
| 13 | 94.10 | 5.90 | 0.859 | 1,456 | 1,251 | 1,083 |  |
| Total |  |  |  |  | 14,053 | 14,053 |  |

Column (2): Market quoted Eurodollar 3-month Certificate of Deposit (CD) futures price.

Column (3): Forward rate as derived from CD futures prices is taken as the realized floating rate in the future.
The forward rate for LIBOR (per annum) can be found from the futures price of the Eurodollar CD futures contract as follows:

$$
100.00 \text { - Futures price }
$$

Column (4): The discount factor is found as follows:
Discount factor in the previous period
$[1+($ forward rate in previous period $\times$ number of days in period/360)]

Column (5): The floating cash flow is found by multiplying the forward rate and the notional amount, adjusted for the number of days in the payment period. That is:

Forward rate previous period $\times$ number of days in period 360

Column (7): This column is found by trial and error, based on a guess of the swap rate. In determining the fixed cash flow, the cash flow must be adjusted for the day count as follows:

## Assumed swap rate $\times$ number of days in period 360

Determining the value of a swap after one year

| $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Period | Futures <br> Price | Forward <br> Rate | Discount <br> Factor | Floating <br> Cash Flow | PV of <br> Floating CF | PV of <br> Fixed CF |
| 1 |  | 5.25 | 1.00000 |  |  |  |
| 2 | 94.27 | 5.73 | 0.986 | 1,370 | 1,352 | 1,284 |
| 3 | 94.22 | 5.78 | 0.972 | 1,448 | 1,408 | 1,225 |
| 4 | 94.00 | 6.00 | 0.958 | 1,445 | 1,385 | 1,195 |
| 5 | 93.85 | 6.15 | 0.944 | 1,516 | 1,432 | 1,190 |
| 6 | 93.75 | 6.25 | 0.929 | 1,554 | 1,445 | 1,172 |
| 7 | 93.54 | 6.46 | 0.915 | 1,579 | 1,446 | 1,153 |
| 8 | 93.25 | 6.75 | 0.900 | 1,668 | 1,502 | 1,159 |
| 9 | 93.15 | 6.85 | 0.885 | 1,706 | 1,510 | 1,115 |
| Total |  |  |  |  | 11,482 | 9,498 |

PV of floating cash flow $\quad \$ 11,482$
PV of fixed cash flow $\quad \$ 9,498$
Value of swap $\$ 1,984$

## Exploiting comparative advantages

A domestic company has comparative advantage in domestic loan but it wants to raise foreign capital. The situation for a foreign company happens to be reversed.


$$
P_{d}=F_{0} P_{f}
$$



Goal: To exploit the comparative advantages in borrowing rates for both companies in their domestic currencies.

## Cashflows between the two currency swap counterparties

 (assuming no intertemporal default)
(maturity)

## Settlement rules

Under the full (limited) two-way payment clause, the nondefaulting counterparty is required (not required) to pay if the final net amount is favorable to the defaulting party.

## Origins of currency swaps

Currency swaps originally were developed by banks in the UK to help large clients circumvent UK exchange controls in the 1970s.

- UK companies were required to pay an exchange equalization premium when obtaining dollar loans from their banks.

How to avoid having to pay this premium?
An agreement would then be negotiated whereby

- The UK organization borrowed sterling and lent it to the US company's UK subsidiary.
- The US organization borrowed dollars and lent it to the UK company's US subsidiary.

These arrangements were called back-to-back loans or parallel loans.

## IBM / World Bank with Salomon Brothers

## as intermediary

- IBM had existing debts in DM and Swiss francs. Due to a depreciation of the DM and Swiss franc against the dollar, IBM could realize a large foreign exchange gain, but only if it could eliminate its DM and Swiss franc liabilities and "lock in" the gain.
- The World Bank was raising most of its funds in DM (interest rate $=12 \%$ ) and Swiss francs (interest rate $=8 \%$ ). It did not borrow in dollars, for which the interest rate cost was about $17 \%$. Though it wanted to lend out in DM and Swiss francs, the bank was concerned that saturation in the bond markets could make it difficult to borrow more in these two currencies at a favorable rate.

World Bank/IBM Currency Swap, 1981


## IBM / World Bank

- IBM was willing to take on dollar liabilities and made dollar payments to the World Bank since it could generate dollar income from normal trading activities.
- The World Bank could borrow dollars, convert them into DM and SFr in FX market, and through the swap take on payment obligations in DM and SFr.


## Remark

1. The swap payments by the World Bank to IBM were scheduled so as to allow IBM to meet its debt obligations in DM and SFr.
2. IBM and the World Bank had AAA-ratings; therefore, the counterparty risk was low.

## Exotic swap products

- Asset swaps
- Total return swaps
- Spread-lock interest rate swaps
- Credit default swaps
- Equity-linked swaps


## Asset swaps

- Combination of a defaultable bond with an interest rate swap. $B$ pays the notional amount upfront to acquire the asset swap package.

1. A fixed coupon bond issued by $C$ with coupon $\bar{c}$ payable on coupon dates.
2. A fixed-for-floating swap.


The asset swap spread $s^{A}$ is adjusted to ensure that the asset swap package has an initial value equal to the notional.

## Asset swaps are more liquid than the underlying defaultable bond.

The Asset Swap may be transacted at the time of the security purchase or added to a bond already owned by the investor.

An asset swaption gives $B$ the right to enter an asset swap package at some future date $T$ at a predetermined asset swap spread sA.

## Example

1. An investor believes CAD rates will rise over the medium term. They would like to purchase CAD 50million 5yr Floating Rate Notes.
2. There are no 5yr FRNs available in the market in sufficient size. The investor is aware of XYZ Ltd 5yr 6.0\% annual fixed coupon Bonds currently trading at a yield of $5.0 \%$. The bonds are currently priced at 104.38.
3. The investor can purchase CAD 50 million Fixed Rate Bonds in the market for a total consideration of CAD 51,955,000 plus any accrued interest. They can then enter a 5 year Interest Rate Swap (paying fixed) with the Bank as follows:

| Notional: <br> Investor <br> Pays: <br> Investor <br> Receives: | CAD 50,000,000 |
| :--- | :--- |
| Up front <br> Payment: | The Bank Pays CAD <br> interest to investor |

The upfront payment compensates the investor for any premium paid for the bonds. Likewise, if the bonds were purchased at a discount, the investor would pay the discount amount to the Bank. This up front payment ensures that the net position created by the Asset Swap is the same as a FRN issued at par so that the initial outlay by the investor is CAD 50 million.

## Asset swap packages

An asset swap package consists of a defaultable coupon bond $\bar{C}$ with coupon $\bar{c}$ and an interest rate swap. The bond's coupon is swapped into LIBOR plus the asset swap rate $s^{A}$. Asset swap package is sold at par.

Remark Asset swap transactions are driven by the desire to strip out unwanted structured features from the underlying asset.

Payoff streams to the buyer of the asset swap package

| time | defaultable bond | swap | net |
| :---: | :---: | :---: | :---: |
| $t=0$ | $-\bar{C}(0)$ | $-1+\bar{C}(0)$ | -1 |
| $t=t_{i}$ | $\bar{c}^{*}$ | $-\bar{c}+L_{i-1}+s^{A}$ | $L_{i-1}+s^{A}+\left(\bar{c}^{*}-\bar{c}\right)$ |
| $t=t_{N}$ | $(1+\bar{c})^{*}$ | $-\bar{c}+L_{N-1}+s^{A}$ | $1^{*}+L_{N-1}+s^{A}+\left(\bar{c}^{*}-\bar{c}\right)$ |
| default | recovery | unaffected | recovery |

* denotes payment contingent on survival.
$s(0)=$ fixed-for-floating swap rate (market quote)
$A(0)=$ value of an annuity paying at the $\$ 1$ (calculated based on observable default free bond prices)

The value of asset swap package is set at par at $t=0$, so that

$$
\bar{C}(0)+\underbrace{A(0) s(0)+A(0) s^{A}(0)-A(0) \bar{c}}_{\text {swap arrangement }}=1
$$

The present value of the floating coupons is given by $A(0) s(0)$. The swap continues even after default so that $A(0)$ appears in all terms associated with the swap arrangement. Solving for $s^{A}(0)$

$$
s^{A}(0)=\frac{1}{A(0)}[1-\bar{C}(0)]+\bar{c}-s(0) .
$$

Rearranging the terms,

$$
\bar{C}(0)+A(0) s^{A}(0)=\underbrace{[1-A(0) s(0)]+A(0) \bar{c}}_{\text {default-free bond }} \equiv C(0)
$$

where the right-hand side gives the value of a default-free bond with coupon $\bar{c}$. Note that $1-A(0) s(0)$ is the present value of receiving $\$ 1$ at maturity $t_{N}$. We obtain

$$
s^{A}(0)=\frac{1}{A(0)}[C(0)-\bar{C}(0)] .
$$

## Pricing

1. From the investors viewpoint, the net cash flows from the Bond plus the Asset Swap are the same as the cash flows from a Floating Rate Note.
2. The yield on the Asset Swap (in the example LIBOR plus 50 bp ), will depend upon the relationship between the Bond yield and the Swap Yield for that currency. When converting a fixed rate bond to floating rate, LOWER swap rates relative to bond yields will result in HIGHER Asset Swap yields. When converting FRNs to fixed rate, HIGHER swap rates relative to bond yields will result in HIGHER Asset Swap yields.

Remark It is a common mistake to assume that the yield over LIBOR on the Asset Swap ( 50 bp in the example above) is merely the difference between the Bond Yield (5\%) and the 5yr Swap yield. It is necessary to price the Asset Swap using a complete Interest Rate Swap pricing model.

## Target Market

Any investor purchasing or holding interest bearing securities. The Asset Swap can either be used to create synthetic securities unavailable in the market, or as an overlay interest rate management technique for existing portfolios. Many investors use Asset Swaps to "arbitrage" the credit markets, as in many instances synthetic FRNs or Bonds produce premium yields compared to traditional securities issued by the same company.

## Total return swap

- Exchange the total economic performance of a specific asset for another cash flow.


Total return comprises the sum of interests, fees and any change-in-value payments with respect to the reference asset.

A commercial bank can hedge all credit risk on a loan it has originated. The counterparty can gain access to the loan on an off-balance sheet basis, without bearing the cost of originating, buying and administering the loan.

The payments received by the total return receiver are:

1. The coupon $\overline{\bar{c}}$ of the bond (if there were one since the last payment date $T_{i-1}$ )
2. The price appreciation $\left(\bar{C}\left(T_{i}\right)-\bar{C}\left(T_{i-1}\right)\right)^{+}$of the underlying bond $C$ since the last payment (if there were only).
3. The recovery value of the bond (if there were default).

The payments made by the total return receiver are:

1. A regular fee of $\operatorname{LIBOR}+s^{\mathrm{TRS}}$
2. The price depreciation $\left(\bar{C}\left(T_{i-1}\right)-\bar{C}\left(T_{i}\right)\right)^{+}$of bond $C$ since the last payment (if there were only).
3. The par value of the bond $C$ if there were a default in the meantime).

The coupon payments are netted and swap's termination date is earlier than bond's maturity.

## Some essential features

1. The receiver is synthetically long the reference asset without having to fund the investment up front. He has almost the same payoff stream as if he had invested in risky bond directly and funded this investment at LIBOR $+s^{\text {TRS }}$.
2. The TRS is marked to market at regular intervals, similar to a futures contract on the risky bond. The reference asset should be liquidly traded to ensure objective market prices for making to market (determined using a dealer poll mechanism).
3. The TRS allows the receiver to leverage his position much higher than he would otherwise be able to (may require collateral). The TRS spread should not be driven by the default risk of the underlying asset but also by the credit quality of the receiver.

## Used as a financing tool

- The receiver wants financing to invest $\$ 100$ million in the reference bond. It approaches the payer (a financial institution) and agrees to the swap.
- The payer invests $\$ 100$ million in the bond. The payer retains ownership of the bond for the life of the swap and has much less exposure to the risk of the receiver defaulting.
- The receiver is in the same position as it would have been if it had borrowed money at LIBOR $+s^{\text {TRS }}$ to buy the bond. He bears the market risk and default risk of the underlying bond.


## Motivation of the receiver

1. Investors can create new assets with a specific maturity not currently available in the market.
2. Investors gain efficient off-balance sheet exposure to a desired asset class to which they otherwise would not have access.
3. Investors may achieve a higher leverage on capital - ideal for hedge funds. Otherwise, direct asset ownership is on on-balance sheet funded investment.
4. Investors can reduce administrative costs via an offbalance sheet purchase.
5. Investors can access entire asset classes by receiving the total return on an index.

## Motivation of the payer

The payer creates a hedge for both the price risk and default risk of the reference asset.

* A long-term investor, who feels that a reference asset in the portfolio may widen in spread in the short term but will recover later, may enter into a total return swap that is shorter than the maturity of the asset. This structure is flexible and does not require a sale of the asset (thus accommodates a temporary short-term negative view on an asset).


## Spread-lock interest rate swaps

Enables an investor to lock in a swap spread and apply it to
an interest rate swap executed at some point in the future.

- The investor makes an agreement with the bank on
(i) swap spread, (ii) a Treasury rate.
- The sum of the rate and swap spread equals the fixed rate paid by the investor for the life of the swap, which begins at the end of the three month (say) spread-lock.
- The bank pays the investor a floating rate. Say, 3-month LIBOR.


## Example

- The current $5 y r$ swap rate is $8 \%$ while the $5 y r$ benchmark government bond rate is $7.70 \%$, so the current spread is 30 bp an historically low level.
- A company is looking to pay fixed using an Interest Rate Swap at some point in the year. The company believes however, that the bond rate will continue to fall over the next 6 months. They have therefore decided not to do anything in the short term and look to pay fixed later.
- It is now six months later and as they predicted, rates did fall. The current 5 yr bond rate is now $7.40 \%$ so the company asks for a 5 yr swap rate and is surprised to learn that the swap rate is $7.90 \%$. While the bond rate fell 30 bp , the swap rate only fell 10 bp . Why?


## Explanations

- The swap spread is largely determined by demand to pay or receive fixed rate.

As more parties wish to pay fixed rate, the "price" increases, and therefore the spread over bond rates increases.

- It would appear that as the bond rate fell, more and more companies elected to pay fixed, driving the swap spread from 30bp to 50bp.
- While the company has saved 10 bp , it could have used a Spread-lock to do better.
- When the swap rate was $8 \%$ and the bond yield $7.70 \%$, the company could have asked for a six month Spread-lock on the $5 y r$ Swap spread.
- While the spot spread was 30 bp , the 6 mth forward Spread was say 35bp.
- The company could "buy" the Spread-lock for six months at 35 bp . At the end of the six months, they can then enter a swap at the then 5 yr bond yield plus 35 bp , in this example a total of $7.75 \%$. The Spread-lock therefore increases the saving from 10bp to 25bp.
- A Spread-lock allows the Interest Rate Swap user to lock in the forward differential between the Interest Rate Swap rate and the underlying Government Bond Yield (usually of the same or similar tenor).
- The Spread-lock is not an option, so the buyer is obliged to enter the swap at the maturity of the Spread-lock.


## Credit default swaps

The protection seller receives fixed periodic payments from the protection buyer in return for making a single contingent payment covering losses on a reference asset following a default.


## Protection seller

- earns investment income with no funding cost
- gains customized, synthetic access to the risky bond


## Protection buyer

- hedges the default risk on the reference asset

1. Very often, the bond tenor is longer than the swap tenor. In this way, the protection seller does not have exposure to the full market risk of the bond.
2. Basket default swap - gain additional yield by selling default protection on several assets.

A bank lends 10 mm to a corporate client at $\mathrm{L}+65 \mathrm{bps}$. The bank also buys 10 mm default protection on the corporate loan for 50 bps .

## Objective achieved

- maintain relationship
- reduce credit risk on a new loan


Default Swap Settlement following Credit Event of Corporate Borrower

## Funding cost arbitrage - Credit default swap

| A-rated institution as Protection Seller | $\stackrel{50 \mathrm{bps}}{\text { annual }}$ | AAA-rated institution as Protection Buyer | LIBOR-15bps <br> as funding cost | Lender to the AAA-rated Institution |
| :---: | :---: | :---: | :---: | :---: |
| funding <br> LIBOR | ost of 50bps | $\begin{gathered} \text { coupon } \\ = \\ \text { LIBOR }+ \end{gathered}$ | 90bps |  |
| Lender to the A-rated Institution |  | BBB risky reference asset |  |  |

The combined risk faced by the Protection Buyer:

- default of the BBB-rated bond
- default of the Protection Seller on the contingent payment

The AAA-rated Protection Buyer creates a synthetic AA-asset with
a coupon rate of LIBOR $+90 \mathrm{bps}-50 \mathrm{bps}=\mathrm{LIBOR}+40 \mathrm{bps}$.
This is better than LIBOR +30 bps , which is the coupon rate of a

AA-asset (net gains of 10bps).

For the A-rated Protection Seller, it gains synthetic access to a BBB-rated asset with earning of net spread of

$$
50 \mathrm{bps}-[(\mathrm{LIBOR}+90 \mathrm{bps})-(\mathrm{LIBOR}+50 \mathrm{bps})]=10 \mathrm{bps}
$$

the A-rated Protection Seller earns 40 bps if it owns the BBB asset directly

In order that the credit arbitrage works, the funding cost of the default protection seller must be higher than that of the default protection buyer.

## Example

Suppose the A-rated institution is the Protection buyer, and assume that it has to pay 60 bps for the credit default swap premium (higher premium since the AAA-rated institution has lower counterparty risk).

The net loss of spread $=(60-40)=20 \mathrm{bps}$.

## Supply and demand drive the price

## Credit Default Protection Referencing a 5 -year Brazilian Eurobond (May 1997)

Chase Manhattan Bank<br>Broker Market<br>JP Morgan<br>240bps<br>285bps<br>325bps

* It is very difficult to estimate the recovery rate upon default.


## Credit default exchange swaps

Two institutions that lend to different regions or industries can diversify their loan portfolios in a single non-funded transaction - hedging the concentration risk on the loan portfolios.


* contingent payments are made only if credit event occurs on a reference asset
* periodic payments may be made that reflect the different risks between the two reference loans


## Counterparty risk

Before the Fall 1997 crisis, several Korean banks were willing to offer credit default protection on other Korean firms.


* Political risk, restructuring risk and the risk of possible future war lead to potential high correlation of defaults.

Advice: Go for a European bank to buy the protection.

## Risks inherent in credit derivatives

- counterparty risk - counterparty could renege or default
- legal risk - arises from ambiguity regarding the definition of default
- liquidity risk - thin markets (declines when markets become more active)
- model risk - probabilities of default are hard to estimate


## Market efficiencies provided by credit derivatives

1. Absence of the reference asset in the negotiation process - flexibility in setting terms that meet the needs of both counterparties.
2. Short sales of credit instruments can be executed with reasonable liquidity - hedging existing exposure or simply profiting from a negative credit view. Short sales would open up a wealth of arbitrage opportunities.
3. Offer considerable flexibilities in terms of leverage. For example, a hedge fund can both synthetically finance the position of a portfolio of bank loans but avoid the administrative costs of direct ownership of the asset.

## Auto-Cancellable Equity Linked Swap

Contract Date: June 13, 2003
Effective Date: June 18, 2003

Termination Date:
The earlier of (1) June 19, 2006 and (2) the Settlement Date relating to the Observation Date on which the Trigger Event takes place (maturity uncertainty).

Trigger Event:
The Trigger Event is deemed to be occurred when the closing price of the Underlying Stock is at or above the Trigger Price on an Observation Date.

Observation Dates:

1. Jun 16,2004 , 2. Jun 16,2005 , 3. Jun 15,2006

Settlement Dates:
With respect to an Observation Date, the 2nd business day after such Observation Date.

Underlying Stock: HSBC (0005.HK)
Notional: HKD 83,000,000.00
Trigger Price: HK\$95.25
Party A pays:
For Calculation Period 1-4: 3-month HIBOR $+0.13 \%$,
For Calculation Period 5-12: 3-month HIBOR - 0.17\%
Party B pays:
On Termination Date, $8 \%$ if the Trigger Event occurred on Jun 16, 2004;
$16 \%$ if the Trigger Event occurred on Jun 16, 2005;
$24 \%$ if the Trigger Event occurred on Jun 15, 2006; or
$24 \%$ if the Trigger Event occurred on Jun 15, 2006; or $0 \%$ if the Trigger Event never occurs.
Final Exchange: Applicable only if the Trigger Event has never occurred
Party A pays: Notional Amount
Party B delivers: $1,080,528$ shares of the Underlying Stock
Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year
Party B pays Party A an upfront fee of HKD1,369,500.00 (i.e. 1.65\% on Notional) on
Jun 18, 2003.

## Model Formulation

- This swap may be visualized as an auto knock-out equity forward with terminal payoff
$1,080,528 \times$ terminal stock price - Notional.
- Modeling of the equity risk: The stock price follows the trinomial random walk. The "clock" of the stock price trinomial tree is based on trading days. When we compute the drift rate of stock and "equity" discount factor, "one year" is taken as the number of trading days in a year.
- The net interest payment upon early termination is considered as knock-out rebate. The contribution of the potential rebate to the swap value is given by the Net Interest Payment times the probability of knock-out.
- The Expected Net Interest Payment is calculated based on today's yield curve. Linear interpolation on today's yield curve is used to find the HIBOR at any specific date. The dynamics of interest rate movement has been neglected for simplicity since only Expected Net Interest Payment (without cap or floor feature) appears as rebate payment.


## Quanto version

Underlying Stock: HSBC (0005.HK)
Notional: USD 10,000,000.00
Trigger Price: HK\$95.25
Party A pays:
For Calculation Period 1-4: 3-month LIBOR
For Calculation Period 5-12: 3-month LIBOR - 0.23\%,

Party B pays:
On Termination Date,
$7 \%$ if the Trigger Event occurred on Jun 16, 2004;
$14 \%$ if the Trigger Event occurred on Jun 16, 2005;
$21 \%$ if the Trigger Event occurred on Jun 15, 2006; or
$0 \%$ if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred Party A pays: Notional Amount
Party B delivers: Number of Shares of the Underlying Stock
Number of Shares: Notional x USD-HKD Spot Exchange Rate on Valuation Date / Trigger Price

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year
Party B pays Party A an upfront fee of USD150,000.00 (i.e. 1.5\% on Notional) on Jun 18, 2003.

## Model Formulation

- By the standard quanto prewashing technique, the drift rate of the HSBC stock in US currency $=r_{H K}-q_{S}-\rho \sigma_{S} \sigma_{F}$,
where
$r_{H K}=$ riskfree interest rate of HKD
$q_{S}=$ dividend yield of stock
$\rho=\begin{gathered}\text { correlation coefficient } \\ \text { and exchange rate }\end{gathered}$
$\sigma_{S}=$ annualized volatility of stock price
$\sigma_{F}=$ annualized volatility of exchange rate
- Terminal payoff (in US dollars)
$=$ Notional / Trigger Price (HKD) x terminal stock price (HKD) Notional.
- The exchange rate $F$ does not enter into the model since the payoff in US dollars does not contain the exchange rate. The volatility of $F$ appears only in the quanto-prewashing formula.


## Worst of two stocks

Contract Date: June 13, 2003
Effective Date: June 18, 2003
Underlying Stock: The Potential Share with the lowest Price Ratio with respect to each of the Observation Dates.

Price Ratio: In respect of a Potential Share, the Final Share Price divided by its Initial Share Price.

Final Share Price: Closing Price of the Potential Share on the Observation Date

|  | Initial | Trigger | Number |
| :--- | :---: | :---: | :---: |
| Potenital Share | Share Price | Price | of Shares |
| HSBC $(0005 . \mathrm{HK})$ | 95.25 | 95.25 | $1,080,528$ |
| HK Electric $(0006 . \mathrm{HK})$ | 29.00 | 29.00 | $3,549,193$ |

Party A pays:
For Calculation Period $1-4$ : 3-month HIBOR $+0.13 \%$,
For Calculation Period 5-12: 3-month HIBOR - 0.17\%,

Party B pays:
On Termination Date, $10 \%$ if the Trigger Event occurred on Jun 16, 2004; 20\% if the Trigger Event occurred on Jun 16, 2005; $30 \%$ if the Trigger Event occurred on Jun 15, 2006; or $0 \%$ if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred Party A pays: Notional Amount
Party B delivers: Number of Shares of the Underlying Stock as shown above

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year

Party B pays Party A an upfront fee of HKD1,369,500.00 (i.e. 1.65\% on Notional) on Jun 18, 2003.

