

Basic convertible bonds calculations

stock price	\$30.00 per share
stock dividend	\$0.50 per share
convertible market price	\$1,000
coupon rate	7.00%
maturity	20 years
conversion price	\$36.37

$$\begin{aligned}\text{Stock dividend yield} &= \text{annual dividend rate} \\ &\quad / \text{current stock price} \\ &= \$0.50 / \$30.00 = 1.67\%\end{aligned}$$

Conversion ratio

= number of shares for which one bond may be exchanged

= par / conversion price

= $\$1,000 / \$36.37 = 27.50$ shares

Conversion value

= equity value or stock value of the convertible

= stock price x conversion ratio

= $\$30.00 \times 27.50 = \825.00

Premium for call right

- An investor who purchases a convertible bond rather than the underlying stock typically pays a premium over the current market price of the stock.
- Why would someone be willing to pay a premium to buy this stock? The market conversion premium per share is related to the price of a call option – limit the downside risk of the convertible bond.

Conversion premium
= (convertible price – conversion value)
/ conversion value
= $(\$1,000 - \$825.00) / \$825.00 = 21.21\%$

Dollar premium
= convertible price – conversion value (expressed
in points)
= $(\$1,000 - \$825.00) / \$1,000 \times 100\%$
= 17.50 points

Conversion premium

In a bullish environment, the enthusiasm of the market boosts conversion premium levels.

- National Semiconductor Corporation (Sept 1995) – coupon rate 6.5 percent and conversion premium of 45 percent.
- 3Com Corporation (Nov., 1994) – coupon rate 10.25 percent and conversion premium of 70 percent. Bondholders are compensated with a high coupon rate while they wait for the stock price to rise.

Break even calculations

$$\begin{aligned} & \text{Break even (years)} \\ &= \text{conversion premium} / (\text{convertible yield} - \text{stock yield}) \\ &= 21.21 / (7.00 - 1.67) = 3.98 \text{ (years)} \end{aligned}$$

This represents the number of years necessary for the stock investor to recover the conversion premium (extra cost of buying the convertible rather than the stock) from the convertible's higher income relative to an instrument of an equivalent amount in the stock.

- After 3.98 years, the convertible has made up, in income alone, the amount of the conversion premium.

Break-even calculations (cont'd)

$$\text{Dollar maintenance} = \frac{\text{market price} - \text{conversion value}}{\text{coupon} - \frac{\text{market price}}{\text{stock price}} \text{ stock dividend}}$$

It measures the time it takes for the convertible yield advantage to pay for its premium compared to an equivalent dollar amount purchased of the underlying stock.

- Some people may choose to use *conversion ratio* instead of market price/ stock price.

Weaknesses of break-even analysis

- It ignores the main advantage of convertible: protection on downside risk on the underlying equity.
- It ignores the margin of safety offered by the convertible with the payment of principal at maturity.

Factors that affect the bond component

- Interest rates
- Credit rating/spreads
- Coupon
- Duration

Factors that affect the warrant component

- Stock performance
- Embedded strike price
- Common dividend yield and dividend growth rate
- Stock volatility
- Life of warrant / call protection

Floor value

The floor value of a convertible bond is the greater of

1. Conversion value
 2. Bond investment value – value as a corporate bond without the conversion option (based on the convertible bond's cash flow if not converted).
- To estimate the bond investment value, one has to determine the required yield on a non-convertible bond with the same quality rating and similar investment characteristics.
 - If the convertible bond does not sell for the greater of these two values, arbitrage profits could be realized.

Put plus stock plus yield advantage

One may treat a convertible bond as yield-enhanced stock plus a put option.

- The put option represents the investment value.
The strike price is the investment value.
- This is just an application of the put-call parity concept.

Bond investment value

- Present value of the interest and principal payments discounted at the straight (non-convertible) bond interest rate

$$\text{bond interest value} = \sum_{t=1}^n \frac{C}{(1+r)^t} + \frac{P}{(1+r)^n}$$

where P = par value, r = discount rate, C = coupon rate,
 n = number of periods to maturity.

<u>Years</u>	<u>present value payment</u>	<u>present value factor</u>	<u> </u>
1 - 20	\$80	8.514	\$681.12
20	\$1,000	0.149	\$149.00
			<u>\$830.12</u>

take $r = 10\%$

Estimation of the discount rate

Use the yield-to-maturity of a similar non-convertible bond as a proxy.

- Ratings are not very responsive to changing financial fundamentals.
- The apparent deterioration of the creditworthiness of an issue will not be reflected in the convertible price because the common stock may be rising due to higher share price volatility.

Duration

Duration is the weighted average of the times that the principal and interest payments are made.

$$\text{duration} = \frac{\sum_{t=1}^n tC_t / (1+i)^t}{\sum_{t=1}^n C_t / (1+i)^t}$$

where t is the time of payment

C_t is the coupon and/or principal payment

i is the market yield.

Duration analysis provides a measure how bond values change with changing interest rates.

Duration analysis applied to convertibles

The approximation for the convertible bond's interest rate sensitivity

$$D^{cv} = D_{adj} \left(1 - \frac{C/I}{2} \right)$$

where C = conversion value and I = investment value.

- The equity component of the convertible bond may dampen the convertible's interest rate sensitivity, depending on the bond's equity participation.

Hence, convertibles trading high above their investment value will be less sensitive to interest rates.

Duration and coupon

- For non-convertible bonds, the duration decreases as their coupon increases. This is because higher coupon bonds deliver more cash flows near the start of the bond's life.
- With convertible feature, the higher coupon rate may lead to lower propensity to convert. The CB then has a longer life, so this leads to higher duration.

These two effects are counteracting.

Interest rate sensitivity

1. The exercise price is a function of the investment value. An increase in interest rates will lower the investment value.
2. However, the exercise price of the embedded call is reduced. A lower exercise price will increase the value of the warrant.

Interest rate sensitivity (cont'd)

	Basic price	Int rate + 1%	Change	Int rate -1%	Change
Investment value	\$847.84	\$812.75	-\$35.09	884.74	\$36.90
Warrant value	\$337.66	\$362.58	+24.92	\$312.72	-\$24.94
Total	\$1185.50	\$1175.33	-\$10.17	\$1197.47	+\$11.92
Percent change			-1.02%		1.19%

Correlation with interest rates

Consider the impact of an increase on interest rate

- The future share price is expected to be higher because of higher drift rate.
- Due to negative correlation between interest rate and share price (say, the S&P 500-stock index has a correlation of about minus 0.5), the share price drops first.

Negative correlations normally lower CB value;
positive correlations make the CB worth more.

In some situation, CBs may have price differences in the range of 10-15% when correlation moves from 1.0 to -1.0 .

Callable feature

Issuer has the right to call back the bond at a pre-specified call price.

- * Upon call, the holder can either convert the bond or redeem at the call price
- * Restrictions on calling may apply; for example, notice period requirement, closing price of stock has been in excess of 150% of the conversion price on any 20 trading days within 30 consecutive days.

Put feature

The put feature allows the holder to sell back the bond to the issuer in return for a fixed sum.

Question: What should be the optimal call policies of the issuer and the optimal conversion strategies of the holder?

Impact of the credit ratings of the issuer on the convertible bond price!

Pricing of risky convertible bonds

One-factor binomial model

- * stock price process follows binomial random walk
- * interest rates to be deterministic

Two discount rates

1. If the convertible is certain to remain a bond, it is appropriate to use a discount rate corresponding to the creditworthiness of the issuer – risky rate.
2. Suppose the bond is certain to be converted, it is then appropriate to use the riskfree rate.

At maturity, the holder will choose the maximum between the par value and the value of stocks received upon conversion.

How to account for the creditworthiness of the issuer?

The discount rate to be used when we roll back is given by

$$pw_u + (1 - p)w_d.$$

Here, p is the probability to a node where the discount rate is w_u and $(1 - p)$ is probability to a node with w_d . The appropriate discount rate is the weighted average of the discounted rates at the nodes in the next time step.

$conv$ = value of stocks received if conversion takes place

$call$ = call price

$roll$ = value given by the rollback

(neither converted nor recalled)

At each node, the optimal strategy of the holder is exemplified by taking the maximum of $\min(roll, call)$ and $conv$.

- The maximum reflects the conversion right, which persists with or without recall by the issuer.
- $\min(roll, call)$ means the bond value can never shoot beyond the call price.

Dynamic programming procedure:

$$\max(\min(roll, call), conv)$$

Alternative dynamic programming procedure:

$$\min(\max(\textit{roll}, \textit{conv}), \max(\textit{call}, \textit{conv}))$$

- The term $\max(\textit{roll}, \textit{conv})$ represents the optimal strategy of the holder.
- Upon recall, the holder chooses to accept the call price or convert into shares. This can be represented by $\max(\textit{call}, \textit{conv})$.

The issuer chooses to recall or to abstain from recalling in order to minimize the option value.

Third dynamic programming procedure:

$$\min(\max(\textit{roll}, \textit{conv}), \textit{call})$$

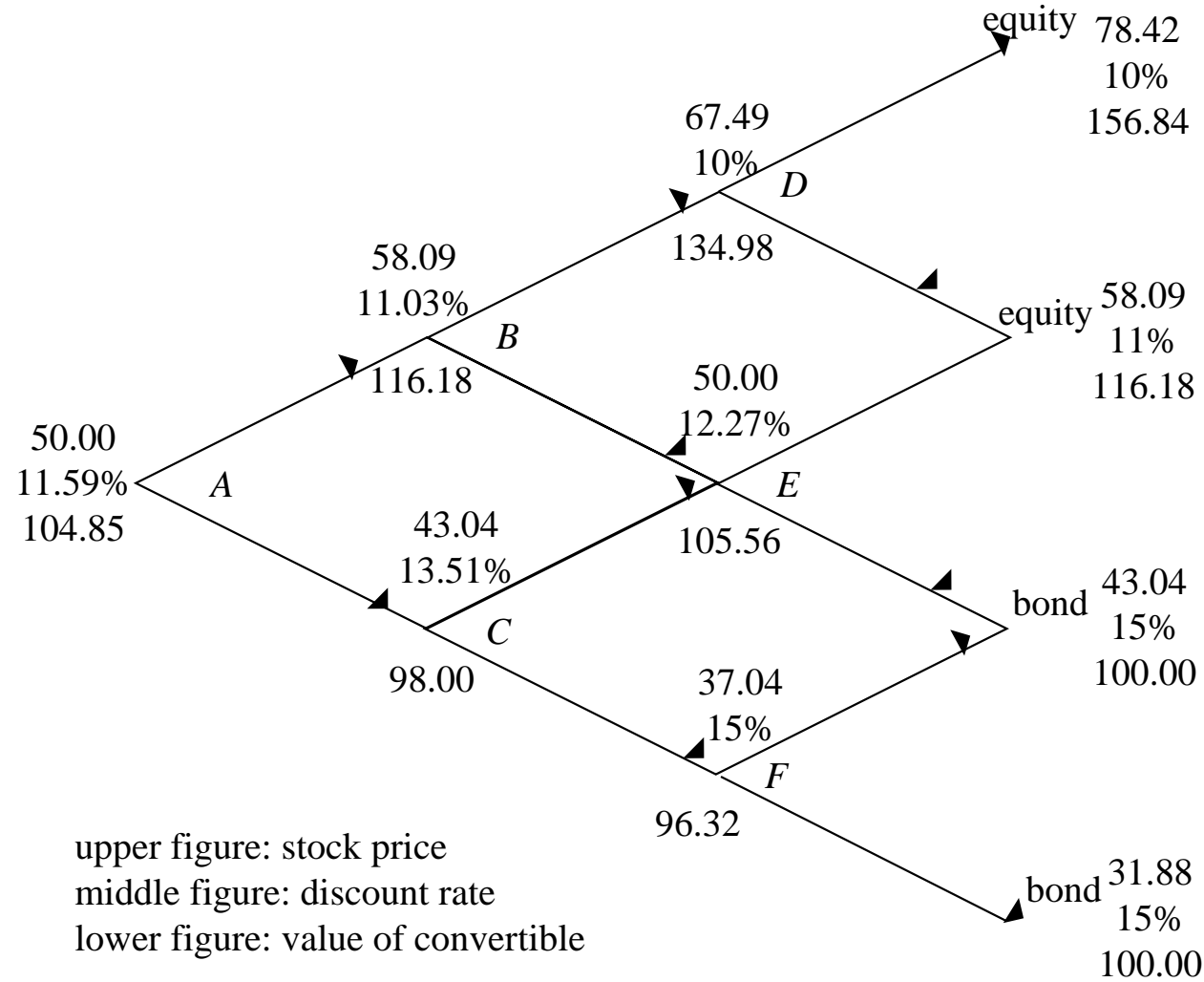
- The term $\max(\textit{roll}, \textit{conv})$ represents the optimal strategy of the holder.
- Even under the optimal strategy adopted by the holder, the bond value is always bounded above by the call price.

Example

A 9-month discount bond issued XYZ company with a face value of \$100. Assume that it can be exchanged for 2 shares of company's stock at any time during the 9 months.

- * It is callable for \$115 at any time.
- * Initial stock price = \$50, $\sigma = 30\%$ per annum and no dividend; risk-free yield curve to be flat at 10% per annum.
- * Yield curve corresponding to bonds issued by the company to be flat at 15%.
- * Tree parameters are: $u = 1.1618$, $d = 0.8607$, $p = 0.5467$,
 $R = e^{0.1\Delta t} = 1.0253$.
- * At maturity, the convertible is worth $\max(100, 2S_T)$.

Binomial tree for pricing a risky convertible bond



At node D

Roll back gives the bond value

$$(0.5467 \times 156.84 + 0.4533 \times 116.18)e^{-0.1 \times 0.25} = 134.98.$$

The bondholder is indifferent to conversion or hold, also the issuer is also indifferent as to whether the bond is called; the correct discount rate at node *D* is 10%.

At node F

The correct discount rate is 15% since the convertible is contain not to be converted if node *E* is reached.

At node E

The correct discount rate is

$$0.5467 \times 10\% + 0.4533 \times 15\% = 12.27\%.$$

The value of convertible at *E*

$$(0.5467 \times 116.18 + 0.4533 \times 100)e^{-0.1227 \times 0.25} = 105.56.$$

The bond should be neither converted nor called.

At node B

The discount rate is

$$0.5467 \times 10\% + 0.4533 \times 12.27\% = 11.03\%$$

and value of convertible is

$$(0.5467 \times 134.99 + 0.4533 \times 105.56)e^{-0.1103 \times 0.25} = 118.34.$$

It is optimal to *call the bond at node B* so that it *causes immediate conversion* and leads to \$116.18. The *discount rate* at node *B* should be *taken to be 10%*, since *conversion* takes place at this node.

At node A

The discount rate is

$$0.5467 \times 10\% + 0.4533 \times 13.51\% = 11.59\%.$$

The convertible value at node *A* is

$$(0.5467 \times 116.18 + 0.4533 \times 98.00)e^{-0.1159 \times 0.25} = 104.85.$$

If the bond has no conversion option, its value is

$$e^{-0.75 \times 0.15} = 89.36.$$

The value of conversion option = $104.85 - 89.36 = 15.49$.