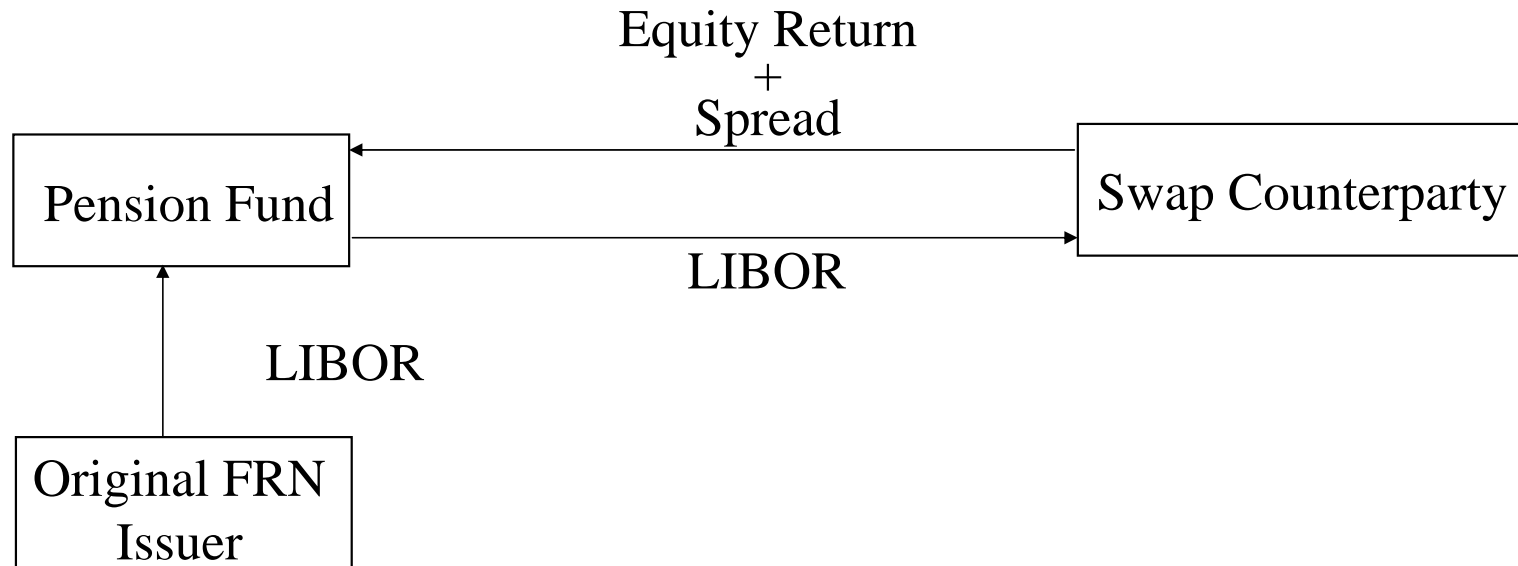# Equity-index-linked swaps

- Equivalent to portfolios of forward contracts calling for the exchange of cash flows based on two different investment rates: a variable debt rate (e.g. 3-month LIBOR) and the return on an equity index (e.g. S& P 500 index).

- It can be based on the total return (i.e. dividends, capital gain or loss) or on the percentage index change for the settlement period plus a fixed spread adjustment.

Altering an Asset Allocation Position with an Equity Swap

Equity Return
+
Spread

| Pension Fund | | Swap Counterparty |

LIBOR

LIBOR

Original FRN
Issuer

A pension fund that currently has a substantial portfolio in floating rate notes. The fund manager wants to convert some debt-based cash flows into equity-based receipts.

Uses of equity-index-linked swaps

1. To take advantage of overall price movements in a specific country's stock market without having to purchase the equity securities directly – reducing both the transaction costs and tracking error associated with actually assembling a portfolio that mimics the index.

2. To create a direct equity investment in a foreign country.

3. To accumulate foreign index returns denominated in their domestic currencies.

Example

- On December 2, the manager of a tactical asset allocation fund that is currently invested entirely in floating-rate debt securities decides to shift a portion of her portfolio to equities.

- Enter into the "receive equity index" side of a one-year equity swap based on movements in the S&P 500 index plus a spread of 10 bps.

- The swap is to have quarterly settlement payments, and the floating-rate side of the agreement is pegged to three-month LIBOR denominated in U.S. dollars.

- At the origination of the swap, the value of the S&P 500 index was 463.11 and three-month LIBOR was 3.50 percent.

- The notional principal of the swap is fixed for the life of the agreement at $50 million.

Net receipt or payment – from the fund manger's perspective –
on each future settlement date.

Settlement Date	Number of Days	S & P Level	LIBOR Level
December 2 (inital year)	-	463.11	3.50%
March 2 (following year)	90	477.51	3.25
June 2	92	464.74	3.75
September 2	92	480.86	4.00
December 2	91	482.59	-

March 2:

  Floating-rate payment =
  (0.0350 – 0.0010) x ($50,000,000) x (90/360) = $425,000
  Equity-index receipt =
  [(477.51 – 463.11)/463.11] x ($50,000,000) = $1,554,706.

  net receipt the fund expects would be ($1,554,706 – $425,000)
= $1,129,706.

June 2:

  Floating-rate payment =
  (0.0325 – 0.0010) x ($50,000,000) x (92/360) = $402,500
  Equity-index receipt =
  [(464.74 – 477.51)/477.51] x ($50,000,000) = -$1,337,145.

  net payment the fund owes would be ($1,337,145 + $402,500)
= $1,739,645.

September 2:

Floating-rate payment =
$(0.0375 - 0.0010)$ x ($50,000,000) x (92/360) = $466,389
Equity-index receipt =
[(480.86 − 464.74)/464.74] x ($50,000,000) = $1,734,303.

So the *net receipt* the fund expects would be
($1,734,303 − $466,389) = $1,267,914.

December 2:

Floating-rate payment =
$(0.0400 - 0.0010)$ x ($50,000,000) x (91/360) = $492,917
Equity-index receipt =
[(482.59 − 480.86)/480.86] x ($50,000,000) = $179,886.

The *net payment* the fund owes would be
($492,917 − $179,886) = $313,031.

Auto-Cancellable Equity Linked Swap

Contract Date: June 13, 2003
Effective Date: June 18, 2003
Termination Date:
The earlier of (1) June 19, 2006 and (2) the Settlement Date relating to the Observation Date on which the Trigger Event takes place (maturity uncertainty).

Trigger Event:
The Trigger Event is deemed to be occurred when the closing price of the Underlying Stock is at or above the Trigger Price on an Observation Date.

Observation Dates:
1. Jun 16, 2004, 2. Jun 16, 2005, 3. Jun 15, 2006

Settlement Dates:
With respect to an Observation Date, the 2nd business day after such Observation Date.

Underlying Stock: HSBC (0005.HK)
Notional: HKD 83,000,000.00
Trigger Price: HK$95.25

Party A pays:
For Calculation Period 1 – 4: 3-month HIBOR + 0.13%, Q, A/365
For Calculation Period 5 – 12: 3-month HIBOR - 0.17%, Q, A/365

Party B pays:
On Termination Date,
8% if the Trigger Event occurred on Jun 16, 2004;
16% if the Trigger Event occurred on Jun 16, 2005;
24% if the Trigger Event occurred on Jun 15, 2006; or
24% if the Trigger Event occurred on Jun 15, 2006; or
0% if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred
Party A pays: Notional Amount
Party B delivers: 1,080,528 shares of the Underlying Stock

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year

Party B pays Party A an upfront fee of HKD1,369,500.00 (i.e. 1.65% on Notional) on
Jun 18, 2003.

Model Formulation

- This swap may be visualized as an auto knock-out equity forward contract with terminal payoff = 1,080,528 x terminal stock price - Notional.

- Modeling of the equity risk: The stock price follows the trinomial random walk. The "clock" of the stock price trinomial tree is based on trading days. When we compute the drift rate of stock and "equity" discount factor, "one year" is taken as the number of trading days in a year.

- The net interest payment upon early termination is considered as knock-out rebate. In simple terms, the contribution of the potential rebate to the swap value is given by the Expected Net Interest Payment times the probability of knock-out.

- The Expected Net Interest Payment is calculated based on today's yield curve. Linear interpolation on today's yield curve is used to find the HIBOR at any specific date. The dynamics of interest rate movement has been neglected for simplicity since only Expected Net Interest Payment (without cap or floor feature) appears as rebate payment.

Quanto version

Underlying Stock: HSBC (0005.HK)
Notional: USD 10,000,000.00
Trigger Price: HK$95.25

Party A pays:
For Calculation Period 1 – 4: 3-month LIBOR, Q,
A/360
For Calculation Period 5 – 12: 3-month LIBOR - 0.23%,
Q, A/360

Party B pays:
On Termination Date,
7% if the Trigger Event occurred on Jun 16, 2004;
14% if the Trigger Event occurred on Jun 16, 2005;
21% if the Trigger Event occurred on Jun 15, 2006; or
0% if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred
Party A pays: Notional Amount
Party B delivers: Number of Shares of the Underlying Stock

Number of Shares: Notional x USD-HKD Spot Exchange Rate on
Valuation Date / Trigger Price

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year

Party B pays Party A an upfront fee of USD150,000.00 (i.e. 1.5% on
Notional) on Jun 18, 2003.

Model Formulation

- By the standard quanto prewashing technique, the drift rate of the HSBC stock in US currency $= r_{HK} - q_S - \rho \, \sigma_S \, \sigma_F$,

  where $\quad r_{HK} =$ riskfree interest rate of HKD

  $q_S =$ dividend yield of stock

  $\rho =$ correlation coefficient between stock price and exchange rate

  $\sigma_S =$ annualized volatility of stock price

  $\sigma_F =$ annualized volatility of exchange rate

- Terminal payoff (in US dollars)

  = Notional / Trigger Price (HKD) x terminal stock price (HKD) - Notional.

- The exchange rate F is the domestic currency price of one unit of foreign currency. The dynamics of F does not enter into the model since the payoff in US dollars does not contain the exchange rate. The volatility of F appears only in the quanto-prewashing formula.

Worst of two stocks

Contract Date: June 13, 2003
Effective Date: June 18, 2003

Underlying Stock: The Potential Share with the lowest Price Ratio with respect to each of the Observation Dates.

Price Ratio: In respect of a Potential Share, the Final Share Price divided by its Initial Share Price.

Final Share Price: Closing Price of the Potential Share on the Observation Date

Potenital Share	Initial Share Price	Trigger Price	Number of Shares
HSBC (0005.HK)	95.25	95.25	1,080,528
HK Electric (0006.HK)	29.00	29.00	3,549,193

Party A pays:
For Calculation Period 1 – 4: 3-month HIBOR + 0.13%, Q, A/365
For Calculation Period 5 – 12: 3-month HIBOR - 0.17%, Q, A/365

Party B pays:
On Termination Date,
10% if the Trigger Event occurred on Jun 16, 2004;
20% if the Trigger Event occurred on Jun 16, 2005;
30% if the Trigger Event occurred on Jun 15, 2006; or
0% if the Trigger Event never occurs.

Final Exchange: Applicable only if the Trigger Event has never occurred
Party A pays: Notional Amount
Party B delivers: Number of Shares of the Underlying Stock as shown
above

Interest Period Reset Date: 18th of Mar, Jun, Sep, Dec of each year

Party B pays Party A an upfront fee of HKD1,369,500.00 (i.e. 1.65%
on Notional) on Jun 18, 2003.

Corridor swaps

- Allow end user to speculate on the volatility of rate changes.

Example

A fund manager currently holding floating-rate debt securities in his portfolio can enter into either of the following three-year, semiannual settlement swaps:

- Traditional: Pay six-month LIBOR, and receive 5.75 percent.

- Corridor: Year 1 Payment: Six-month LIBOR for days when 4 percent $\leq$ LIBOR $\leq$ 5 percent, 0 on all other days;

Year 2 Payment: Six-month LIBOR
for days when 5 percent $\leq$ LIBOR $\leq$
6 percent, 0 on all other day;
Year 3 Payment: Six-month LIBOR
for days when 6 percent $\leq$ LIBOR $\leq$
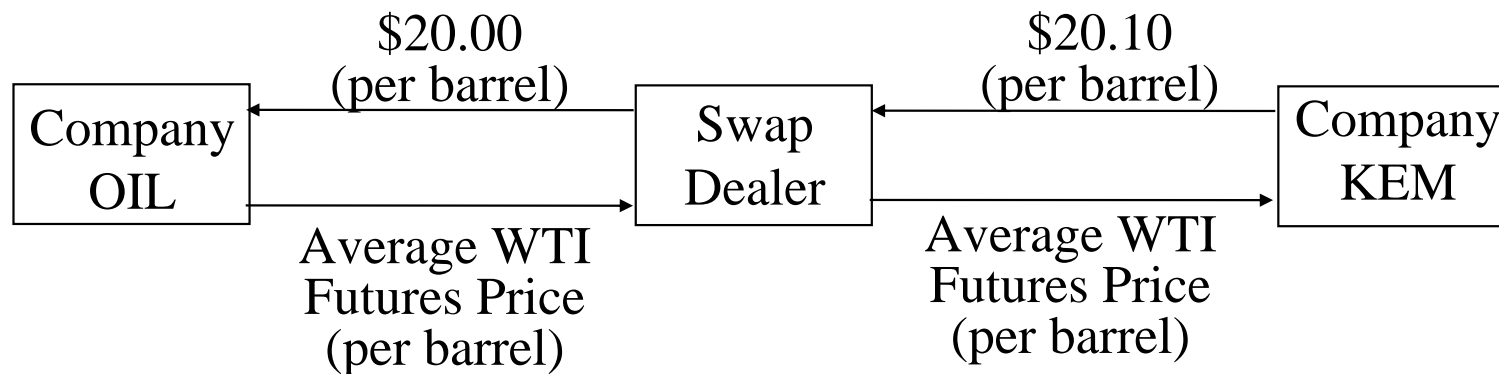7 percent, 0 on all other days; and

Receive fixed rate of 4.50 percent.

Although the traditional swap would allow the manager
to lock in a fixed income of 5.75 percent, the corridor
arrangement offers the possibility of an enhanced
return if LIBOR falls outside of the prescribed range
during a substantial portion of the settlement period.

Commodity swaps

- To fix the price of a commodity over a certain period of time.

Example

Oil Oriented Commodity Swap

$20.00
(per barrel)
$20.10
(per barrel)

Company OIL   Swap Dealer   Company KEM

Average WTI
Futures Price
(per barrel)

Average WTI
Futures Price
(per barrel)

Use of commodity swaps

- It is often prudent for producers (e.g., Company OIL) to enter into receive-fixed commodity swaps to remove price volatility at those times when the operation is particularly vulnerable.

- Commodity swaps can also be used to reverse forward positions previously transacted if the counterparty to those agreements changes its view of the future prices.

By their agreements with the swap deals, OIL is effectively fixing the price of its oil sales at $20.00 a barrel and KEM is fixing the price of its purchase at $20.10.

- If average WTI (West Texas Intermediate) settlement price is $20.75:
  OIL pays to Dealer:
    ($20.75 − $20.00) x (500,000) = $375,000,
  KEM receives from Dealer:
    ($20.75 − $20.10) x (500,000) = $325,000

- If average WTI settlement price is $19.40:
  OIL receives from Dealer:
    ($20.00 − $19.40) x (500,000) = $300,000
  KEM pays to Dealer:
    ($20.10 − $19.40) x (500,000) = $350,000

Mark-to-market swaps

- To reduce the amount of actual credit risk either of the two counterparties to the transaction that must carry during the remaining life of the contract.

- On a settlement date, the counterparties make their respective fixed- and floating-rate payments on the existing swap and then liquidate the remaining portion of the existing swap using the new swap fixed rate.

Mark-to-Market Swap Mechanics

Assume LIBOR and Future Swap Fixed-Rate Paths

Date 0	LIBOR = 4.00%	2.0-year swap fixed rate = 6.00%
Date 1	LIBOR = 4.25%	1.5-year swap fixed rate = 5.75%
Date 2	LIBOR = 4.75%	1.0-year swap fixed rate = 6.25%
Date 3	LIBOR = 5.00%	0.5-year swap fixed rate = 5.00%

Settlement-Date Cash Flows

Date 1   Swap settlement: $(6\% - 4\%)(0.5)(\$100,000,000) = \$1,000,000$

Mark-to-market settlement:

$$\sum_{t=1}^{3}[(0.06-0.0575)(0.5)(100,000,000)] \div (1.02875)^{t} = \$354,428$$

Loan payment: $(4.00\% + 1.00\%)(0.5)(\$100,000,000)$

$= \$2,500,000$

Total payment: $\$3,854,428$

Date 2 Swap settlement: (5.75% - 4.25%)(0.5)($100,000,000)
$$= \$750,000$$

Mark-to-market settlement:

$$\sum_{t=1}^{2}[(0.0575-0.0625)(0.5)(100,000,000)] \div (1.03125)^{t} = -\$477,502$$

Loan payment: (4.25% + 1.00%)(0.5)($100,000,000)
$$= \$2,625,000$$

Total payment: $2,897,498

Indexed amortizing rate swaps

- An IAR swap is a traditional fixed/floating interest rate swap that has been altered to mimic the performance of a mortgage portfolio.

- The end user of the swap will receive cash flows similar to a pool of fixed-rate mortgages and pay cash flows linked to LIBOR.

- The fixed rate received (underlying Treasury yield plus the swap spread) represents the long position in mortgages.

- The floating rate paid (LIBOR) represents the short position in deposits.

Example

A recently negotiated, indexed, amortizing rate swap contract has the following terms:

- Maturity   Three years

- Notional principal   USD 50 million

- Fixed rate   6.50% (actual/365 day count)

- Floating rate index   Six-month LIBOR (actual/360 day count)

- Lockout period   One year

- Payment and amortization frequency

  Semiannually

- Amortization schedule

  If LIBOR < 3.5 percent,
  100 percent amortization
  If LIBOR > 7.5 percent,
  0 percent amortization
  If 3.5% < LIBOR < 7.5%,
  amortize 0.25 percent of notional
  per basis point

- "Clean up" call

  When principal has amortized to
  15 percent of original principal,
  swap terminates

Amortization Schedule for IAR Swap

Period (months)	LIBOR (%)	Reduction (%)	Outstanding Notional Principal ($million)
0-6	6.00	Lockout	50.00
12-Jun	6.75	Lockout	50.00
Dec-81	6.25	31.25	34.38
18-24	5.50	50.00	17.19
24-30	4.00	62.50	6.45
30-36	4.50	-	Swap terminated

- The reduction to $34.38 million on the first amortization date is obtained by

  $50 million x [1 – (750 bps – 625 bps)] x 0.25 per bp.

- The swap terminates at the end of 24-30 month period when the clean-up call limit of 0.15 x 50 millions = $7.5 million is reached.
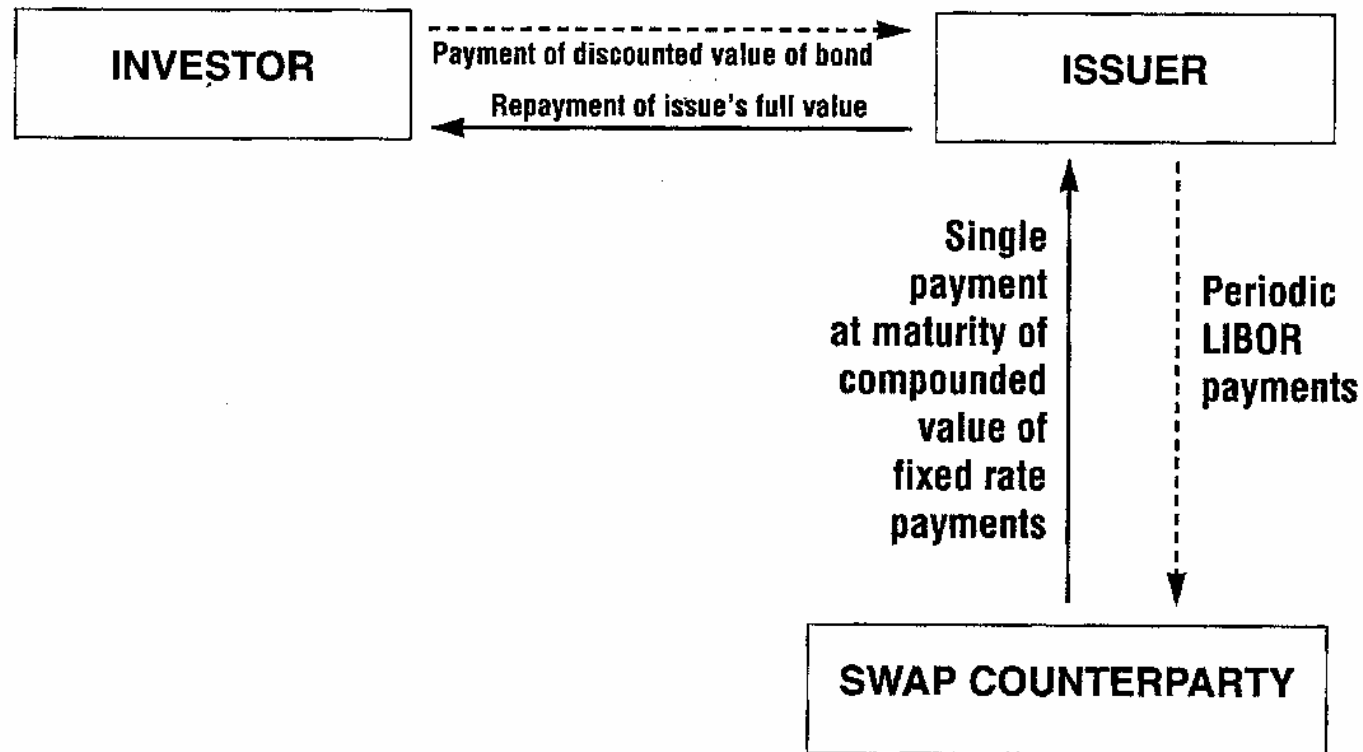
Zero coupon swaps

The fixed payments are compounded and made at the end or the beginning of the swap. They are primarily tax driven structures.

Example

Japan had one of the largest markets in zero coupon swaps in the 1980s since zero coupon bonds were treated very favorably by local tax authorities. A corporate issues a zero coupon bond and enter into a swap with a bank, which would take on the responsibility for the coupon payment and take floating payments from the corporate.

- Zero coupon swaps present greater credit risks since one counterparty is receiving payments on a far less regular basis – can be seen as a loan. Such risk should be structured into the pricing.

A Zero-Coupon Swap on the Back of a Zero-Coupon Bond

| INVESTOR | Payment of discounted value of bond → | ISSUER |
| | ← Repayment of issue's full value | |

Single payment at maturity of compounded value of fixed rate payments (↑)

Periodic LIBOR payments (↓)

SWAP COUNTERPARTY

Forward swaps

Forward swaps are executed now but begin at a preset future date.

Uses They allow asset and liability managers to implement their view of the yield curve.

- Corporations may wish to lock into forward rates in the belief that they will be lower than the spot rate at a future date but at the same time may wish to leave their liabilities floating at an attractive lower rate for a period.

- Municipalities have used them to lock in rates for future debt refinancing.

Suppose a corporation wants to enter into a swap beginning one year's time for a period of 4 years (one-year-by-four-year swap), the swap house will have to enter into two offsetting swaps immediately to hedge its position.
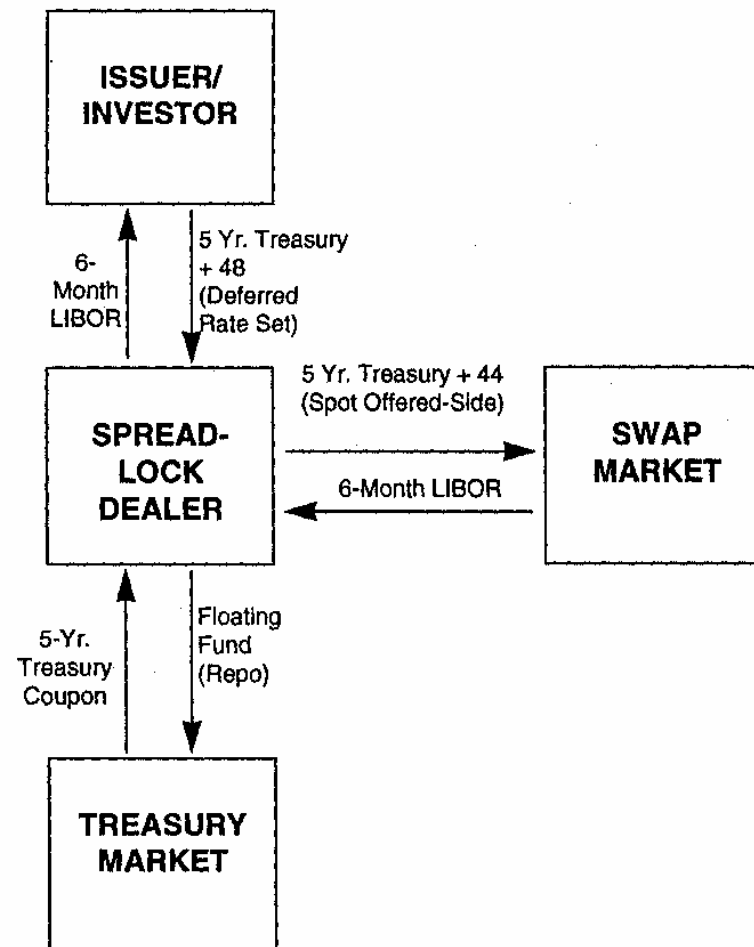
A FORWARD SWAP HEDGE
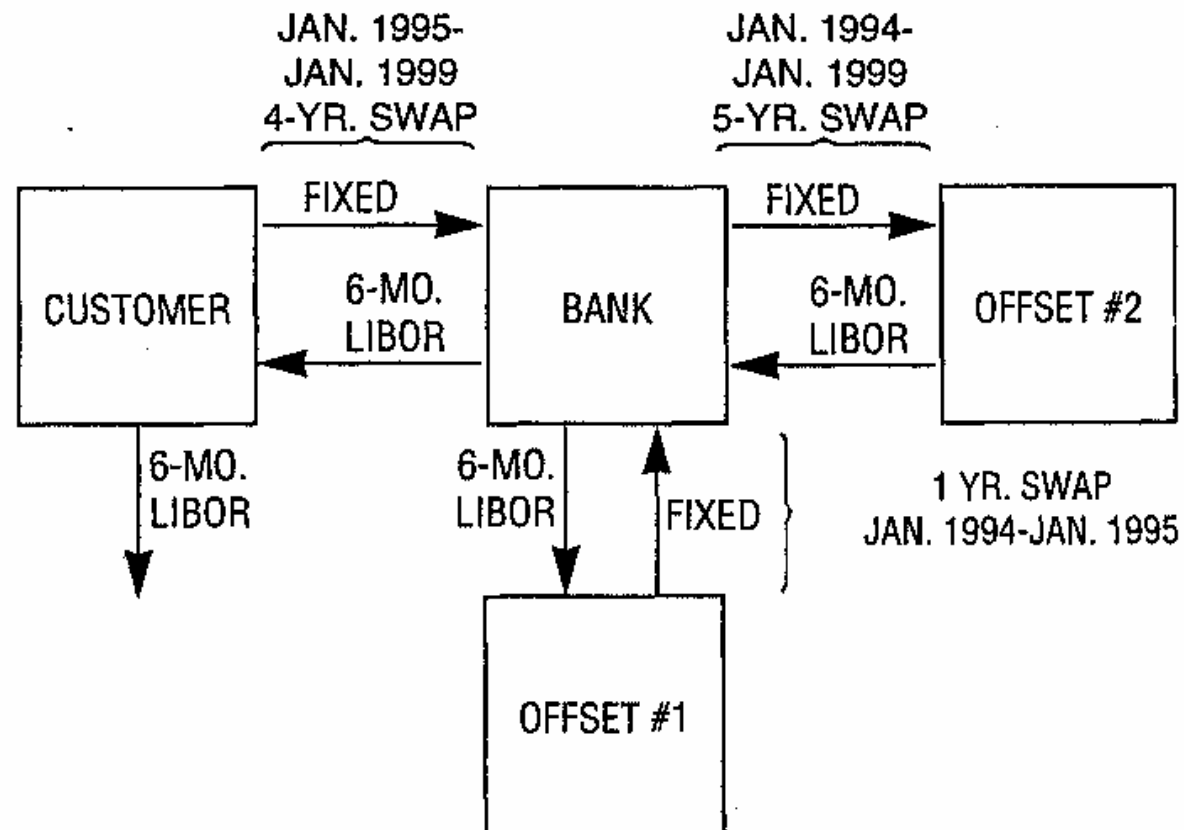
Spread-lock interest rate swaps

Enables an investor to lock in a swap spread and apply it to

an interest rate swap executed at some point in the future.

- The investor makes an agreement with the bank on

(i) swap spread, (ii) a Treasury rate.

- The sum of the rate and swap spread equals the fixed rate paid by the investor for the life of the swap, which begins at the end of the three month (say) spread-lock.

- The bank pays the investor a floating rate. Say, 3-month LIBOR.

    Two hedges are required: The swap trade hedges the spread-lock dealer with respect to changes in the swap spread, and the Treasury transaction hedges the dealer with respect to the changes in the 5-year Treasury rate.

SPREAD-LOCK INTERST RATE SWAP

ISSUER/
INVESTOR

6-
Month
LIBOR

5 Yr. Treasury
+ 48
(Deferred
Rate Set)

SPREAD-
LOCK
DEALER

5 Yr. Treasury + 44
(Spot Offered-Side)

6-Month LIBOR

SWAP
MARKET

5-Yr.
Treasury
Coupon

Floating
Fund
(Repo)

TREASURY
MARKET

LIBOR-in-arrears swaps

The floating rate for each period of the swap is set at the end of

the payment period rather than at the beginning.

- The flow at 3 months is determined by the 3-month against 6-month forward rate, and the flow at 6 months by the 6-month against 9-month forward, and so on.

Pricing issue

The difference in the price between a plain vanilla swap and a LIBOR-in-arrears swap is approximately the difference between the front 3-month period and the forward rate for the last arrears set (24 against 27 months), amortized over the term of the swap. In a positively sloped curve this means that the fixed rate payer will have to pay a higher rates because he is theoretically receiving the higher LIBOR rate.

LIBOR yield curve and associated implied forward rates

Maturity	Current LIBOR	Forward date	Forward rates
6	4.00%	-	-
12	4.10%	6	4.2%
18	4.20%	12	4.4%
24	4.40%	18	5.0%
30	4.60%	24	5.4%

With an upward sloping yield curve, the implied forward LIBOR rates also increase with successively distant investment dates.

LIBOR-in-arrears swaps (cont'd)

Floating rate determination for two-year traditional and arrears swap

Settlement Period (months)	Traditional Swap LIBOR	Arrears Swap LIBOR	Difference
0- 6	4.00%	4.20%	+0.20%
6-12	4.20%	4.40%	+0.2%
12-18	4.40%	5.00%	+0.6%
18-24	5.00%	5.40%	+0.4%

Constant maturity (yield curve) swaps

To exchange cash flows based on the difference between interest rates at two different points on a given yield curve.

- For example, an investor might agree to receive 6-month LIBOR and to pay the yield on 10-year Treasuries minus a spread.

Uses

They allow investors to take a view on the future shape of the yield curve without worrying about convexity or duration.

The fair value of a yield curve swap can be calculated by looking at the forward curves for the shorter and longer maturity. The appropriate swap spread is the constant spread that will balance the present value of the projected long-end and short-end payments.

Accrual swaps

- Interest on one side accrues only when the floating reference rate is within a certain range.

Example

A fixed rate Q is exchanged for 3-month LIBOR every quarter. The fixed rate accrues only on days when 3-month LIBOR is below 8% per annum.

- The fixed-rate payer's position is equivalent to a regular swap plus a series of binary options, one for each day of the life of the swap.