This assignment is related to the comprehension of the following paper. You are required to derive some of the formulas and give your critical views on the validity of the model.

"Investment under uncertainty: The case of replacement and investment decision," David C. Mauer and Steven H. Ott, Journal of Financial and Quantitative Analysis, vol 30(4) p.581-605 (1995).

The paper analyzes the determinants of sequential replacement investment decisions with maintenance and operation cost uncertainty and realistic tax effects. The authors determine the best time to replace a deteriorating asset with a new one that will produce the same product or service. Their model allows for
(i)   stochastic maintenance and operation cost,
(ii)   salvage value that fluctuates with the above cost and tax effects,
(iii)   tax effects include depreciation tax shields, investment tax credit on the purchase price of a new asset, taxation of the capital gain / loss on the sale of replaced asset,
(iv)   uncertainty about the arrival of a technological innovation.
Factors that affect the optimal time between replacements include
(i) volatility of cost, (ii) purchase price of a new asset, (ii) corporate tax rate, (iv) depreciation rate, (v) technological uncertainty, (vi) tax policy.

Please respond to the following questions in your report.

1. Derive the results in eqs. (3)-(5) using the knowledge of the first passage time of a Brownian motion.
2. Derive governing equation (7) using contingent claims pricing theory. How do you understand the term "risk-adjusted drift rate of cost"?
3. Do you agree with the authors' choice of reflecting boundary condition as stated in eq. (10)? Can you propose an alternative boundary condition?
4. The "smooth pasting condition" in eq. (11) is said to be related to the minimization procedure. Give an interpretation of this result.
5. Derive the result on expected replacement cycle as stated in eq. (12). Note that the second term is seen as the adjustment due to the reflecting barrier.
6. Based on the results presented in Section II.B, give a summary of the dependence of the replacement barrier and expected replacement cycle on various parameters in the model. Comment on whether there are results derived from the theoretical model that are counter-intuitive.
7. Explain the relation between the value functions as stated in eq. (14).
8. Derive the governing equations given as eqs (16)-(17). Give the justification of the auxiliary conditions as stated in eqs (18)-(22).
9. Section IIB examines the dependence of the replacement policies on the uncertainty of technological innovation. Give a brief summary of their findings and provide your intuitive understanding of the phenomena.
10. Supply all the necessary auxiliary conditions for the two governing equations under the stochastic tax law model [see eqs (25-26)].

11. Tax policy uncertainty can result in either an increase or a decrease in replacement investment. Give a summary of the arguments presented in Section IVB that explain such phenomenon.

Finally, you are also strongly encouraged to add your comments on (i) validity of the assumptions made in the model, (ii) technical accuracy in their results and interpretations, (iii) potential extension of the model.