Topic 1A

Nature of credit risk

Spread is the yield above the riskfree Treasury rate.

Credit risk consists of two components: default risk and spread risk.

1. Default risk (違約風險) : any non-compliance with the exact specification of a contract.

2. Spread risk: reduction in market value of the contract / instrument due to changes in the credit quality of the debtor / counterparty.

   – price or yield change of a bond as a result of credit rating downgrade

Event of default

1. Arrival risk – timing of the event, modeled by a $\boxed{\text{stopping time } \tau}$

* A stopping rule is defined such that one can determine whether a stochastic process stops or continued, given the information available at that time.

2. Magnitude risk – loss amount / recovery value

   Loss amount = par value (possibly plus accrued interest) –

   market value of a defaultable bond

* The event $\{\tau = t\} \in \mathcal{F}_t$, that is, $\mathcal{F}_t$-measurable ; τ is the random stopping time.
For example, a Target Redemption Note is terminated once the target coupon amount has been collected.

Risk elements

1. Exposure at default / recovery rates – both are random variables
2. Default probability (characterization of the random time of default)
3. Transition probabilities – the process of changing the creditworthiness is called credit migration.

Credit event

Occurs when the calculation agent is aware of publicly available information as to the existence of a credit condition.

- Credit condition means either a payment default or a bankruptcy (清盤) event in respect of the issuer.

- Publicly available information means information that has been published in any two or more internationally recognized published or electronically displayed financial news sources.

* Chapter 11 Bankruptcy

- a chapter of the US Bankruptcy code
- a Company is protected from creditors while it restructures its business, usually by downsizing and narrowing focus.
- keep the organization in tact while seeking protection from creditors.

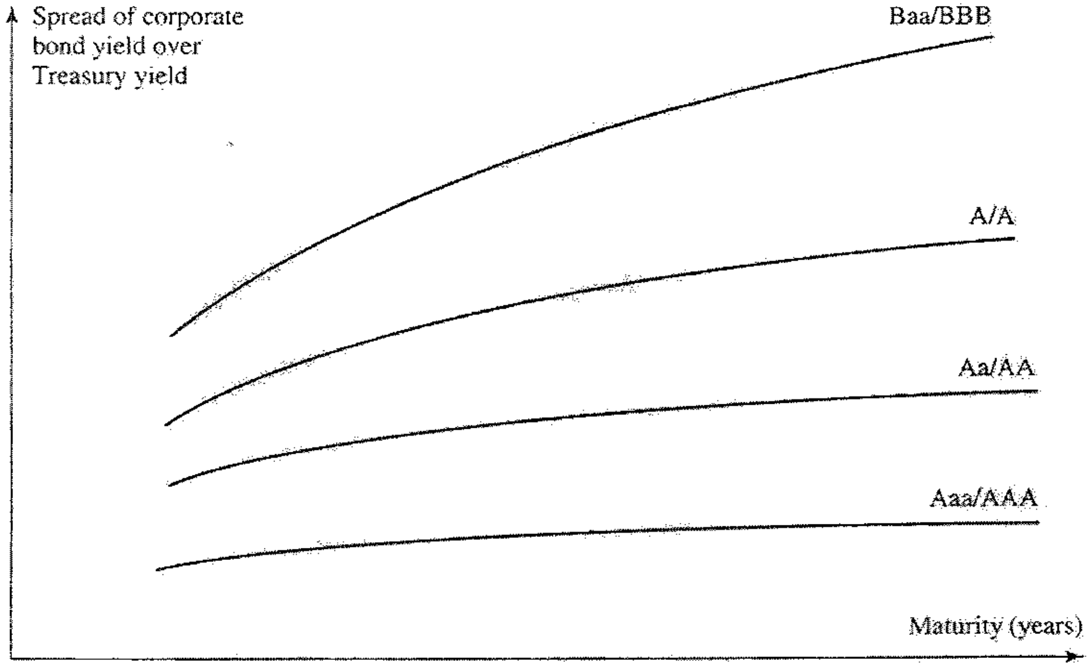
The price of a corporate bond must reflect not only the spot rates for default-free bonds but also a risk premium to reflect default risk and any options embedded in the issue.

Credit spreads: compensate investor for the risk of default on the underlying securities

Construction of a credit risk adjusted yield curve is hindered by

1. The general absence in money markets of liquid traded instruments on credit spread. For liquidly traded corporate bonds, we may have good liquidity on trading of credit default swaps whose underlying is the credit spread.

2. The absence of a complete term structure of credit spreads as implied from traded corporate bonds. At best we only have infrequent data points.



Spread over Treasury zero curve for corporate bond yields

- The spread increases as the rating declines. It also increases with maturity.

- The spread tends to increase faster with maturity for low credit ratings than for high credit ratings.

Bonds – securitized versions of loans and tradeable

Payment structure

1. Upfront payment by the investor to purchase the bond.

2. On the coupon dates, the investor receives coupon (fixed or floating) from the issuer.

3. On bond's maturity date, the issuer pays the par value and the final coupon payment.

- Embedded with various option features
  - callable, puttable, conversion, reset

Bundles of risks embedded

- duration and convexity (sensitivity to the interest rate movement)
- credit risk: risk of default and risk of volatility in credit spreads
- early termination due to recall by issuer: may be hedgeable by swaption
- liquidity: bid-ask spread

Ratings of bonds based on financial variables

- Using the Moody's system, the best rating is Aaa. Next comes Aa, A, Baa, Ba, B and Caa, etc. Only bonds with ratings of Baa or above are called *investment grade*.

Moody's		S & P	
Aaa (not sub-divided)		AAA (not sub-divided)	
	Aa1		AA+
Aa	Aa2	AA	AA
	Aa3		AA-
A		A	
Baa		BBB	
Ba		BB	
B		B	
etc		etc	

Recovery rates

Amounts recovered on corporate bonds as a percent of par value from Moody's Investor's Service

Class	Mean (%)	Standard derivation (%)
Senior secured	52.31	25.15
Senior unsecured	48.84	25.01
Senior subordinated	39.46	24.59
Subordinated	33.17	20.78
Junior subordinated	19.69	13.85

The amount recovered is estimated as the market value of the bond one month after default.

- Bonds that are newly issued by an issuer must have seniority below that of existing bonds issued earlier by the same issuer.

Credit ratings migration

Over time, bonds are liable to move from one rating category to another. The probabilities in the table are based on historical data and are therefore real world probabilities.

One-year transition matrix of percentage probabilities

	Rating at year-end							
Initial rating	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**Default**
AAA	93.66	5.83	0.40	0.09	0.03	0.00	0.00	0.00
AA	0.66	91.72	6.94	0.49	0.06	0.09	0.02	0.01
A	0.07	2.25	91.76	5.18	0.49	0.20	0.01	0.04
BBB	0.03	0.26	4.83	89.24	4.44	0.81	0.16	0.24
BB	0.03	0.06	0.44	6.66	83.23	7.46	1.05	1.08
B	0.00	0.10	0.32	0.46	5.72	83.62	3.84	5.94
CCC	$\boxed{0.15}$ *	0.00	0.29	0.88	1.91	10.28	61.23	25.26
Default	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Sources: Standard & Poor's, January 2001.

* This may represent an isolated case where a CCC-bond moves to AAA within one year.

Modeling default of single name

Market's assessment of the default risk of the obligor (assuming some form of market efficiency – information is aggregated in the market prices). The sources are

- market prices of bonds and other defaultable securities issued by the obligor
- prices of CDS's referencing this obligor's credit risk

How to construct a clean term structure of credit spreads from observed market prices?

⋆ Based on no-arbitrage pricing principle, a model that is based upon and calibrated to the prices of traded assets is immune to simple arbitrage strategies using these traded assets.

Market instruments used in bond price-based pricing

- At time t, the defaultable and default-free zero-coupon bond prices of all maturities $T \geq t$ are known. These defaultable zero-coupon bonds have no recovery at default.

- Information about the probability of default over all time horizons as assessed by market participants are fully reflected when market prices of default-free and defaultable bonds of all maturities are available.

Risk neutral probabilities

The financial market is modeled by a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathcal{F}, Q)$, where Q is the risk neutral probability measure.

- All probabilities and expectations are taken under Q. Probabilities are considered as state prices.

  1. For constant interest rates, the discounted Q-probability of an event A at time T is the price of a security that pays off \$1 at time T if A occurs.
  2. Under stochastic interest rates, the price of the contingent claim associated with A is $E_Q[\beta(T)\mathbf{1}_A]$, where $\beta(T)$ is the discount factor. This is based on the risk neutral valuation principle and the money market account $M(T) = \dfrac{1}{\beta(T)} = e^{\int_t^T r_u\,du}$ is used as the numeraire.

Indicator functions

For $A \in \mathcal{F}$, $\mathbf{1}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{otherwise} \end{cases}$.

τ = random time of default; $I(t)$ = survival indicator function

$$I(t) = \mathbf{1}_{\{\tau > t\}} = \begin{cases} 1 & \text{if } \tau > t \\ 0 & \text{if } \tau \leq t \end{cases}.$$

$B(t,T)$ = price at time t of zero-coupon bond paying off \$1 at T

$\overline{B}(t,T)$ = price of defaultable zero-coupon bond if $\tau > t$;

$$I(t)\overline{B}(t,T) = \begin{cases} \overline{B}(t,T) & \text{if } \tau > t \\ 0 & \text{if } \tau \leq t \end{cases}.$$

Monotonicity properties on the bond prices

1. $0 \leq \overline{B}(t,T) < B(t,T), \quad \forall t < T$

2. Starting at $\overline{B}(t,t) = B(t,t) = 1$,

$$B(t,T_1) \geq B(t,T_2) > 0 \quad \text{and} \quad \begin{array}{l} \overline{B}(t,T_1) \geq \overline{B}(t,T_2) \geq 0 \\ \forall t < T_1 < T_2, \tau > t. \end{array}$$

Independence assumption

$\{B(t,T) | t \leq T\}$ and τ are independent under $(\Omega, \mathcal{F}, Q)$ (not the true measure).

Implied probability of survival in $[t, T]$– based on market prices of bonds

$$B(t,T) = E_Q\left[e^{-\int_t^T r_u\,du}\right] \quad \text{and} \quad \overline{B}(t,T) = E_Q\left[e^{-\int_t^T r_u\,du} I(T)\right].$$

Invoking the independence between defaults and the default-free interest rates

$$\overline{B}(t,T) = E\left[e^{-\int_t^T r_u\,du}\right] E[I(T)] = B(t,T)P(t,T)$$

implied survival probability over $[t,T] = P(t,T) = \dfrac{\overline{B}(t,T)}{B(t,T)}$.

- The *implied default probability* over $[t,T], P_{def}(t,T) = 1-P(t,T)$.

- Assuming $P(t,T)$ has a right-sided derivative in T, the *implied density of the default time*

$$Q[\tau \in (T, T+dT] | \mathcal{F}_t] = -\frac{\partial}{\partial T} P(t,T)\, dT.$$

- If prices of zero-coupon bonds for all maturities are available, then we can obtain the implied survival probabilities for all maturities (complementary distribution function of the time of default).

Probability of default assuming no recovery

Define

$y(T)$:	Yield on a T-year corporate zero-coupon bond
$y^*(T)$:	Yield on a T-year risk-free zero-coupon bond
$Q(T)$:	Probability that corporation will default between time zero and time T
τ :	Random time of default

- The value of a T-year risk-free zero-coupon bond with a principal of 100 is $100e^{-y^*(T)T}$ while the value of a similar corporate bond is $100e^{-y(T)T}$.

- Present value of expected loss from default is

$$\begin{aligned} & 100\{E[e^{-\int_0^T r_u\,du}] - E[e^{-\int_0^T r_u\,du}\mathbf{1}_{\{\tau>T\}}] \\ = \; & 100[e^{-y^*(T)T} - e^{-y(T)T}]. \end{aligned}$$

There is a probability $Q(T)$ that the corporate bond will be worth zero at maturity and a probability $1 - Q(T)$ that it will be worth 100. The value of the bond is

$$\{Q(T) \times 0 + [1 - Q(T)] \times 100\}e^{-y^*(T)T} = 100[1 - Q(T)]e^{-y^*(T)T}.$$

The yield on the bond is $y(T)$, so that

$$100e^{-y(T)T} = 100[1 - Q(T)]e^{-y^*(T)T}$$

or

$$Q(T) = 1 - e^{-[y(T)-y^*(T)]T}.$$

Example

Suppose that the spreads over the risk-free rate for 5-year and a 10-year BBB-rated zero-coupon bonds are 130 and 170 basis points, respectively, and there is no recovery in the event of default.

$$\begin{aligned} Q(5) &= 1 - e^{-0.013\times 5} = 0.0629 \\ Q(10) &= 1 - e^{-0.017\times 10} = 0.1563. \end{aligned}$$

The probability of default between five years and ten years is $Q(5;10)$ where

$$Q(10) = Q(5) + [(1 - Q(5)]Q(5;10)$$

or

$$Q(5;10) = \frac{0.01563 - 0.0629}{1 - 0.0629}$$

- In the event of a default, the bondholder receives a proportion R of the bond's no-default value.

- If there is no default, the bondholder receives 100.

- The bond's no-default value is $100e^{-y^*(T)T}$ and the probability of a default is $Q(T)$.

- The value of the bond is

$$[1 - Q(T)]100e^{-y^*(T)T} + Q(T)100Re^{-y^*(T)T}$$

so that

$$100e^{-y(T)T} = [1 - Q(T)]100e^{-y^*(T)T} + Q(T)100Re^{-y^*(T)T}.$$

This gives

$$Q(T) = \frac{1 - e^{-[y(T)-y^*(T)]T}}{1 - R}.$$

When $R=1$, we must have $y(T) = y^*(T)$.

Forward probability of default

Conditional probability of default in the second year, given that the corporation does not default in the first year.

Year	Cumulative default probability (%)	Default probability in year (%)
1	0.2497	0.2497
2	0.9950	0.7453
3	2.0781	1.0831
4	3.3428	1.2647
5	4.6390	1.2962

$$0.002497 + (1 - 0.002497) \times 0.007453 = 0.009950$$

$$0.009950 + (1 - 0.009950) \times 0.010831 = 0.020781$$