Topic 2A

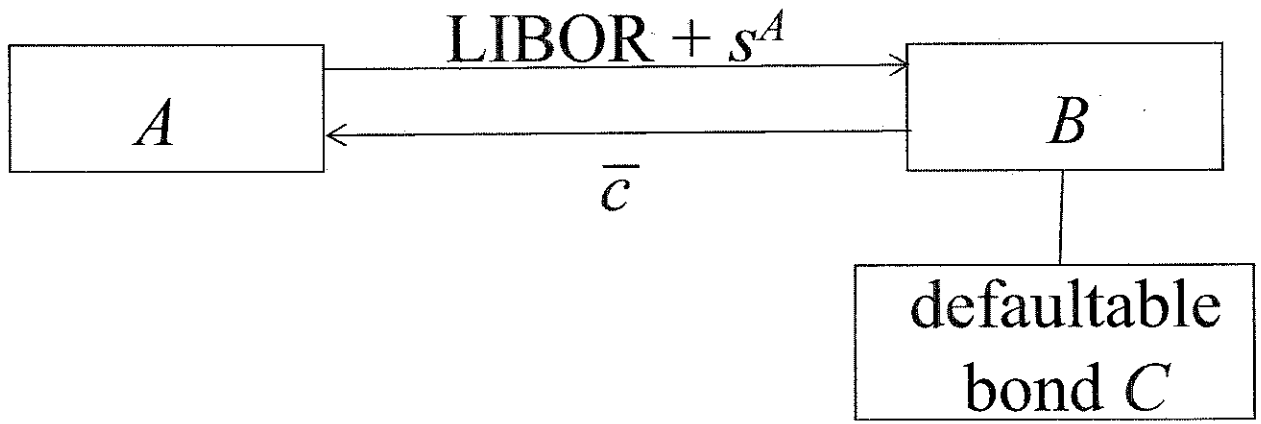
Asset swap

- Combination of a defaultable bond with an interest rate swap.

  B pays the notional amount upfront to acquire the asset swap package.

1. A fixed coupon bond issued by C with coupon $\overline{c}$ payable on coupon dates.

2. A fixed-for-floating swap.



The interest rate swap continues even after the underlying bond defaults.

The asset swap spread s^A is adjusted to ensure that the asset swap package has an initial value equal to the notional (at par value).

- Asset swap package is sold at par.

Asset swaps are more liquid than the underlying defaultable bonds. Provide the flexibility to strip out unwanted coupon stream from the underlying risky bond.

- Asset swaps are done most often to achieve a more favorable payment stream.

For example, an investor is interested to acquire the defaultable bond issuer by a company but he prefers floating rate coupons instead of fixed rate. The whole package of bond and interest rate swap is sold.

1. Default free bond

$$C(t) = \text{time-}t \text{ price of default-free bond with fixed-coupon } \bar{c}$$

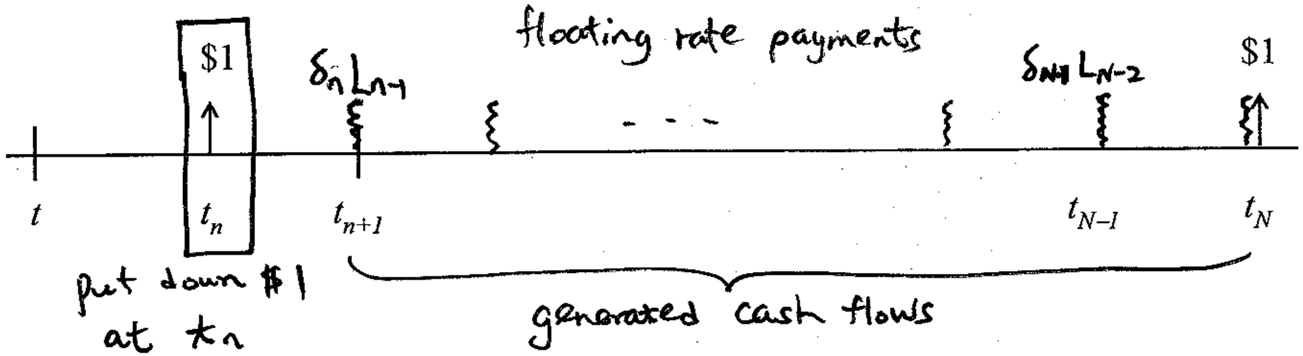
2. Defaultable bond

$$\overline{C}(t) = \text{time-}t \text{ price of defaultable bond with fixed-coupon } \bar{c}$$

The difference $C(t) - \overline{C}(t)$ reflects the premium on the potential default risk of the defaultable bond.

Let $B(t, t_i)$ be the time-t price of a unit par zero coupon bond maturing on t_i. Write δ_i as the accrual period over (t_{i-1}, t_i) using a certain day count convention. Note that δ_i differs slightly from the actual length of the time period $t_i - t_{i-1}$.

Time-t value of sum of floating coupons paid at $t_{n+1}, \cdots, t_N = B(t, t_n) - B(t, t_N)$. This is because \$1 at t_n can generate all floating coupons over $t_{n+1}, \cdots, t_N$, plus \$1 par at t_N.



3. Interest rate swap (tenor is $[t_n, t_N]$; reset dates are $t_n, \cdots, t_{N-1}$ while payment dates are $t_{n+1}, \cdots, t_N$)

$$
\begin{aligned}
s(t) &= \text{forward swap rate at time } t \text{ of a standard fixed-for-floating} \\
&= \frac{B(t, t_n) - B(t, t_N)}{A(t; t_n, t_N)}, \quad t \le t_n
\end{aligned}
$$

where $A(t; t_n, t_N) = \sum_{i=n+1}^{N} \delta_i B(t, t_i)$ = value of the payment stream paying δ_i on each date t_i. The first swap payment starts on t_{n+1} and the last payment date is t_N.

Payoff streams to the buyer of the asset swap package $(\delta_i = 1)$

time	defaultable bond	interest rate swap	net
$t = 0^\dagger$	$-\overline{C}(0)$	$-1 + \overline{C}(0)$	-1
$t = t_i$	$\overline{c}^*$	$-\overline{c} + L_{i-1} + s^A$	$L_{i-1} + s^A + (\overline{c}^* - \overline{c})$
$t = t_N$	$(1 + \overline{c})^*$	$-\overline{c} + L_{N-1} + s^A$	$1^* + L_{N-1} + s^A + (\overline{c}^* - \overline{c})$
default	recovery	unaffected	recovery

$\star$ denotes payment contingent on survival.

† The value of the interest rate swap at $t = 0$ is not zero. The sum of the values of the interest rate swap and defaultable bond is equal to par at $t = 0$.

The asset swap buyer pays \$1 (notional). In return, he receives

1. risky bond whose value is $\bar{C}(0)$;
2. floating leg payments at LIBOR;
3. fixed leg payments at $S^A(0)$;

while he forfeits

4. fixed leg payments at $\bar{c}$.

The two streams of fixed leg payments can be related to annuity. The floating leg payments can be related to swap rate times annuity.

The additional asset spread s^A serves as the compensation for bearing the potential loss upon default.

$s(0)$ = fixed-for-floating swap rate (market quote)

$A(0)$ = value of an annuity paying at \$1 per annum (calculated based on observable default free bond prices)

The value of asset swap package is set at par at $t = 0$, so that

$$\overline{C}(0) + \underbrace{A(0)s(0) + A(0)s^A(0) - A(0)\overline{c}}_{\text{swap arrangement}} = 1.$$

The present value of the floating coupons is given by $A(0)s(0)$. The swap continues even after default so that $A(0)$ appears in all terms associated with the swap arrangement.

Solving for $s^A(0)$

$$s^A(0) = \frac{1}{A(0)}[1 - \overline{C}(0)] + \overline{c} - s(0). \qquad (A)$$

The asset spread s^A consists of two parts [see Eq. (A)]:

(i) one is from the difference between the bond coupon and the par swap rate, namely, $\overline{c} - s(0)$;

(ii) the difference between the bond price and its par value, which is spread as an annuity.

Hedge based pricing – approximate hedge and replication strategies

Provide hedge strategies that cover much of the risks involved in credit derivatives – independent of any specific pricing model.

Rearranging the terms,

$$\overline{C}(0) + A(0)s^A(0) = \underbrace{[1 - A(0)s(0)] + A(0)\overline{c}}_{\text{default-free bond}} \equiv C(0)$$

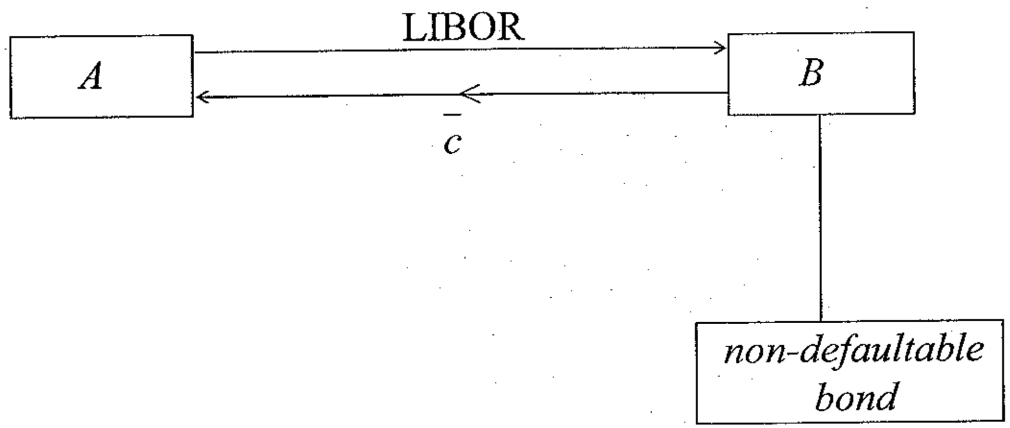
where the right-hand side gives the value of a default-free bond with coupon $\overline{c}$. Note that $1 - A(0)s(0)$ is the present value of receiving \$1 at maturity t_N. We obtain

$$s^A(0) = \frac{1}{A(0)}[C(0) - \overline{C}(0)]. \qquad (B)$$

- The difference in the bond prices is equal to the present value of annuity stream at the rate $s^A(0)$.

Alternative proof

A combination of the non-defaultable counterpart (bond with coupon rate $\overline{c}$) plus an interest rate swap (whose floating leg is LIBOR while the fixed leg is $\overline{c}$) becomes a par floater. Hence, the new asset package should also be sold at par.



The buyer is guaranteed to receive LIBOR floating rate interests plus par.

Value of interest rate swap $= A(0)[S(0) - \overline{c}]$;

value of interest rate swap $+ C(0) = 1$

so $C(0) = 1 - A(0)S(0) + A(0)\overline{c}$.

- The two interest swaps with floating leg at LIBOR $+ s^A(0)$ and LIBOR, respectively, differ in values by $s^A(0)A(0)$.

- Let $V_{swap\_L+s^A}$ denote the value of the swap at $t = 0$ whose floating rate is set at LIBOR $+ s^A(0)$. Both asset swap packages are sold at par. We then have

$$1 = \overline{C}(0) + V_{swap\_L+s^A} = C(0) + V_{swap\_L}.$$

  Hence, the difference in $C(0)$ and $\overline{C}(0)$ is the present value of the annuity stream at the rate $s^A(0)$, that is,

$$C(0) - \overline{C}(0) = V_{swap\_L+s^A} - V_{swap\_L} = s^A(0)A(0).$$

Third alternative proof

Replicate the interest rate swap using the default-free coupon bond with coupon $\bar{c}_i - s^A$.

Let $C'(0)$ denote the time-0 price of the default-free coupon bond with coupons $\bar{c}_i - s^A$.

Since the asset swap is sold at par, we have

$$\underbrace{\text{Value of interest rate swap}}_{1 - C'(0)} + \bar{C}(0) = 1$$

so that $C'(0) = \bar{C}(0)$. Note that holding \$1 at time 0 generates the floating LIBOR interest payments plus \$1 at par at maturity date of the bond. The par at maturity cancels with the negative position of the default free coupon bond.

3 bonds

$C(0)$ = default free bond with fixed coupon $\bar{c}$

$\bar{C}(0)$ = defaultable bond with fixed coupon $\bar{c}$

$C'(0)$ = default free bond with fixed coupon $\bar{c} - S^A(0)$

Apparently, the default risk can be replicated by the annuity of notional equals asset spread $S^A(0)$.

$$S^A(0) = \frac{C'(0) - C(0)}{A(0)} = \frac{\bar{C}(0) - C(0)}{A(0)}.$$

The asset swap holder receives this annuity of $S^A(0)$ as a compensation of the potential default risk.

In-progress asset swap

- At a later time $t > 0$, the prevailing asset spread is

$$s^A(t) = \frac{C(t) - \overline{C}(t)}{A(t)},$$

  where $A(t)$ denotes the value of the annuity over the remaining payment dates as seen from time t.

  As time proceeds, $C(t) - \overline{C}(t)$ will tend to decrease to zero, unless a default happens*. This is balanced by $A(t)$ which will also decrease.

- The original asset swap with $s^A(0) > s^A(t)$ would have a positive value. Indeed, the value of the asset swap package at time t equals $A(t)[s^A(0) - s^A(t)]$. This value can be extracted by entering into an offsetting trade.

*A default would cause a sudden drop in $\overline{C}(t)$, thus widens the difference $C(t) - \overline{C}(t)$.