MATH 4321 – Game Theory

Mid-term Test, 2019

Time allowed: 80 minutes *Instructor: Prof. Y. K. Kwok*

[points]

1. A sprinter hurts her leg today. An x-ray suggests that it is broken with probability 0.2. The problem is deciding whether she should participate in next week's tournament. If she runs, she thinks she will win with probability 0.1. If the leg is broken and she runs, then it will be further damaged and the payoffs are as follows:

   +100 if she wins the race and the leg is not broken

   +50 if she wins and the leg is broken

   0 if she loses and the leg is not broken

   −50 if she loses and the leg is broken

   −10 if she does not run and the leg is broken

   0 if she does not run and the leg is not broken

   (a) Draw the decision tree for this problem. What is the best choice of action and its expected payoff? **[4]**

   (b) Suppose the sprinter can gather exact information by having more tests. Find the value of perfect information about the state of her leg. **[4]**

2. Consider the following payoff matrix A for a two-person zero-sum game:

$$A = \begin{pmatrix} -4 & 2 & -1 \\ -4 & 1 & 4 \\ 0 & -1 & 5 \end{pmatrix}.$$

   Note that A^{-1} does not exist since the last row equals the second row minus the first row.

   (a) Use dominance to solve for the optimal mixed strategies though A has no inverse. Find the value of the game. **[6]**

   (b) Verify the Equality of Payoff Theorem once you have found the saddle point mixed strategies. **[3]**

   Hint Find the row and column strategies that are dominated so that it suffices to consider a reduced 2×2 matrix for optimal mixed strategies.

3. A strategy profile P is said to be weakly Pareto-dominating another profile Q if all players have higher or equal payoff under profile Q, with strict inequality for at least one player. Prove or disprove: If a strategy profile weakly Pareto-dominates all other strategy profiles, then it must be a Nash equilibrium. **[5]**

4. Player 1 is a police officer who must decide whether to patrol the streets or to hang out at the coffee shop. His payoff from hanging out at the coffee shop is 10, while his payoff from patrolling the streets depends on whether he catches a robber, who is player 2. If the robber prowls the streets then the police officer will catch him and obtain a payoff of 20. If the robber stays in his hideaway then the officer's payoff is 0. The robber must choose between staying hidden or prowling the streets. If he stays hidden then his payoff is 0, while if he prowls the streets his payoff is -10 if the officer is patrolling the streets and 10 if the officer is at the coffee shop.

   (a) Write down the matrix form of this two-person nonzero game. **[2]**

   (b) Find and plot the best response function of each player. **[4]**

   (c) Find all possible Nash equilibriums of this game, which may be pure or mixed. Also, compute the value of the game to both players at each Nash equilibrium. **[2]**

   Hint The number of Nash equilibriums can be one, two or three.

5. Suppose two merchants have to choose a location along the straight road. They may choose any point in $\{1, 2, \cdots, n\}$. Assume there is exactly one customer at each of these points and a customer will always go to the nearest merchant. If the two merchants are equidistant to a customer then they share that customer, that is, $\frac{1}{2}$ the customer goes to each store. For example, if $n = 11$ and if the player I chooses location 3 and player II chooses location 8, then the payoff to I is 5 and the payoff to II is 6.

   (a) Suppose $n = 5$. Find the bimatrix and find the Nash equilibrium. **[4]**

   (b) Find the Nash equilibrium in general if $n = 2k + 1$, that is, n is an odd integer. Can we find a Nash equilibrium when n is even? Explain your answer. **[6]**

— End —