# **1.1 Nature of credit risk**

Credit risk consists of two components: default risk and spread risk

- 1. Default risk: any non-compliance with the exact specification of a contract.
- 2. Spread risk: reduction in market value of the contract / instrument due to changes in the credit quality of the debtor / counterparty.
  - price or yield change of a bond as a result of credit rating downgrade

#### Event of default

- 1. Arrival risk timing of the event
- 2. Magnitude risk loss amount / recovery value

#### Risk elements

- 1. Exposure at default / recovery rates both are random variables
- 2. Default probability
- 3. Transition probabilities the process of changing the creditworthiness is called credit migration.

### **Credit event**

Occurs when the calculation agent is aware of publicly available information as to the existence of a credit condition.

- Credit condition means either a payment default or a bankruptcy event in respect of the issuer.
- Publicly available information means information that has been published in any two or more internationally recognized published or electronically displayed financial news sources

*Payment default* means, subject to a dispute in good faith by the issuer, either the issuer fails to pay any amount due of the reference asset, or any other present or future indebtedness of the issuer for or in respect of money borrowed or raised or guaranteed.

*Cross-default clauses* on debt contracts are such that when the firm misses a single payment on a debt, it is declared in default on all its obligations.

*Bankruptcy* event means the declaration by the issuer of a general moratorium (legal authorization of delay of payment of debt) or rescheduling of payments on any external indebtedness. In practice, bankruptcy corresponds to the situation where the firm is liquidated, and the proceeds from the asset sale is distributed to the various claim holders according to pre-specified priority rules.

## **Bonds – securitized versions of loans and tradeable**

#### Payment structure

- 1. Upfront payment by the investor to purchase the bond.
- 2. On the coupon dates, the investor receives coupon (fixed or floating) from the issuer.
- 3. On bond's maturity date, the issuer pays the par value and the final coupon payment.

#### Bundles of risks embedded

- duration and convexity (sensitivity to the interest rate movement)
- credit risk: risk of default and risk of volatility in credit spreads
- early termination due to recall by issuer
- liquidity

Static hedge versus dynamic hedge: How to manage duration, convexity and callability risk independent of the bond position?

# Pricing of credit derivatives and rating of credit linked notes whose payoff depends on certain credit event.

- Unambiguous definition of the credit event bankruptcy, downgrade, restructuring, merger, payment default, etc.
- Possibility of default default probability and hazard rate.
- •Recovery value and settlement risk.
- Correlation of defaults between obligors / risky assets.

#### Remarks

- The *variability* of default risk within a loan portfolio can be substantial. The highest default probability is significantly larger than the smallest default probability.
- 2. The *correlation* between default risks of different borrowers is generally low (that is, low joint default frequency), though it can be significant for related companies, and smaller companies within the same domestic industry sector.
- 3. The lack of correlation would increase the difficulty of hedging portfolio default risk with tradable instruments. The best resort to reduce default risk is *diversification*.

## **Modeling default risk**

#### Cumulative risk of default

This measures the total default probability of an obligor over the term of the obligation.

Some basic techniques

- Credit ratings, if the companies have been rated.
- Calculate key accounting ratios using the firm's financial data, then compared with the comparable median for rated firms allow a rating equivalent to be determined.
- KMV model based on stock price dynamics (for listed companies).

Credit spread: compensate investor for the risk of default on the underlying securities

spread = yield on the loan – riskfree yield

Construction of a credit risk adjusted yield curve is hindered by

- 1. The absence in money markets of liquid traded instruments on credit spread.
- 2. The absence of a complete term structure of credit spreads. At best we only have infrequent data points.

## Term structure of credit spreads

The price of a corporate bond must reflect not only the spot rates for default-free bonds but also a risk premium to reflect default risk and any options embedded in the issue.

Simple approach

- 1. Take the spot rates that are used to discount the cash flows of corporate bonds to be the Treasury sport rates plus a *constant* credit spread.
- 2. Since the credit spread is expected to increase with maturity, we need a *term structure for credit spreads*.

#### Difficulty

Unlike Treasury securities, there are no issuers that offer a sufficiently wide range of corporate zero-coupon securities to construct a zero-coupon spread curve.

Туре	Maturity	Price per \$1 par	Yield (%)
Treasury	1 year	0.930	7.39
Corporate	1 year	0.926	7.84
Treasury	2 years	0.848	8.42
Corporate	2 years	0.840	8.91

- For the one-year corporate security, the 4-cent difference produces a credit spread of 45 basis points.
- Price of corporate zero = Price of Treasury zero × (1 – probability of default) so probability of default of one-year security = $1 - \frac{0.926}{0.930} \approx 0.0043$ probability of default of two-year security =  $1 - \frac{0.848}{0.840} \approx 0.0095$ .
- Forward probability of default: conditional probability of default in the second year, given that the corporation does not default in the first year.

## **Survival function**

T = continuous random variable that measures the default time

$$F(t) = P[T \le t], \qquad t \ge 0$$

Survival function = S(t) = 1 - F(t) = P[T > t]

density function 
$$= f(t) = F'(t) = -S'(t)$$
  
 $= \lim_{\Delta \to 0} \frac{P[t \le T < t + \Delta]}{\Delta}.$ 

 $_{t}q_{x}$  = conditional probability that risky security will default within the next t years conditional on its survival for x year

$$= P[T - x \le t \mid T > x], \quad t \ge 0$$

$$p_x = 1 - p_x, \qquad t \ge 0.$$
$$S(t) = p_0$$

For t = 1, we write

$$p_x = P[T - x > 1 | T > x]$$

- $\mathbf{q}_x = P[T x \le 1 \mid T > x]$ 
  - = marginal default probability
  - = probability of default in the next year conditional on the survival until the beginning of the year

A credit curve is simply the sequence of  $q_0, q_1, \dots, q_n$  in discrete models.

## Hazard rate function

Gives the instantaneous default probability for a security that has survived up to time *x* 

$$h(x)\Delta x = \frac{f(x)}{1 - F(x)}\Delta x \approx \frac{F(x + \Delta x) - F(x)}{1 - F(x)} = P[x < T \le x + \Delta x \mid T > x]$$

so that

So that  

$$h(x) = -\frac{S'(x)}{S(x)}, \text{ giving } S(t) = e^{-\int_0^t h(s) \, ds}.$$

$${}_t p_x = e^{-\int_0^t h(s+x) \, ds}$$

$${}_t q_x = 1 - e^{-\int_0^t h(s+x) \, ds}.$$
Also,  $F(t) = 1 - S(t) = 1 - e^{-\int_0^t h(s) \, ds}$  and  $f(t) = S(t) h(t).$ 

When the hazard rate is constant, then

$$f(t) = h \ e^{-ht}.$$

## **Reduced form approach**

*Default* is modeled as a *point process*. We are not interested in the event itself but the sequence of random times at which the events occur.

- Over  $[t, t + \Delta t]$  in the future, the probability of default, conditional on no default prior to time *t*, is given by  $h_t \Delta t$ , where  $h_t$  is referred to as the hazard rate process.
- Let  $\Gamma$  denote the time of default Conditional probability of default over  $[t, t + \Delta t]$ , given survival up to time *t*, is

$$P_r\left[t < \Gamma \le t + \Delta t \middle| \ \Gamma > t\right] = h_t \Delta t.$$

## **Comparison between reduced form model and structural model**

This is an alternative to Merton's structural model. In Merton's model, the default occurs when the value of the firm falls below a pre-specified deterministic threshold (liabilities of the firm). In this case, the default time is then *predictable*.

- The default occurs as a complete surprise. It allows to add some randomness to the default threshold.
- It loses the micro-economic interpretation of the default time (the model comes from the reliability theory), but traders do not care for the purpose of pricing.

• Survival probability

$$P_r[\Gamma > t] = \exp\left(-\int_0^t h_s \ ds\right).$$

- Suppose we write  $P_r[t < \Gamma \le t + \Delta t] = f_t \Delta t$ , where  $f_t$  is the density function of the default time, we then have  $f_t = h_t \exp\left(-\int_0^t h_s \, ds\right)$ .
- Probability of surviving until time *t*, given survival up to  $s \le t$ ,  $P_r[\Gamma > t | \Gamma > s] = \frac{P_r[\Gamma > t]}{P_r[\Gamma > s]} = \exp\left(-\int_s^t h_s \, du\right).$
- Default in  $(t, t + \Delta t]$ , conditional on no default up to time *s*,  $P_r[t < \Gamma \le t + \Delta t | \Gamma > s] = h_t \exp\left(-\int_s^t h_u \, du\right) \Delta t.$

Suppose  $h_t$  is a random process with dependence on the history of a vector of macro-economic/firm specific random variables. Then

$$P_r[\Gamma > t] = E\left[\exp\left(-\int_0^t h_s \ ds\right)\right]$$

where the expectation is taken over all possible paths of the Brownian process.

#### Information set

Let  $G_t$  denote the information set or filtration such that  $h_t$  is a process adapted to  $G_t$ . Also, we let  $I_t$  denote the information set which tells whether default has occurred. The union of  $G_t$  and  $I_t$  is the full information  $\mathcal{F}_t$  that contains information about the path history of the state variable process and the default history.

# Value of credit-risky discount bond

 $B_t$  = value of a unit initialized money market account

$$= \exp\left(\int_{0}^{t} r_{s} ds\right)$$
 [without default risk],  
where  $r_{s}$  is in general stochastic.

 $Q_{ab}$  = value at time *a* of a credit - risky discount bond that matures at time *b* 

$$= E_{a} \left[ \exp \left( -\int_{a}^{b} r_{s} ds \right) \mathbf{1}_{\{\Gamma > b\}} \right]$$
$$= B_{a} E_{a} \left[ \frac{\mathbf{1}_{\{\Gamma > b\}}}{B_{b}} \right]$$

where  $E_a$  denotes the conditional expectation (in risk neutral measure) given the full information set  $F_a$  up to time a. We need to specify the process that drives the occurrence of default.

Value of default free coupon-bearing bond (continuous model)

$$\frac{dB}{dt} + k(t) = r(t)B, \qquad t < T$$
$$B(T) = F$$
$$B(t) = e^{-\int_{t}^{T} r(s) ds} \left[ F + \int_{t}^{T} k(u)e^{-\int_{u}^{T} r(s) ds} du \right]$$

Over time increment dt, change in bond value is  $\frac{dB}{dt}dt$  and coupon received is k(t) dt. The above sum must equal to the riskless return r(t)B(t) dt

