2.1 Merton's firm value model

- Built upon a stochastic process of the firm's value. [This is not the book value of the assets, but more like the value that the firm can be sold – including good view.]
- Aim to provide a link between the prices of equity and all debt instruments issued by one particular firm
- Default occurs when the firm value falls to a low level such that the issuer cannot meet the par payment at maturity or coupon payments.

Potential applications

1. Relative value trading between shares and debt of one particular issuer.
2. Default risk assessment of a firm based upon its share price and fundamental (balance sheet) data.

Credit valuation model

1. Credit risk should be measured in terms of *probabilities and mathematical expectations*, rather than assessed by qualitative ratings.

2. Credit risk model should be based on current, rather than historical measurements. The relevant variables are the *actual market values rather than accounting values*. It should reflect the development in the borrower's credit standing through time.

3. An assessment of the future earning power of the firm, company's operations, projection of cash flows, etc., has already been made by the aggregate of the market participants in the stock market. The stock price will be the first to reflect the changing prospects. The challenge is how to *interpret the changing share prices* properly.

4. The various liabilities of a firm are *claims on the firm's value*, which often take the form of options, so the credit model should be consistent with the theory of option pricing.

Firm value

- The value of a firm is the value of its business as a going concern. The firm's business constitutes its assets, and the present assessment of the future returns from the firm's business constitutes the current value of the firm's assets.
- The value of the firm's assets is different from the bottom line on the firm's balance sheet. When the firm is bought or sold, the value traded is the ongoing business. The difference between the amount paid for that value and the amount of book assets is usually accounted for as the "good will".
- The value of the firm's assets can be measured by *the price at which the total of the firm's liabilities can be bought or sold*. The various liabilities of the firm are *claims on its assets*. The claimants may include the debt holders, equity holder, etc.
- market value of firm asset

  = market value of equity + market value of bonds

  = share price $times$ no. of shares + sum of market bond prices

Assumptions in the Merton model

1. The firm asset value V_t evolves according to

$$\frac{dV}{V} = \mu\,dt + \sigma\,dZ$$

   $\mu =$ instantaneous expected rate of return.

2. The liabilities of the firm consist only of a single debt with face value F. The debt is assumed not to have coupon nor embedded option feature.

3. The debt is viewed as a contingent claim on the firm's asset. Default can be triggered only at maturity and this occurs when $V_T < F$.

4. Upon default, the firm is liquidated at zero cost and all the proceeds from liquidation are transferred to the debt holder.

The terminal payoff of this contingent claim can be expressed as

$$\min(V_T, F) = F - \underbrace{(F - V_T)^+}_{\text{put payoff}}, \quad x^+ = \max(x, 0).$$

The debt holders have sold a put to the issuer – right to put the firm assets at the price of face value F.

Under the setting of the Black-Scholes framework (frictionless & competitive market)

- unlimited access to short selling and no indivisibilities of assets,
- borrowing and lending through a money-market account can be done at the same riskless, continuously compounded rate r,
- agents are price takers, that is, trading in assets has no effect on prices,
- no transaction costs.

The ability to form a perfectly hedged portfolio is a sufficient conditon for the derivation of a pricing equation that is free of preferences.

Let $\overline{B}(V,t)$ denote the value of the risky debt at time t, then

$$\frac{\partial \overline{B}}{\partial t} + \frac{\sigma^2}{2} V^2 \frac{\partial^2 \overline{B}}{\partial V^2} + rV \frac{\partial \overline{B}}{\partial V} - r\overline{B} = 0,$$
$$\overline{B}(V,T) = F - \max(F - V_T, 0).$$

The solution is given by

$$\overline{B}(V,\tau) = Fe^{-r\tau} - P_E(V,\tau), \quad \tau = T - t$$
$$P_E(V,\tau) = Fe^{-r\tau} N(-d_2) - VN(-d_1) = \text{ expected loss}$$
$$d_1 = \frac{\ln \frac{V}{F} + \left(r + \frac{\sigma^2}{2} \right) \tau}{\sigma \sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau}.$$

Equity value $E(V,\tau) = V - \overline{B}(V,\tau)$

$$= V - [Fe^{-r\tau} - P_E(V,\tau)] = C_E(V,\tau),$$

by virtue of the put-call parity relation. The result is not surprising since the shareholders have the contingent claim $\max(V_T - F, 0)$ at maturity.

Risk neutral valuation gives the relative value of the contingent claims in term of the value of the underlying asset.

Write the expected loss as

$$N(-d_2)\left[Fe^{-r(T-t)} - \frac{N(-d_1)}{N(-d_2)}V\right],$$

where $\dfrac{N(-d_1)}{N(-d_2)}$ is considered as the *expected discounted recovery rate*.

$\overline{B}$ = present value of par − default probability × expected discounted loss given default

and

default probability $= N(-d_2) = P_r[V_T \leq F]$.

Numerical example

Data

$V_t = 100, \sigma_V = 40\%, \ell_t =$ quasi-debt-leverage ratio $= 60\%$,

$T - t = 1$ year and $r = \ln(1 + 5\%)$.

Calculations

1. Given $\ell_t = \dfrac{Fe^{-r(T-t)}}{V} = 0.6$,

   then $F = 100 \times 0.6 \times (1 + 5\%) = 63$.

2. Discounted expected recovery value

$$= \frac{N(-d_1)}{N(-d_2)} V = \frac{0.069829}{0.140726} \times 100 = 49.62.$$

3. Expected discounted shortfall amounts $= 63 - 49.62 = 10.38$.

4. Cost of default = put value

$$= N(-d_2) \times \text{ expected discounted shortfall}$$

$$= 14.07\% \times 10.38 = 1.46;$$

   value of credit risky bond is given by $60 - 1.46 = 58.54$.

Term structure of credit spreads

The yield to maturity of the risky debt $Y(\tau) = -\frac{1}{\tau} \ln \frac{\overline{B}(V, \tau)}{F}$.

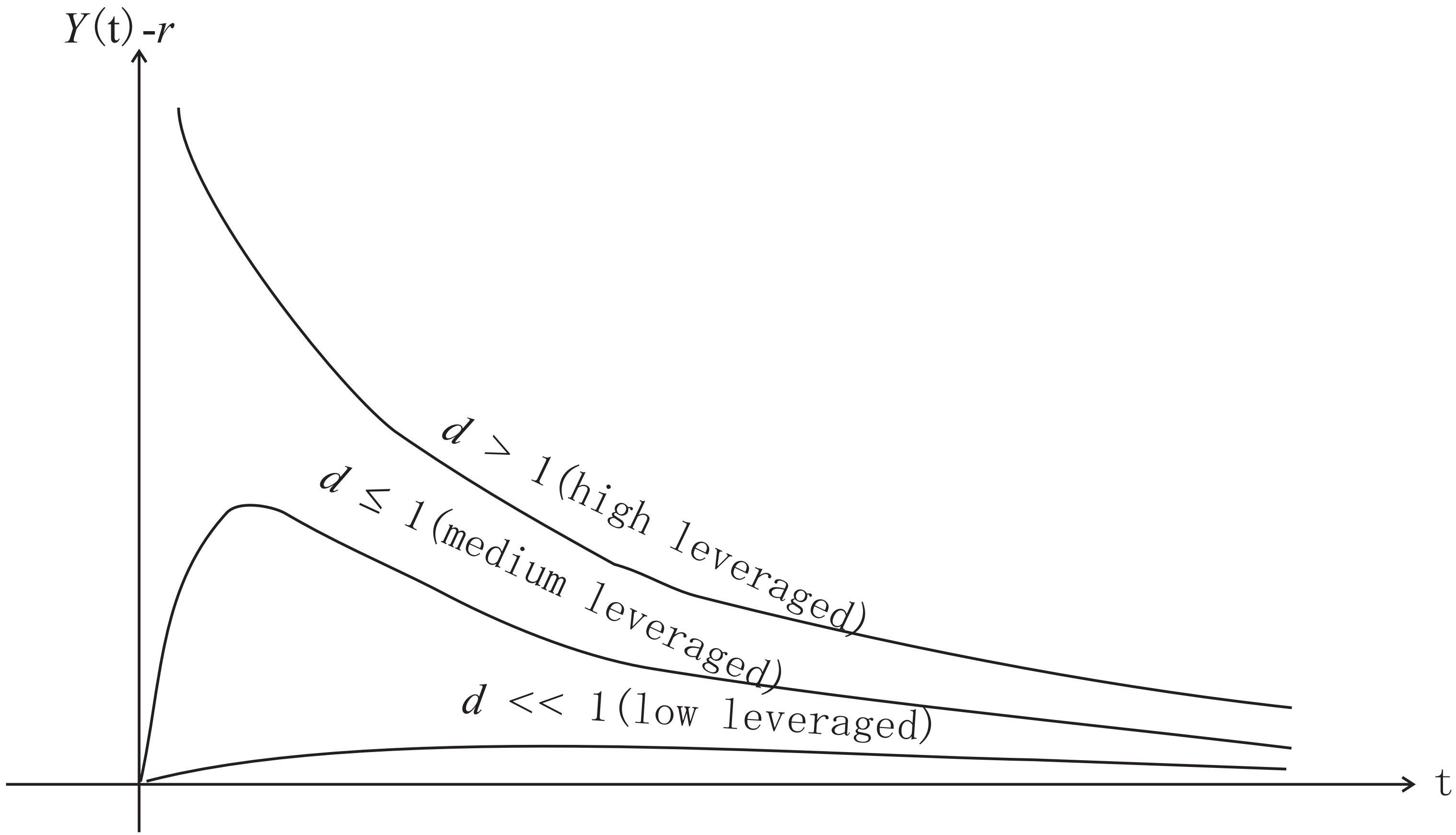
Credit spread $= Y(\tau) - r = -\frac{1}{\tau} \ln \left(N(d_2) + \frac{1}{d} N(d_2) \right)$, where

$$d = \frac{Fe^{-r\tau}}{V} = \text{“quasi” debt-to-asset ratio,}$$

$$d_1 = \frac{\ln d}{\sigma\sqrt{\tau}} - \frac{\sigma\sqrt{\tau}}{2}, \quad d_2 = -\frac{\ln d}{\sigma\sqrt{\tau}} - \frac{\sigma\sqrt{\tau}}{2}.$$

The credit spread $Y(\tau) - r$ is a function of d and $\sigma^2\tau$.

The time-dependent behaviors of the credit spread depends on whether $d \geq 1$ or $d < 1$ (see Figure).



- As time approaches maturity, the credit spread always tends to zero when $d \leq 1$ but tends toward infinity when $d > 1$.

- At times far from maturity, the credit spread has low value for all values of d since sufficient time has been allowed for the firm value to have a higher potential to grow beyond F.

Time dependent behaviors of credit spreads

- Downward-sloping for highly leveraged firms.
- Humped shape for medium leveraged firms.
- Upward-sloping for low leveraged firms.

Possible explanation

- For high-quality bonds, credit spreads widen as maturity increases since the upside potential is limited and the downside risk is substantial.

Remark

Most banking regulations do not recognize the term structure of credit spreads. When allocating capital to cover potential defaults and credit downgrades, a one-year risky bond is treated the same as a ten-year counterpart.

Shortcomings

1. Default can never occur by surprise since the firm value is assumed to follow a diffusion process – may be partially remedied by introducing jump effect into the firm value process.

2. Actual spreads are larger than those predicted by Merton's model.

3. Default premiums are shown to be inversely related to firm size as revealed from empirical studies. In Merton's model, $Y(\tau) - r$ is a function of d and $\sigma^2\tau$ only.

Reference

H.Y. Wong and Y.K. Kwok, "Jump diffusion model for risky debts: quanlity spread differentials," *International Journal of Theoretical and Applied Finance*, vol. 6(6) (2003) p.655-662.

Example – Risky commodity-linked bond

- A silver mining company offered bond issues backed by silver. Each $\$1,000$ bond is linked to 50 ounces of silver, pays a coupon rate of 8.5% and has a maturity of 15 years.
- At maturity, the company guarantees to pay the holders either $\$1,000$ or the market value of 50 ounces of silver.

Rationale The issuer is willing to share the potential price appreciation in exchange for a lower coupon rate or other favorable bond indentures.

Terminal payoff of bond value

$$\overline{B}(V,S,T) = \min(V, F + \max(S - F, 0)),$$

where V is the firm value, r is the interest rate, S is the value of 50 ounces of silver, F is the face value.

2.2 Extensions of structural approach of pricing risking debts

1. *Interest rate uncertainty*

   Debts are relatively long-term interest rate sensitive instruments. The assumption of constant rates is embarrassing.

   - Jump-diffusion process of the firm value.
   - Allows for a jump process to shock the firm value process.
   - Remedy the realistic small short-maturity spreads in pure diffusion model. Default may occur by surprise.

2. *Bankruptcy– triggering mechanism*

   Black-Cox (1976) assume a cut-off level whereby intertemporal default can occur. The cut-off may be considered as a safety covenant which protects bondholder or liability level for the firm below which the firm bankrupts.

3. *Deviation from the strict priority rule*

   Empirical studies show that the absolute priority rule is enforced in only 25% of corporate bankruptcy cases. The write-down of creditor claims is usually the outcome of a bargaining process which results in shifts of gains and losses among corporate claimants relative to their contractual rights.

Quality spread differentials between fixed rate debt and floating rate debt

- In *fixed rate debts*, the par paid at maturity is fixed.

- A *floating rate debt* is similar to a *money market account*, where the par at maturity is the sum of principal and accured interests. The amount of accrued interests depends on the realization of the stochastic interest rate over the life of bond.

  What is the appropriate proportion of debts into either fixed rate or flating rate debts?

  1. Balance sheet duration
  2. Current interest rate environment
  3. Peer group practices.

Whether the default premiums demanded by investors are equal for both types of debts?

Related question: Deos the swap rate in an interest rate swap depend on which party is serving as the fixed rate payer?

Empirical studies reveal that the yeeld premiums for fixed rate debts are in general higher than those for floating rate debts. Why? On the other hand, when the yield curve is upward sloping, floating rate debt holders should demand a higher floating spread.

Interest rate dynamics

The short rate r_t follows the Vasicek model

$$dr_t = k(\theta - r_t)\,dt + \sigma_r\,dZ_r.$$

The price of the default free discount bond with unit par is given by

$$P = a(T)e^{-rb(T)}$$

where

$$\begin{aligned} b(T) &= \frac{1 - e^{-kT}}{k}, \\ a(T) &= \exp\left(-\left(\theta + \frac{\sigma\lambda}{k} - \frac{\sigma^2}{2k^2}\right)[T - b(T)]\right) - \frac{\sigma_r^2 b(T)^2}{4k}, \end{aligned}$$

λ is the markt price of risk. The dynamics of P is given by

$$\frac{dP}{P} = r\,dt + \sigma_P\,dZ_r, \quad \text{where} \quad \sigma_P = b(T)\sigma_r.$$

Firm value process

Under risk neutral meausre Q

$$\frac{dV}{V} = r\,dt + \sigma_r\,dZ_V.$$

Fixed rate debt

The price of fixed rate debt B_X of face amount F_X is the present value of the maturity payoff

$$\min(V(T), F_X) = V(T) - \max(V(T) - F_X, 0).$$

Now, $B_X = PE_{Q^T}[\min(V(T), F_X)]$, where Q^T is the forward measure. Under Q^T, the variance of the firm value expressed in units of P is given by

$$\sigma_X^2 = \int_0^T [\sigma_P^2(t) + 2\rho\sigma_V\sigma_P(t) + \sigma_V^2]\, dt, \quad \text{where} \quad \rho\, dt = dZ_r\, dZ_V.$$

Performing for the expectation calculations

$$B_X = VN(-h_X) + PF_X N(h_X - \sigma_X)$$

where

$$h_X = -\frac{\ln k_X}{\sigma_X} + \frac{\sigma_X}{2} \quad \text{and} \quad k_X = \frac{PF_X}{V}.$$

Floating rate debt

A zero floating rate bond with a stochastic face amount

$$F_L(T) = D_L e^{\int_0^T r(t)\,dt} = D_L Y(T)$$

where D_L is a constant and $Y(t)$ is the money market account. The dynamics of $Y(t)$ is

$$dY(t) = r(t)Y(t)\,dt, \quad \text{and} \quad Y = 1.$$

We use $Y(t)$ as the numeraire and define $M(t) = V(t)/Y(t)$. By Ito's lemma

$$d\ln M = -\frac{\sigma_V^2}{2}\,dt + \sigma_V\,dZ_V.$$

Note that $\ln M(T)|M = V$ is normally distributed with mean $\mu_M = \ln V - \frac{\sigma_V^2 T}{2}$. The price of the floating rate loan

$$\begin{aligned} B_L &= V - E_{Q^*}[(M(T) - D_L)^+] \\ &= VN(-h_L) + D_L(h_L - \sigma_V\sqrt{T}) \end{aligned}$$

where $Q^*_{h_L}$ is the meausre iwth $Y(t)$ as the numeraire

$$h_L = \frac{-\ln D_L/V}{\sigma_V\sqrt{T}} + \frac{\beta_V\sqrt{T}}{2}.$$

Default premiums

Yield on the risky fixed rate debt

$$Y_X = \frac{1}{T} \ln \frac{F_X}{B_X}$$

and the default premium $\pi_X = Y_X - R_X$ where $R_X = -\frac{\ln P}{T}$, we have

$$\pi_X = -\frac{1}{T} \ln \left[N(h_X - \sigma_X) + \frac{N(-h_X)}{k_X} \right].$$

The default free yield of floating rate debt is

$$R_L = \frac{1}{T} \int_0^T r(t)\, dt.$$

The default premium $\pi_L = Y_L - R_L$ so that

$$\pi_L = -\frac{1}{T} \ln \left[N(h_L - \sigma_V \sqrt{T}) + \frac{N(-h_L)}{k_L} \right].$$

Though R_L and F_L are random variable but the default premium π_L is a deterministic functions.

Examination of fixed-floating differential

A firm is assumed to have the choice to issue between fixed rate and floating rate debt to raise the same dollar amount B.

$$B = VN(-h_X) + PF_X N(h_X - \sigma_X)$$
$$B = VN(-h_L) + D_L N(h_L - \sigma_V \sqrt{T}).$$

Solve for F_X^* and D_L^* such that

$$F_X^* = \frac{B - VN(-h_X)}{PN(h_X^* - \sigma_X)} \quad \text{where} \quad h_X^* = \frac{\ln \frac{V}{PF_X^*}}{\sigma_X} \quad \frac{\sigma_X}{2}$$

$$D_L^* = \frac{B - VN(-h_L^*)}{N(h_L^* - \sigma_V \sqrt{T})} \quad \text{where} \quad h_L^* \frac{\ln \frac{V}{D_L^*}}{\sigma_V \sqrt{T}} + \frac{\sigma_V \sqrt{T}}{2}.$$

The default premiums evaluated at B are

$$\pi_X|_{B_X=B} = \frac{1}{T} \ln \frac{PF_X^*}{B} \quad \text{and} \quad \pi_L|_{B_L=B} = \frac{1}{T} \ln \frac{D_L^*}{B}.$$

Fixed-floating quality differential = DIF $= \pi_X|_{B_X=B} - \pi_L|_{B_X=B} = \dfrac{1}{T} \ln \dfrac{PF_X^*}{D_L^*}.$

The quality differential is zero if and only if

$$PF_V^* = D_L^* \quad \text{or} \quad k_X^* = k_L^*$$

where

$$\begin{aligned} k_X^* &= \frac{PF_X^*}{V} = \text{ quasi-debt ratio for fixed rate debt} \\ k_L^* &= \frac{D_L^*}{V} = \text{ quasi-debt ratio for floating rate debt.} \end{aligned}$$

Lemma

$$k_X^* > k_L^* \Longleftrightarrow \sigma_X^2 > \sigma_V^2 T \quad \text{and} \quad k_X^* = k_L^* \Longleftrightarrow \sigma_X^2 = \sigma_V^2 T.$$

Proposition 1

When $\rho \geq 0$, DIF > 0

Proposition 2

When $\rho < 0$, DIF is generally positive.

Reference

M. Ikeda, "Default premiums and quality spread differentials in a stochastic interest rate economy," Advances in Futures and options Research, vol. 8 (1995) p.175-202.

Black-Cox model (1976)

Impact of various bond indenture provisions on risky debt valuation

1. *Inter-temporal default* (safety covenants)

   If the firm value falls to a specified level, the bondholders are entitled to force the firm into bankruptcy and obtain the ownership of the assets.

2. *Subordinated bonds*

   Payments can be made to the junior debt holders only if the full promised payment to the senior debt holders has been made.

claim	$V < P$	$P \leq V \leq P + Q$	$V > P + Q$
senior bond	V	P	P
junior bond	0	$V - P$	Q
equity	0	0	$V - P - Q$

$P =$ par value of senior bond

$Q =$ par value of junior bond

Longstaff-Schwartz model (1995)

Interest rate uncertainty

Vasicek interest rate process: $dr = a(c - r)\,dt + \sigma_r\,dZ_r$

Bankruptcy-triggering mechanism

Threshold value $\nu(t)$ for the firm value at which financial distress occurs; take $\nu(t) = K =$ constant.

Briys and de Varenne model

"Valuing risky fixed debt: an extension," *Journal of Financial and Quantitative Analysis*, vol. 32, p. 239-248 (1997).

Assume the existence of a unique probability measure Q (risk neutral measure) under which the continuously discounted price of any security is a Q-martingale.

Under Q, the short rate r_t follows

$$dr_t = a(t)[b(t) - r_t]\,dt + \sigma_r(t)\,dW_t.$$

For Gaussian type interest rate models, the dynamics of $B(t,T)$ under Q is

$$\frac{dB}{B} = r_t\,dt - \sigma_P(t,T)\,dW_t$$

where

$$\sigma_P(t,T) = \sigma_r(t)\int_t^T \exp\left(-\int_t^u a(s)\,ds\right)\,du.$$

Under Q, the firm value process V_t follows

$$\frac{dV_t}{V_t} = r_t\,dt + \sigma_V\left[\rho\,dW_t + \sqrt{1-\rho^2}\,d\widetilde{W}_t\right],$$

where W_t and $\widetilde{W}_t$ are orthogonal Wiener processes.

The default-trigger barrier $\overline{K}(t) = \alpha F B(t,T), 0 \leq \alpha \leq 1, F$ is the face value.

Define the first passage time of the process V_u through the barrier $\overline{K}(u), t \leq u \leq T$.

$$T_{V,\overline{K}} = \inf\left\{u \geq t : V_u = \overline{K}(u) = \alpha F B(t,T)\right\}.$$

The price as of time t of the risky zero coupon bond is

$$\begin{aligned}\overline{B}(r,t;T) &= E_Q\left[\exp\left(-\int_t^T r_u\,du\right)\right.\\ &\left.\left\{f_1 \alpha F \mathbf{1}_{\{T_{V,\overline{K}}<T\}} + F\mathbf{1}_{\{T_{V,\overline{K}}\geq T, V_T \geq F\}} + f_2 V_T \mathbf{1}_{\{T_{V,\overline{K}}\geq T, V_T < F\}}\right\}\right]\end{aligned}$$

where f_1 and f_2 are recovery rates.

Advantages

The term structure of corporate spreads is affected by the presence of safety covenant and the violations of the absolute priority rule.

- Larger corporate spreads than those derived by Merton's model.
- Corporate spreads will exhibit more amplex structures since there are more parameters in the model.

Solution under Ho-Lee interest rate dynamics

Assume the one-factor Gaussian term structure model of Ho-Lee, where the risk neutral dynamics of the default free zero coupon bond with maturity T is governed by

$$\frac{dB}{B} = r\,dt + \sigma_r(T-t)\,dW_1.$$

Rewrite the firm value dynamics as

$$\frac{dV}{V} = r\,dt + \sigma_V(\rho\,dW_1 + \sqrt{1-\rho^2}\,dW_2),$$

where W_1 and W_2 are uncorrelated Wiener processes.

T-forward measure

$$\overline{B} = B(1 - \widetilde{c}P_r^T[\tau < T]) = B[1 - \widetilde{c}(1 - P_r^T[\tau \geq T])],$$

where P_r^T denotes the probability under the T-forward measure and $\widetilde{c}B$ is the bankruptcy cost.

Let $\widetilde{V} = V/B$, then

$$\{\tau \geq T\} \Longleftrightarrow \{\widetilde{V} \geq \overline{K}, \quad t \leq T\}$$

and under P^T

$$\frac{d\widetilde{V}}{\widetilde{V}} = [\rho\sigma_V - \sigma_r(T-t)]\, dW_1^T + \sigma_V\sqrt{1-\rho^2}\, dW_2^T \equiv \sigma(t)\, d\widehat{W},$$

where

$$\sigma^2(t) = \sigma_V^2 - 2\rho\sigma_V\sigma_r(T-t) + \sigma_V^2(T-t)^2$$

Time change

Consider the process M given by

$$dM = \sigma(t)d\widehat{W},$$

its quadratic variation is given by

$$\langle M\rangle = Q(t) = \int_0^t \sigma(s)^2\, ds.$$

The value of M_t at time t can be represented as the value of a Brownian motion $W_{Q(t)}$ at time $Q(t)$. We write the time-changed Brownian motion by

$$M_t = W_{Q(t)}.$$

Define $Y_{Q(t)} = \widetilde{V}_t$, where

$$\widetilde{V}_t = \int_0^t \widetilde{V}_u \, dM_u = \int_0^{Q(t)} Y_u \, dW_u = Y_{Q(t)} \quad \text{and} \quad \frac{dY}{Y} = dW.$$

Y follows a lognormal random walk itself and it does not have any time-dependent volatility. Let $x = \ln Y$, then $dx = -\frac{1}{2}\, dt + dW$.

Hitting probability

$$
\begin{aligned}
P_r\left[\left\{\widetilde{V} \geq \widetilde{K}, t \leq T\right\}\right] &= P_r\left[\left\{\ln\frac{Y}{Y_0} \geq \ln\frac{\overline{K}}{Y_0}, t \leq Q(T)\right\}\right] \\
&= N\left(\frac{k + \frac{Q(T)}{2}}{\sqrt{Q(T)}}\right) - e^{-2k} N\left(\frac{-k + \frac{Q(T)}{2}}{\sqrt{Q(T)}}\right),
\end{aligned}
$$

where $k = \ln\frac{Y_0}{\overline{\overline{K}}}$. Lastly, the price of the risky-coupon bond is given by

$$\overline{B} = B[1 - \widetilde{C}(1 - P)],$$

where

$$P = N\left(\frac{k + \frac{Q(T)}{2}}{\sqrt{Q(T)}}\right) - e^{-2k}\left(\frac{-k + \frac{Q(T)}{2}}{\sqrt{Q(T)}}\right)$$

with $k = \ln\frac{V_0}{B_0\overline{\overline{K}}}$ and $Q(T) = \sigma_V^2 T - \rho\sigma_V\sigma_r T^2 + \frac{\sigma_r^2}{3}T^3$.

Towards dynamic capital structure: Stationary leverage ratios

"Do credit spreads reflect stationary leverage ratio?" P. Collin-Dufresne and R.S. Goldstein, *Journal of Finance*, vol. 56 (5), p.1929-1957 (2001).

Background

- Firms adjust outstanding debt levels in response to changes in firm value, thus generating mean-reverting leverage ratios.
- Develop a structural model of default with stochastic interest rates that generates stationary leverage ratios (exogenous assumption on future leverage).
- Empirical studies show the support for the existence of target leverage ratios within an industry. Theoretical dynamic models of optimal capital structure find that firm value is maximized when a firm acts to keep its leverage ratio within a certain band.

Assume that under the risk neutral measure Q,

$$\frac{dV_t}{V_t} = (r - \delta)\, dt + \sigma\, dW_t$$

where δ is the payout rate. The default threshold changes dynamically over time. Let k_t denote the log-default threshold,

$$dk_t = \lambda(y_t - \nu - k_t)\, dt, \quad \text{where } y_t = \ln V_t.$$

- When $k_t < y_t - \nu$, the firm acts to increase k_t. That is, firms tend to issue debt when their leverage ratio falls below some target.

Define the log-leverage $\ell_t = k_t - y_t$, then

$$d\ell_t = \lambda(\bar{\ell} - \ell_t)\, dt - \sigma\, dZ_t,$$

where

$$\bar{\ell} = \frac{-r + \delta + \frac{\sigma^2}{2}}{\lambda} - \nu.$$

2.3 Counterparty risks of swaps and debt negotiation models

Credit risk of defaultable currency swaps

- A financial swap is the exchange of cashflows based on an underlying index under some prescribed terms.
- Two major sources of risks
  - rate risk (change in interest rate or exchange rate)
  - credit risk (either party may default)

  The swap default risk is two-sided.
- Maximum loss associated with the credit risk is measured by the swap's replacement cost.

Question How much spread is appropriate to cover the swap credit risk of the swap counterparty?

Reference

H.Yu and Y.K. Kwok, "Contingent claim approach for analyzing the credit risk of defaultable currency swaps," AMS/IP Studies in Advanced Mathematics, vol. 26, p.79-92 (2002).

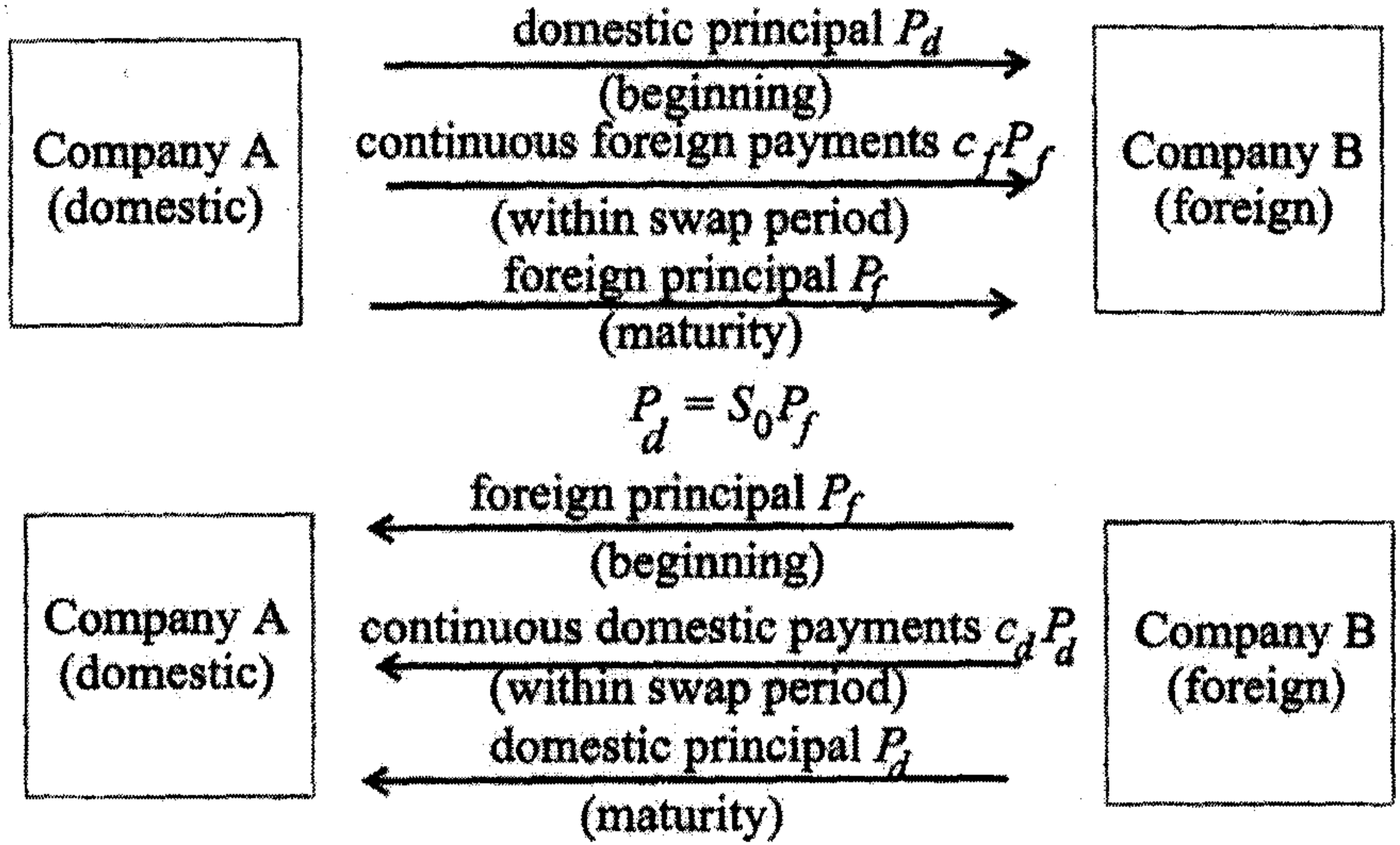
Considerations in credit risk analysis

The settlement payment to the swap counterparty upon inter-temporal default depends on the settlement clauses in the swap contract.

- Under the full (limited) two-way payment clause, the non-defaulting counterparty is required (not required) to pay if the final net amount is favorable to the defaulting party.
- When the swap is favorable to the non-defaulting party, it receives only the fraction $1 - w$ of the market quotation value of the swap agreement.

Remark

It is too simplfied to estimate the spreads on the higher-rated and lower-rated swaps from the quality spread on bonds of similar credit categories.



Cashflows between the two currency swap counterparties, assuming no intertemporal default.

Cashflows between currency swap counterparties

- Domestic company A has comparative advantage in borrowing domestic loan but it wants to raise foreign capital (reverse situation for its counterparty B).
- For simplicity of analysis, we assume that the exchange payments are continuous.
- The swap rates are chosen such that the value of the swap contract is set to be zero at initiation.
- When the firm value F of company A falls to the threshold level H, company A is forced to reorganize.
- Under the risk neutral measure Q, the dynamics of the exchange rate S and firm value F of company A are governed by
$$\begin{aligned}\frac{dS}{S} &= (r_d - r_f)\,dt + \sigma_S\,dZ_S \\ \frac{dF}{F} &= r_d\,dt + \sigma_F\,dZ_F\end{aligned}$$
where r_d and r_f are the domestic and foreign riskfree interest rates and $dZ_S\,dZ_F = \rho\,dt$.

$$V(S,t) = \text{value at time } t \text{ of the riskfree currency swap to company } B$$
$$\overline{V}(S,F,t) = \text{value at time } t \text{ of the defaultable swap to company } B$$

Governing equations

(i) $$\frac{\partial V}{\partial t} + \frac{\sigma_S^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} + (r_d - r_f)\frac{\partial V}{\partial S} + (P_f c_f S - P_d c_d) - r_d V = 0,$$
$$0 < S < \infty, t > 0,$$

with terminal payoff

$$V(S,T) = P_f S - P_d.$$

(ii) $$\frac{\partial \overline{V}}{\partial t} + \frac{\sigma_S^2}{2} S^2 \frac{\partial^2 \overline{V}}{\partial S^2} + \rho\sigma_S\sigma_F SF \frac{\partial^2 \overline{V}}{\partial S \partial F} + \frac{\sigma_F^2}{2} F^2 \frac{\partial^2 \overline{V}}{\partial F^2},$$
$$+[r_d F - (P_f C_f S - P_d C_d)]\frac{\partial \overline{V}}{\partial F} + (r_d - r_f) S \frac{\partial \overline{V}}{\partial S} + (P_f C_f S - P_d C_d) - r_d V = 0$$
$$0 < S < \infty, \quad H < F < \infty, t > 0.$$

Prescription of auxiliary conditions

1. Limited two-way settlement

- $\overline{V}(S,F,T) = \begin{cases} P_f S - P_d, & F > H \\ (1-w)\max(P_f S - P_d, 0), & F \leq H \end{cases}.$
- $\lim_{F\to\infty} \overline{V}(S,F,t) = V(S,t)$ for all t.
- $\overline{V}(S,H,t) = (1-w)\max(V(S,t),0)$
- When $S = 0$, it will stay at that level for all later times. The foreign payments become worthless, and the swap contract behaves like a bond where B pays the continuous payments $c_d P_d$ and final par value P_d.

  Present value of the sum of these cashflows

  $$= P_d \left\{ e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}] \right\}.$$

  $$\overline{V}(0,F,t) = -P_d \left\{ e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}] \right\} \mathbf{1}_{\{F>H\}}.$$

2. Full two-way settlement

   B has to honor the swap contract even when A becomes default.

   - $\overline{V}(S,F,T) = \begin{cases} P_f S - P_d & F > H \\ P_f S - P_d & F \le H \text{ and } P_f S - P_d \le 0 \\ (1-w)(P_f S - P_d) & F \le H \text{ and } P_f S - P_d > 0 \end{cases}$.
   - $\overline{V}(S,H,t) = \begin{cases} (1-w)V(S,t) & V(S,t) > 0 \\ V(S,t) & V(S,t) \le 0 \end{cases}$.
   - $\overline{V}(0,F,t) = -P_d\left\{ e^{-r_d(T-t)} + \frac{c_d}{r_d}[1 - e^{-r_d(T-t)}] \right\}$.