2.4 Industrial implementation: KMV model

Expected default frequency

- *Expected default frequency* (EDF) is a forward-looking measure of actual probability of default. EDF is firm specific.

- KMV model is based on the structural approach to calculate EDF (credit risk is driven by the firm value process).

  - It is best when applied to publicly traded companies, where the value of equity is determined by the stock market.

  - The market information contained in the firm's stock price and balance sheet are translated into an implied risk of default.

- Accurate and timely information from the equity market provides a continuous credit monitoring process that is difficult and expensive to duplicate using traditional credit analysis.

- Annual reviews and other traditional credit processes cannot maintain the same degree of "on guard" that EDFs calculated on a monthly or a daily basis can provide.

Key features in KMV model

1. Distance to default ratio determines the level of default risk.

   - This key ratio compares the firms net worth $E(V_T) - d^*$ to its volatility.
   - The net worth is based on values from the equity market, so it is both timely and superior estimate of the firm value.

2. Ability to adjust to the credit cycle and ability to quickly reflect any deterioration in credit quality.

3. Work best in highly efficient liquid market conditions.

Three steps to derive the actual probabilities of default:

1. Estimation of the market value and volatility of the firm asset value.

2. Calculation of the distance to default, an index measure of default risk.

3. Scaling of the distance to default to actual probabilities of default using a default database.

- Changes in EDF tend to anticipate at least one year earlier than the downgrading of the issuer by rating agencies like Moodys and S & Ps
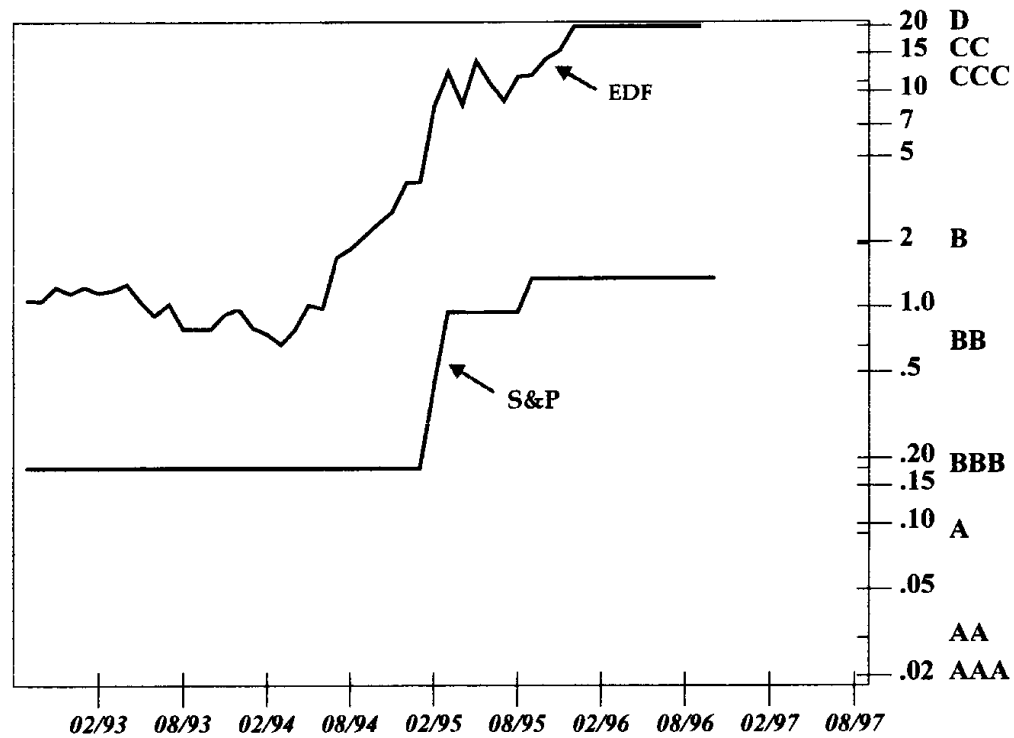
Distribution of terminal firm value at maturity of debt

$$V_T = V_0 \exp\left(\left(\mu - \frac{\sigma_v^2}{2}\right)T + \sigma_v \sqrt{T} Z_T\right)$$
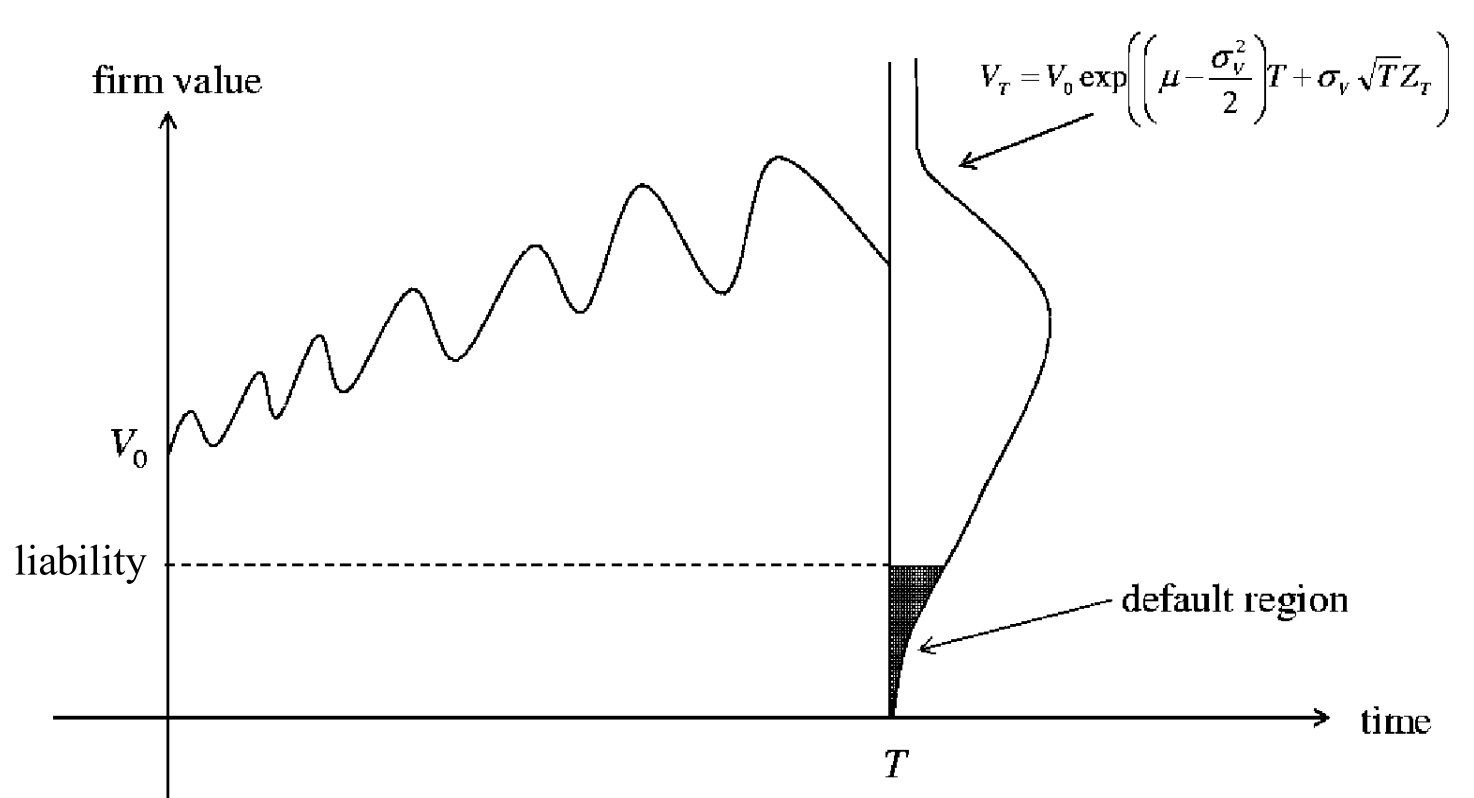
- According to KMV's empirical studies, log-asset returns confirm quite well to a normal distribution, and σ_V stays relatively constant.

- From the sample of several hundred companies, firms default when the asset value reaches a level somewhere between the value of total liabilities and the value of the short-term debt.

Distance to default

Default point, $d^* = $ short-term debt $ + \dfrac{1}{2} \times$ long-term debt. Why $\dfrac{1}{2}$? Why not!

From $V_T = V_0 \exp\left(\left(\mu - \dfrac{\sigma^2}{2}\right)T + \sigma_V Z_T\right)$, the probability of finishing below D at date T is

$$N\left(-\frac{\ln\frac{V_0}{D} + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma_V \sqrt{T}}\right).$$

Distance to default is defined by

$$d_f = \frac{E(V_T) - d^*}{\widehat{\sigma}_V \sqrt{T}} = \frac{\ln\frac{V_0}{d^*} + \left(\mu - \frac{\widehat{\sigma}_V^2}{2}\right)T}{\widehat{\sigma}_V \sqrt{T}},$$

where V_0 is the current market value of firm, μ is the expected rate of return on firm value and $\widehat{\sigma}_V$ is the annualized firm value volatility. The probability of default is a function of the firm's capital structure, the volatility of the asset returns and the current asset value.

Estimation of firm value V and volatility of firm value σ_V

- Usually, only the price of equity for most public firms is directly observable. In some cases, part of the debt is directly traded.

- Using option pricing approach:
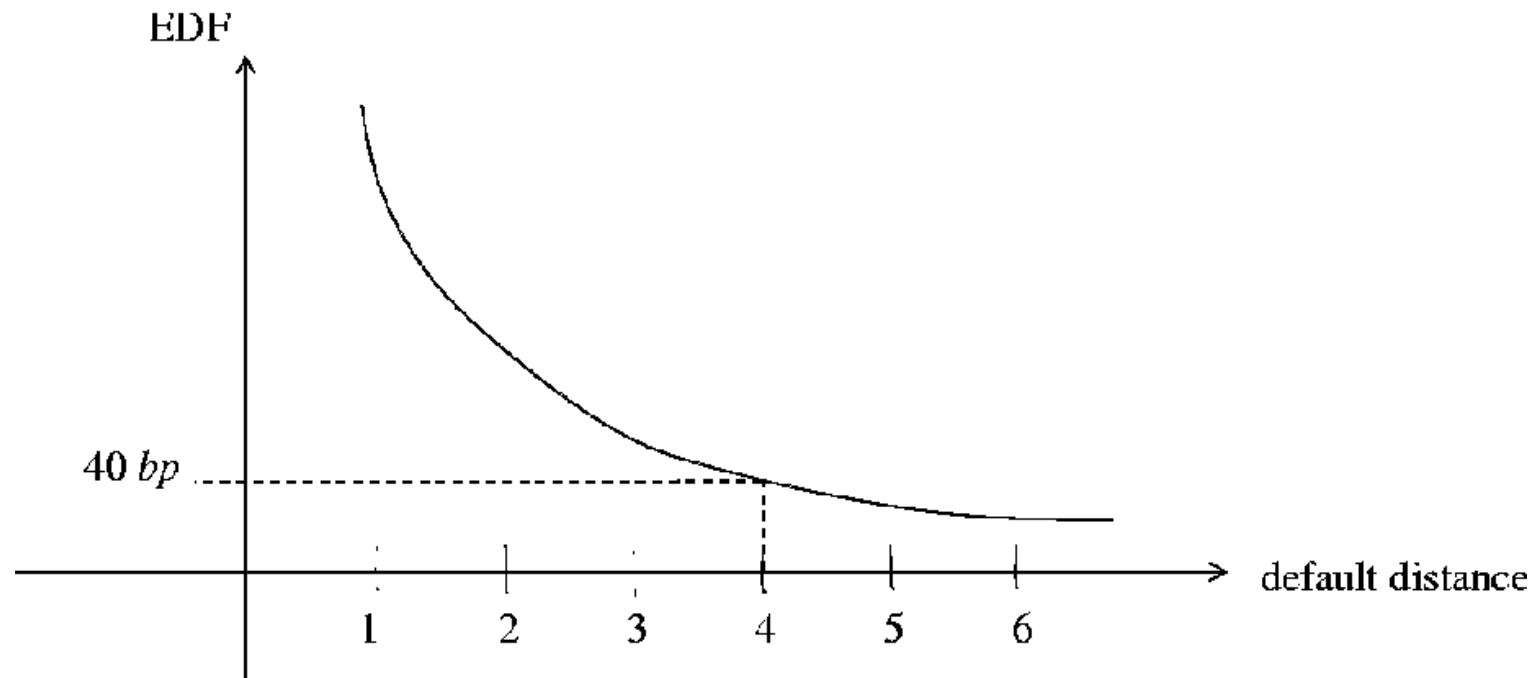
    equity value, $E = f(V, \sigma_V, K, c, r)$

    volatility of equity, $\sigma_E = g(V, \sigma_V, K, c, r)$

    where K denotes the leverage ratio in the capital structure, c is the average coupon paid on the long-term debt, r is the riskfree rate. Actually, the relation between σ_E and σ_V is obtained via the Ito lemma: $E\sigma_E = \dfrac{\partial f}{\partial V} V \sigma_V$.

- Solve for V and σ_V from the above 2 equations.

Probabilities of default from the default distance

Based on historical information on a large sample of firms, for each time horizon, one can estimate the proportion of firms of a given default distance (say, $d_f = 4.0$) which actually defaulted after one year.

Example *Federal Express* (dollars in billion of US$)

	November 1997	*February 1998*
Market capitalization (price × shares outstanding)	$7.9	$7.3
Book liabilities	$4.7	$4.9
Market value of assets	$12.6	$12.2
Asset volatility	15%	17%
Default point	$3.4	$3.5
Default distance	$\dfrac{12.6 - 3.4}{0.15}$	$\dfrac{12.2 - 3.5}{0.17}$
EDF	$0.06\%(6bp) = AA^-$	$0.11\%(11bp) = A^-$

The causes of changes for an EDF are due to variations in the *stock price, debt level* (leverage ratio), and *asset volatility*.

Weaknesses of the KMV approach

- It requires some *subjective estimation* of the input parameters.

- It is difficult to construct theoretical EDFs without the *assumption of normality* of asset returns.

- *Private firms EDFs* can be calculated only by using some compabability analysis based on accounting data.

- It does not *distinguish* among different types of long-term bonds according to their seniority, collateral, covenants or convertibility.

Conversion of actual EDF into risk neutral EDF

When we price credit derivatives, we need to have the information on the risk neutral probabilities of default. Under Q

$$\frac{dV_t^*}{V_t^*} = r\,dt + \sigma\,dZ_t.$$

Let $\quad \widehat{Q}_T \quad = \quad Pr[V_T^* \leq DPT_T]$

$\qquad\qquad = \quad$ probability that the asset value at T
$\qquad\qquad\qquad$ falls below DPT_T under Q

$\qquad\qquad = \quad N(-d_2^*)$

where

$$d_2^* = \frac{\ln\frac{V_0}{DPT_T} + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}.$$

On the other hand, $EDF_T = N(-d_2)$ where $d_2 = \dfrac{\ln\frac{V_0}{DPT_T} + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}.$

Now, $-d_2 + \dfrac{(\mu - r)\sqrt{T}}{\sigma} = -d_2^*$ so that

$$\widehat{Q}_T = N\left[N^{-1}(EDF_T) + \frac{\mu - r}{\sigma}\sqrt{T}\right].$$

Since $\mu \geq r$, we have $\widehat{Q}_T \geq EDF_T$.

According to $CAPM$, $\mu - r = \beta\pi$

where $\beta =$ beta of the asset with the market

$$= \rho\frac{\sigma}{\sigma_M} = \frac{\text{cov}(R, R_M)}{\text{var}(R_M)},$$

Here, R and R_M are the return of firm asset and market portfolio,
$\rho =$ correlation coefficient between the asset returns and
market's return
$\pi =$ market risk premium for one unit of beta risk $= \mu_M - r$.

Finally, $\dfrac{\mu - r}{\sigma} = \dfrac{\beta\pi}{\sigma} = \rho\dfrac{\pi}{\sigma_M} = \rho U$, where $U = \dfrac{\mu_M - r}{\sigma_M}$ is the market Sharpe ratio. The market Sharpe ratio is the excess return per unit of market volatility for the market portfolio.