MATH690 – Credit Risk Models

Homework One

Spring, 2005 *Course Instructor: Prof. Y.K. Kwok*

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1. Suppose that the risk-free zero curve is flat at 7% per annum with continuous compounding and that defaults can occur at times 1 year, 2 years, and 3 years in a three-year credit default swap. Suppose that the recovery rate is 30% and the default probabilities at times 1 year, 2 years and 3 years are $0.0224, 0.0247$ and 0.0269, respectively. What is the credit default swap spread? Assume that payments are made semi-annually and that the accrued interest on the reference bond is always zero at the time of a default.

2. "A long forward contract subject to credit risk is a combination of a short position in a no-default put and a long position in a call subject to credit risk." Explain this statement.

   Hint: A forward is a combination of a long European call and a short European put. Examine the scenario where the forward expires in-the-money.

3. Suppose that the risk-free zero curve is flat at 6% per annum with continuous compounding and that defaults can occur at times 1 year, 2 years, 3 years, and 4 years in a four-year plain vanilla credit default swap with semiannual payments. Suppose that the recovery rate is 20% and the probabilities of default at times 1 year, 2 years, 3 years and 4 years are $0.01, 0.015, 0.02$, and 0.025, respectively. The reference obligation is a bond paying a coupon semiannually of 8% per year. Defaults always take place immediately before coupon-payment dates on this bond. What is the credit default swap spread?

4. You have the option to enter into a five-year credit default swap at the end of one year for a swap spread of 100 basis points. The principal is $100 million. Payments are made on the swap semiannually. The forward swap spread for the period between year 1 and year 6 is 90 basis points, the volatility of the forward swap spread is 15%, LIBOR is flat at 5% (continuously compounded). The risk-neutral probability of a default by the reference entity during the first year is 0.015. What is the value of the option? Assume that the option ceases to exist if there is a default during the first year.

5. Consider a defaultable coupon bond with coupon dates $t_1, t_2, \cdots, t_n$, where t_n is the maturity date T. The coupon payment is c and the par is unity. The i^{th} coupon payment is only made if the bond has not defaulted at time $t_i, i = 1, 2, \cdots, n$. If the bond defaults prior to maturity, a recovery rate δ of the par is paid at the default time T_d. Assuming T_d to be independent of $r(t)$ under the risk neutral measure Q. Show that the value of this defaultable coupon bond is given by

$$\begin{aligned} D(r,t;T) &= \sum_{i=1}^{n} cB(r,t;t_i)P[T_d > t_i] + B(r,t;T)P[T_d > T] \\ &\quad + \int_t^T \delta B(r,t;u)q(u)\,du, \end{aligned}$$

   where $q(t)$ is the probability density of the default time.

6. Consider the Hull-White model for the credit default swap, show that an upper and a lower bound on the risky bond price maturing at t_j are given by

$$B_j \leq B_j^0 - \sum_{i=1}^{j-1} q_i \beta_{ij}$$

and

$$B_j \geq B_j^0 - \sum_{i=1}^{j-1} q_i \beta_{ij} - \frac{\beta_{jj}}{t_j - t_{j-1}} \left[1 - \sum_{i=1}^{j-1} q_i (t_i - t_{i-1}) \right],$$

respectively.

Hint: Note that default probability densities must be non-negative and the cumulative probability of default must be less than one.

7. We would like to highlight the differences between entering a total return swap (TRS) and an outright purchase. We assume that **B** is the total return receiver, and **A** the total return payer. Using the example in the following table, we compare:
   (i) An outright purchase of the **C**-bond at $t = 0$ with a sale at $t = T_N$. **B** finances this position with debt that is rolled over at LIBOR, maturing at T_N.
   (ii) A position as a total return receiver in a TRS with **A**.

Time	Defaultable bond	TRS payments		
		Funding	Returns	Marking to market
$t=0$	$-\overline{C}(0)$	0	0	0
$t=T_i$	$\overline{c}$	$-\overline{C}(0)(L_{i-1}+s^{TRS})$	$+\overline{c}$	$+\overline{C}(T_i)-\overline{C}(T_{i-1})$
$t=T_N$	$(\overline{C}(T_N)+\overline{c})$	$-\overline{C}(0)(L_{N-1}+s^{TRS})$	$+\overline{c}$	$+\overline{C}(T_N)-\overline{C}(T_{N-1})$
Default	Recovery	$-\overline{C}(0)(L_{i-1}+s^{TRS})$	0	$-(\overline{C}(T_{i-1})-\text{Recovery})$

The above table shows the payoff streams of a total return swap to the total return receiver **B** (the payoffs to the total return payer **A** are the converse of these). The TRS is unwound upon default of the underlying bond. Day count fractions are set to one $\delta_1 = 1$.

First we note that **B** receives the coupon payments of the underlying security at the same time in both positions. Thus we need not consider these payments and we set $\overline{c} = 0$.

Second, the debt service payments in strategy (a) and the LIBOR part of the funding payment in the TRS (strategy (b)) coincide, too. Thus these payments cancel, too.

(a) Explain why bonds that are initially traded at a discount to par should command a positive TRS spread s^{TRS}, while bonds that are traded above par should have a negative TRS spread s^{TRS}.

(b) Also, why the TRS rate s^{TRS} does *not* reflect the default risk of the underlying bond. If the underlying bond is issued at par and the coupon is chosen such that its price before default is always at par (assuming constant interest rates and spreads), then the TRS rate should be zero, irrespective of the default risk that the bond carries.