Review of Topics — Basic Concepts of Probability Theory

1. Sample space S
   - discrete: $S_1 = \{1, 2, \cdots, n\}$
   - continuous: $S_2 = \{x : 0 \le x \le 1\}$

2. Event $E \subset S$
   - sure event: $E_1 = S$
   - impossible event: $E_2 = \phi$

3. Corollary 5

$$
\begin{aligned}
P[A \cup B] &= P[A] + P[B] - P[A \cap B] \\
P[A \cup B \cup C] &= P[A] + P[B] + P[C] \\
&\quad - P[A \cap B] - P[A \cap C] - P[B \cap C] \\
&\quad + P[A \cap B \cap C]
\end{aligned}
$$

4. $P[A \cap B] = P[A]P[A|B]$

5. Conditional probability

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

6. Mutually exclusiveness

$$
\begin{aligned}
& A \cap B = \phi \quad \text{i.e. } A \text{ and } B \text{ are disjoint} \\
\Rightarrow \quad & P[A \cap B] = D \text{ and } P[A \cup B] = P[A] + P[B]
\end{aligned}
$$

7. Independence
   - A and B are independent if

$$P[A \cap B] = P[A]P[B]$$

   - A, B and C are independent if

$$
\begin{aligned}
P[A \cap B] &= P[A]P[B] \\
P[A \cap C] &= P[A]P[C] \\
P[B \cap C] &= P[B]P[C]
\end{aligned}
$$

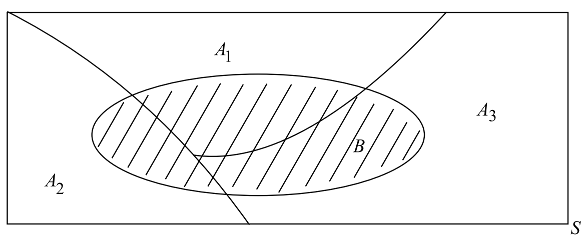
   and

$$P[A \cap B \cap C] = P[A]P[B]P[C]$$

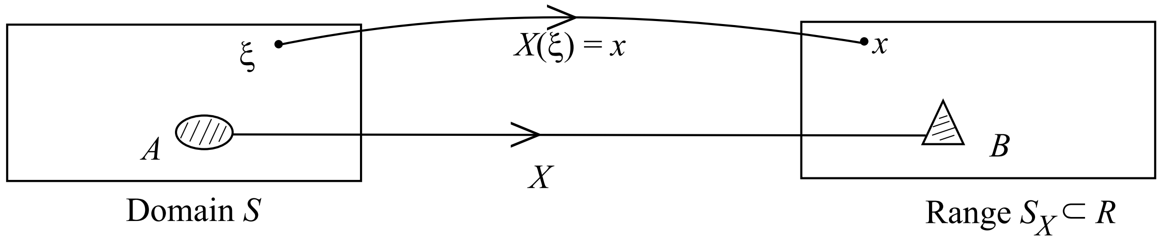
8. Bayes' Theorem

   $\{A_1, A_2, A_3\}$ is a partition of S. For $i = 1, 2, 3$

$$
\begin{aligned}
& P[A_i|B] \\
= \; & \frac{P[B \cap A_i]}{P[B]} \\
= \; & \frac{P[A_i]P[B|A_i]}{P[A_1]P[B|A_1] + P[A_2]P[B|A_2] + P[A_3]P[B|A_3]}
\end{aligned}
$$



9. Random variable X



10. Events A and B are equivalent if

$$P[A] = P[B].$$

Usually, given B, A is computed by

$$A = X^{-1}(B).$$

11. cdf: $F_X(x) = P[X \le x], \quad -\infty < x < \infty$

12. $P[a < X \le b] = F_X(b) - F_X(a)$

13. $P[X = b]$
   - $F_X(x)$ is continuous at $x = b$

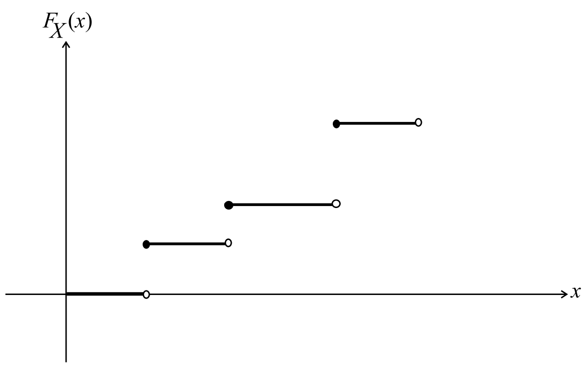
$$P[X = b] = 0$$

   - Otherwise

$$P[X = b] = F_X(b) - F_X(b^-)$$

14. Three types of random variables
   - Discrete random variables

$$F_X(x) = \sum_k P_X(x_k) u(x - x_k)$$



- Continuous random variables
- Mixed type

$$F_X(x) = \theta F_D(x) + (1-\theta)F_C(x), \quad 0 < \theta < 1$$

15. pdf of continuous $F_X(x)$

$$f_X(x) = \frac{d}{dx}F_X(x) \geq 0$$

- $F_X(x) = \int_{-\infty}^{x} f_X(t)\ dt$

16. pdf of discrete $F_X(x)$
    - Define $u(x) = \int_{-\infty}^{x} \delta(t)\ dt$
    - Dirac function $\delta(x)$:

$$\delta(x) = \frac{d}{dx}u(x)$$

17. For both continuous and discrete random variables:
    - Conditional cdf of X given A

$$F_X(x|A) = \frac{P[\{X \leq x\} \cap A]}{P[A]}, \quad P[A] > 0$$

- Conditional pdf of X given A

$$f_X(x|A) = \frac{d}{dx}F_X(x|A)$$