Interaction of the conversion and call rights

The conversion and call rights are common embedded optionality in financial instruments. For example, a convertible bond entitles her holder to *convert* the bond into common shares of the bond issuer's company. On the other hand, in order to cap the unlimited upside potential of the bond value, the bond indenture usually includes a clause where the bond issuer can *call* back the bond at a predetermined call price. Upon calling, the bondholder either chooses to receive the cash amount equivalent to the call price or to convert into the common shares (this is called forced conversion). For these two rights, the convertible feature is the right conferred to the bondholder while the callable feature is the right held by the bond issuer. What should be the optimal policies adopted by the bondholder to convert and the bond issuer to call? How do these two rights interact with each other?

The conversion right can have broader interpretation. The *reset* feature may be considered as the right to convert from the original derivative to a new derivative with the reset contract terms. The *early exercise* feature of an American call option may be visualized as the conversion from the option into the underlying asset minus the strike price. The right to convert may be at any moment during the life of the derivative or be limited to some pre-set dates. The conversion can be either voluntary (chosen optimally by the holder) or automatic once certain scenarios occur. For example, the strike price of an option is automatically reset once the asset price breaches some threshold value on a pre-set reset date. If the conversion right is voluntary, then the pricing problem requires the determination of the critical asset price (a time dependent function) at which the derivative should be optimally converted.

For the callable feature, the optimal calling policy by the issuer is to call back the derivative once the derivative value reaches the call price K, so that the derivative value is essentially capped by K. The *callable* feature is closely related to the *capped* feature. For example, in an American capped call option, the payoff is given by $\max[\min(S_T,L) - X,0]$, where S_T is the terminal asset price, L is the cap and X is the strike price. Here, the option payoff is capped by $L - X$. Consider the two derivatives: the callable American call option with call price K and strike price X, and the American capped call option with cap L. The callable option will be called back by the issuer once the asset price S reaches $K + X$. This is because the issuer will not tolerate the value of the callable option to go above K. Imagine that if the option is not called, the option value would be higher than K when $S > K + X$ since the intrinsic value of an American call option is $S - X$. On the other hand, the holder of the American capped call will choose to exercise prematurely when the asset price S hits L since the payoff will not be increased even when S shoots above L. Here, we observe the equivalence of L and $K + X$. Interestingly, the holder optimally chooses to exercise the American capped call prematurely at $S = L$ while the optimal decision to call back the callable American call at $S = K + X$ is made by the issuer.

What would be the interaction of the early exercise right and the call right in the callable American call option? Let $B_{call}(t)$ denote the critical asset price boundary above which the callable American call option should be either optimally exercised or called back, and let $B(t)$ be the critical asset price boundary at which the non-callable counterpart should be optimally exercised. Within the time interval where $B(t) < K + X$, the call feature is rendered superfluous and the callable American call would be optimally exercised by the holder when S hits $B(t)$. However, at those times where $B(t) > K + X$, there will be no voluntary early exercise by the holder. This is because once S hits $K + X$, the callable American call will be called back by the issuer and the holder is forced to exercise. We then have (see the figure)

$$B_{call}(t) = \min[B(t), K + X].$$

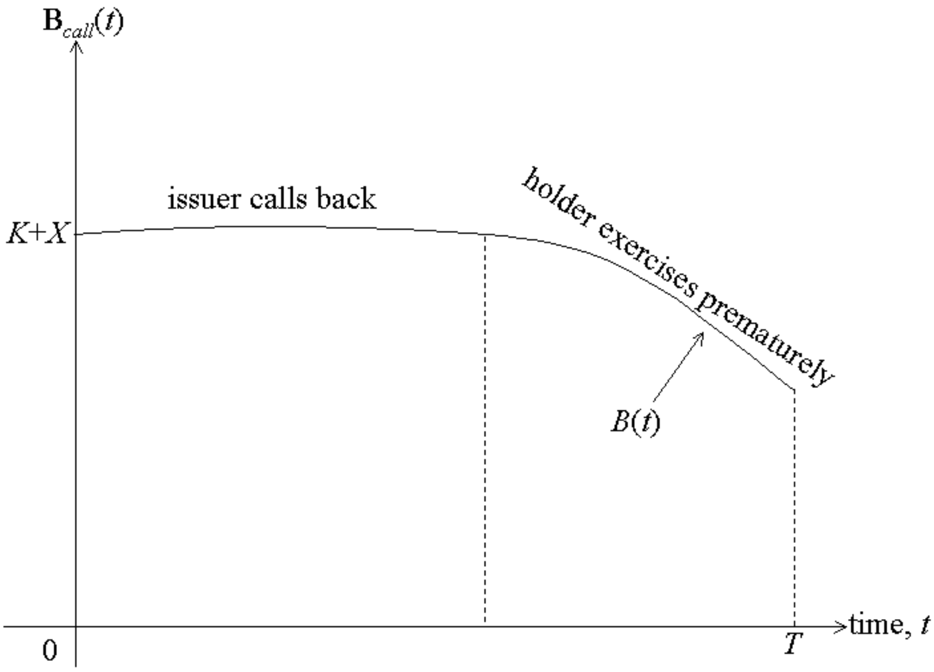

Figure: The critical asset price boundary $B_{call}(t)$ of a callable American call option.

The American capped call option has similar behavior of the critical asset price boundary, except that $K + X$ is replaced by L. As the final remark, one can follow the above argument to deduce the critical asset price boundary for a derivative with both conversion and call rights. We would expect that the critical asset price associated with the combined rights is given by the minimum of the two critical asset prices, each of them corresponds to only one right.

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