Compound options

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A compound option is simply an option on an option. The exercise payoff of a compound option involves the value of another option. A compound option then has two expiration dates and two strike prices. Take the example of a European style call on a call. On the first expiration date T_1, the holder has the right to buy a new call using the strike price X_1. The new call has expiration date T_2 and strike price X_2. Let the current time be time 0, S be the underlying asset price and $c(S,\tau;X)$ denote the value of a call with time to expiry τ and strike price X. Let c_{call} denote the value of the call on a call at the current time. On the first expiration date T_1, the value of the call on a call is given by

$$c_{call} = \max[X_1, c(S, T_2 - T_1; X_2)].$$

Let S^* be the critical asset price such that $c(S, T_2 - T_1; X_2) = X_1$. When $S > S^*$, we have $c(S, T_2 - T_1; X_2) > X_1$, and accordingly, the holder should exercise the call at T_1. The value of the call on a call at the current time depends on the joint probability that the asset price is above S^* at T_1 and above X_2 at T_2. Under the lognormal assumption of the underlying asset price process, the price formula for c_{call} is given by

$$c_{call} = Se^{-qT_2} N_2\left(a_+, b_+; \sqrt{T_1 / T_2}\right) - X_2 e^{-rT_2} N_2\left(a_-, b_-; \sqrt{T_1/T_2}\right) - X_1 e^{-rT_1} N(a_-),$$

where

$$a_+ = \frac{\ln(S/S^*) + \left(r - q + \sigma^2/2\right)T_1}{\sigma\sqrt{T_1}} \qquad a_- = a_+ - \sigma\sqrt{T_1},$$

$$b_+ = \frac{\ln(S/X_2) + \left(r - q + \sigma^2/2\right)T_2}{\sigma\sqrt{T_2}}, \qquad b_- = b_+ - \sigma\sqrt{T_2}.$$

Here, r is the interest rate, q is the dividend yield, σ is the volatility, and $N_2(x,y;\rho)$ is the two-dimensional cumulative normal distribution function with correlation coefficient ρ. For overlapping Brownian increments, the correlation coefficient is seen to be $\sqrt{T_1/T_2}$. The first term in the price formula for c_{call} gives the risk neutral expectation of the asset value conditional on $S > S^*$ at T_1 and $S > X_2$ at T_2, the second term give the expected cash payment upon exercising at T_2 and the last term is the expected cash payment upon exercising at T_1.

Other examples of compound options are call on a put, put on a call and put on a put. Their respective price formulas are given by

$$c_{put} = X_2 e^{-rT_2} N_2(-a_-, -b_-; \sqrt{T_1/T_2}) - Se^{-qT_2} N_2(-a_+, -b_+; \sqrt{T_1/T_2}) - X_1 e^{-rT_1} N(-a_-)$$

$$p_{call} = X_2 e^{-rT_2} N_2(-a_-, b_-; -\sqrt{T_1/T_2}) - Se^{-qT_2} N_2(-a_+, b_+; -\sqrt{T_1/T_2}) + X_1 e^{-rT_1} N(-a_-)$$

$$p_{put} = Se^{-qT_2} N_2(a_+, -b_+; -\sqrt{T_1/T_2}) - X_2 e^{-rT_2} N_2(a_-, -b_-; -\sqrt{T_1/T_2}) + X_1 e^{-rT_1} N(a_-).$$

A generalization of compound option is the chooser option where the holder on the first expiration date T_1 can choose whether the option is a call or a put (Rubinstein, 1992). Both the call and put have the same expiration date T_2 and strike price X. On the first expiration date T_1, the value of the chooser option is given by

$$V_{chooser} = \max[c(S, T_2 - T_1; X), p(S, T_2 - T_1; X)].$$

It can be shown that the chooser option can be replicated by the combination of a call option with strike price X and expiration date T_2 and $e^{-q(T_2 - T_2)}$ unit of put option with strike price $Xe^{-(r-q)(T_2 - T_1)}$ and expiration date T_1.

The pricing of many other derivative instruments can be modeled as compound options. By visualizing the underlying stock as an option on the firm value, an option on stock of a levered firm that expires earlier than the maturity date of the debt issued by the firm can be regarded as a compound option on the firm value (Geske, 1979). On the expiration of the option (the first expiration date of the compound option), the holder chooses to acquire the stock or otherwise. The decision depends on whether the stock as a call on the firm value is more valuable than the strike price. Another example is the pricing of American options with discrete dividends (Whaley, 1982). At time right before a dividend payment date, the holder chooses to exercise the American option to receive the stock (plus the dividend payment right after the dividend date) or continue to hold the option. This is like a chooser option with the dividend payment date as the choose date. The holder makes the choice of holding the American call option or receiving the exercise payoff of the American call. The decision depends on the relative magnitude of the dividend payment and the insurance value associated with the continued holding of the American call. Bonds with extended maturities can be modeled as compound options. On the first maturity date, the bondholder chooses to extend the maturity of the bond or receive the face value. The financial decision depends on the relative magnitude of the face value and the present value of the extended bond on the first maturity date (Ananthanarayanan and Schwartz, 1980). Lastly, in real options applications, multi-stage investment project can be modeled as compound American options. The exercise of a presently available real option opens the opportunity for the implementation of future real options (Alvarez and Stenbacka, 2001).

References

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