Optimal Calling Policies in Convertible Bonds

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Abstract: *Effective numerical algorithms are developed to evaluate the impact of the soft call and hard call constraints, notice period requirement and other factors on the optimal issuer's calling policy in convertible bonds. Our results show that the critical stock price at which the issuer should optimally call the convertible bond depends quite sensibly on these constraints and requirements.*

Keywords: *Convertible bonds, optimal calling policies*

1 INTRODUCTION

A convertible bond is a hybrid financial instrument with attributes of both fixed income securities and equity. The embedded conversion option allows the bondholder to convert into the equity of the issuer at any time at the contractual conversion price. The conversion component gives the holder the possibility to benefit from future capital appreciation in the company's equity, while the fixed income component provides a return floor. The issuer pays discrete coupon payments to the holder at regular intervals, usually at a lower coupon rate. Most convertible bonds have the call provision that could be used by the issuer to manage the debt-equity ratio of his company. Upon issuer's call, the holder can either redeem the bond at the call price or convert into stocks (known as forced conversion).

To protect the conversion privilege from being called away too soon, the bond indenture commonly contains the hard call constraint that restrains the issuer to initiate the call at the early life of the bond. Additionally, to avoid market manipulation by the issuer, the soft call constraint requires the stock price to be above the trigger price (usually chosen to be 30% to 50% higher than the conversion price) for a consecutive or cumulative period in order that the issuer can initiate the call. Upon calling by the issuer, there is a call notice period requirement whereby the holder can make the decision to redeem or convert at the end of the notice period.

Convertible bond valuation models have been quite extensively studied in the past decades [see Nyborg's paper [1] for a survey of the models]. In these contingent claim models, besides the inclusion of stochastic interest rate, they either use the firm value of the issuer or the stock price as the underlying state variable for modeling the equity component. To model the credit risk of a convertible bond, Tsiveriotis and Fernandes [2] incorporated the issuer's debt spread into the pricing model by solving a set of coupled equations, one for the bond part of the convertible bond, and the other for the whole bond value, using different discount rates for the equity and bond components.

In this paper, we construct numerical algorithms that model accurately the embedded features in a convertible bond and use them to explore the various factors that affect the optimal calling policy and conversion policy. It would be interesting to examine the sensitivity of these embedded features on the critical stock price at which it is optimal to call or convert. The value of a convertible bond consists of two parts, the equity component and bond component. The bond component is sensitive on interest rate and default risk of the issuer. The value of the equity component is dependent on the stock price, or alternatively on the firm value of the issuer's firm. In one-factor models, the interest rate and default spread are assumed to be deterministic. If we are mainly interested in the analysis of the equity component, the simplification in one-factor models can be considered acceptable. The stochastic state variable in a one-factor model can be either the stock price or the firm value. The firm value model naturally incorporates the dilution effect upon conversion of the convertible bond. If the dilution effect is not significant (say, the particular convertible bond constitutes only a minute portion of the whole capital structure of the firm), then the use of stock price as the underlying state variable may be more appropriate. Compared to the firm value models, the stock price models avoid the prescription of the capital structure of the firm. Also, the estimation of the parameter values in the stock price model is easier. For example, the stock price volatility is more directly observable compared to the firm value volatility. Also, the conversion value and payoff structures of the convertible bond depend directly on the stock price.

In this work, the one-factor models are employed to analyze the impact of the soft call and hard call constraints, notice period requirement, the interaction of the call right and conversion right, etc. To model the default risk of the convertible bond, we assume deterministic credit spread and use two different interest rates for discounting. The risky interest rate is the sum of riskless interest rate and credit spread. The equity component and the bond component are discounted by riskless interest rate and risky interest rate, respectively.

This paper is organized as follows. In the next section, we show the formulation of the one-factor model. The details of the valuation algorithms are presented, emphasizing on the treatment of coupon payments and call policies. In particular, we propose effective techniques to deal with the soft call and hard call constraints, notice period requirement, etc. Also, we illustrate how to incorporate different discount rates for the bond and equity components. In Section 3, we use the one-factor model to analyze the interaction of the call and conversion policies, impact of soft call, hard call and notice period requirements on the optimal calling policies. The paper is ended with conclusive summaries in the last section.

2 CONSTRUCTION OF NUMERICAL ALGORITHMS

By following the standard contingent claim approach, we develop the governing equations for the convertible bond value for the one-factor model. Our one-factor model assumes deterministic interest rates and deterministic credit spreads. Special precautions are taken to prescribe the auxiliary conditions that model the various embedded features in the convertible bonds.

2.1 Differential equation formulation

We adopt the stock price S as the underlying stochastic state variable in our one-factor model. Under the risk measure, S is assumed to follow the lognormal process

$$\frac{dS}{S} = (r-q)dt + \sigma_S dZ_S, \tag{1}$$

where r is the riskless interest rate, q and σ_S are the dividend yield and volatility of the stock price, respectively, and dZ_S is the standard Wiener process. All parameters can be deterministic function of time t.

Let $V(S,t)$ denote the value of the convertible bond. The convertible bond value consists of two components: equity and bond. Recall that for bonds without the convertible feature, it is common to model the credit risk associated with issuer's default by adding a credit spread s to the riskless interest rate in discounting the bond value in time. We follow the approach by Tsiveriotis and Fernandes [2] that only the bond component of the convertible bond value is discounted by the *risky* interest rate r_B, where $r_B = r+s$. In this model, we assume s to be deterministic in time. We let $V_B(S,t)$ denote the bond component of the convertible bond value so that $V(S,t) - V_B(S,t)$ is the corresponding equity component. The convertible bond pays periodic coupon payments c_i at time $t_i, i = 1, 2, \cdots, N$. The external cash payouts may be represented by $c(t) = \sum_{i=1}^{N} c_i \delta(t-t_i)$. The convertible bond value $V(S,t)$ and its bond component $V_B(S,t)$ are governed by the following coupled system of equations

$$\begin{aligned}&\frac{\partial V}{\partial t} + \frac{\sigma_S^2}{2}S^2\frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} \\ &\quad - r(V-V_B) - r_B V_B + c(t) = 0, \qquad (2a)\\ &\quad 0 < S < S^*, 0 < t < T,\end{aligned}$$

$$\begin{aligned}&\frac{\partial V_B}{\partial t} + \frac{\sigma_S^2}{2}S^2\frac{\partial^2 V_B}{\partial S^2} + (r-q)S\frac{\partial V_B}{\partial S} \\ &\quad - r_B V_B + c(t) = 0, \qquad (2b)\\ &\quad 0 < S < S^*, 0 < t < T,\end{aligned}$$

where T is the maturity date of the bond and S^* is some critical stock price at which the convertible bond is terminated prematurely. The early termination can be effected either by optimal conversion by the holder or optimal calling by the issuer.

The embedded option features in a convertible bond are characterized by the prescription of the auxiliary conditions in the pricing model, the details of which are discussed below.

(i) Terminal payoff on maturity date T

The terminal values of V and V_B are given by

$$\begin{aligned}V(S,T) &= (P+c_N)\mathbf{1}_{\{P+c_N \geq nS\}} \\ &\quad + nS\mathbf{1}_{\{P+c_N < nS\}} \qquad (3a)\\ V_B(S,T) &= (P+c_N)\mathbf{1}_{\{P+c_N \geq nS\}} \qquad (3b)\end{aligned}$$

where $\mathbf{1}_A$ is the indicator function for the event A. Here, P denotes the par value of the bond, c_N is the last coupon payment and n is the number of units of stock to be exchanged for the bond upon conversion. It is a common practice in convertible bond indenture that the accrued interest will not be paid upon voluntary conversion.

(ii) Conversion policy

Since the bondholders have the right to convert the bond into n units of stock at any time, the intrinsic value of the bond always stays equal or above the conversion value. Upon conversion, the value of the bond component vanishes identically. We then have

$$V(S,t) \geq nS \quad \text{when the convertible bond remains alive,} \tag{4a}$$

$$V(S,\bar{t}) = nS \quad \text{and} \quad V_B(S,\bar{t}) = 0, \tag{4b}$$

where $\bar{t}$ is the time of optimal conversion chosen by the bondholders.

(iii) Calling policy

Let $[T_c, T], T_c > 0$, denote the callable period, that is, the bond cannot be called during the earlier part of the bond life $[0, T_c]$. Upon calling, the bondholders can decide whether to redeem the bond for cash or convert into shares at the end of the notice period of t_n days. Let $\widehat{t}$ denote the date of call so that $\widehat{t} + t_n$ is the conversion decision date for the bondholders. The bondholders essentially replace the original bond at time $\widehat{t}$ by a new convertible bond that expires at later time $\widehat{t} + t_n$ with payoff $\max(nS, K + \widehat{c})$, where $\widehat{c}$ is the accrued interest from the last coupon date to $\widehat{t} + t_n$ and K is the pre-specified call price of the convertible bond. Let $V_{new} = V_{new}(S, t; K, t_n)$ denote the value of this new convertible bond. Obviously, this option value V_{new} becomes the convertible bond value upon calling. When there is no soft call requirement (a constraint that is related to stock price movement over a short period prior to calling), the convertible bond value should be capped by V_{new}. The convertible bond should be called once its value reaches $V_{new}(S, t; K, t_n)$. We then have

(a) within the callable period

$$V(S, t) \leq V_{new}(S, t; K, t_n) \tag{5a}$$

(b) on the call date

$$V(S, \widehat{t}) = V_{new}(S, t; K, t_n), \quad V_B(S, \widehat{t}) = 0. \tag{5b}$$

When there is a soft call requirement, it is possible that $V(S, t)$ stays above $V_{new}(S, t; K, t_n)$.

(iv) Coupon payments

By no arbitrage argument, there is a drop in bond value that equals the coupon amount c_i across a coupon payment date $t_i, i = 1, 2, \cdots, N$. We have

$$V(S, t_i^+) = V(S, t_i^-) - c_i, \quad V_B(S, t_i^+) = V_B(S, t_i^-) - c_i, \quad i = 1, 2, \cdots, N. \tag{6}$$

2.2 Numerical algorithms

We employ finite difference algorithms to solve for $V(S, t)$ and $V_B(S, t)$ as governed by Eqs. (2.2a,b) and subject to the auxiliary conditions stated in Eqs. (2.3–2.6). As usual, we adopt the log-transformed variable $x = \ln S$ and time to expiry $\tau = T - t$. Let V_j^m and $\widetilde{V}_j^m$ denote the numerical approximation of $V(x, \tau)$ and $V_B(x, \tau)$ at the grid point $x = j\Delta x$ and $\tau = m\Delta t$, respectively, where Δx and Δt are the respective stepwidth and time step. The numerical schemes take the basic form

$$\widetilde{V}_j^{m+1} = p_u \widetilde{V}_{j+1}^m + p_m \widetilde{V}_j^m + p_d \widetilde{V}_{j-1}^m - r_B \widetilde{V}_j^m c_i 1_{\{E_i\}} \tag{7a}$$

$$V_j^{m+1} = p_u V_{j+1}^m + p_m V_j^m + p_d V_{j-1}^m - r(V_j^m - \widetilde{V}_j^m) - r_B \widetilde{V}_j^m + c_i 1_{\{E_i\}}. \tag{7b}$$

The risk neutral probabilities of upward jump, zero jump and downward jump of $x = \ln S$ are given by

$$p_u = \frac{1}{2\lambda^2} + \frac{\left(r - q - \frac{\sigma_S^2}{2}\right)\sqrt{\Delta t}}{2\lambda\sigma_S}, \quad p_m = 1 - \frac{1}{\lambda^2}, \quad p_d = \frac{1}{2\lambda^2} - \frac{\left(r - q - \frac{\sigma_S^2}{2}\right)\sqrt{\Delta t}}{2\lambda\sigma_S}, \tag{8}$$

respectively, where $\Delta x = \lambda\sigma_S\sqrt{\Delta t}$. It is common to choose $\lambda = \sqrt{3}$. Here, E_i denotes the event that the coupon payment c_i is paid at t_i. When the payment date t_i is bracketed between time levels $m\Delta t$ and $(m+1)\Delta t$, the bond values V_j^{m+1} and $\widetilde{V}_j^{m+1}$ are increased by an extra amount c_i. The initial values V_j^0 and $\widetilde{V}_j^0$ are given by

$$\widetilde{V}_j^0 = \begin{cases} P + c_N & \text{if} \quad x_j \leq \ln \frac{P + c_N}{n} \\ 0 & \text{otherwise} \end{cases}, \tag{9a}$$

$$V_j^0 = \begin{cases} P + c_N & \text{if} \quad x_j \leq \ln \frac{P + c_N}{n} \\ nS & \text{otherwise} \end{cases}. \tag{9b}$$

Interaction of the callable and conversion features

We recall that upon the issuance of the notice of call, the bondholder essentially receives a new convertible bond that replaces the original one. This new bond has maturity life equals the length of the notice period and par equals the call price K plus the accrued interest amount $\widehat{c}$. The conversion ratio remains the same but there are no intermediate conversion right and coupon payment. With the provision of the early conversion privilege, the bondholder chooses the maximum of continuation value V_{cont} and conversion value $V_{conv} = nS$ if there is no recall. Upon recall of the bond, the original bond becomes the above new convertible bond. The issuer adopts the optimal policy of either to recall or abstain from recalling so as to minimize the bond value with reference to the possible actions of the bondholder. The following dynamic programming procedure effectively summarizes the above arguments

$$V_j^n = \min(V_{new}, \max(V_{cont}, V_{conv})), \tag{10}$$

where V_{cont} is the continuation value as computed by numerical scheme (7b). Once conversion into shares takes place, the bond component becomes zero. When the calling right is non-operative (say, during the period under the hard call constraint) and only conversion right exists, the above dynamic programming procedure reduces to

$$V_j^n = \max(V_{cont}, V_{conv}). \tag{11}$$

To incorporate the soft call requirement, we model the associated Parisian feature using the forward shooting grid approach proposed in Kwok-Lau's paper [3], where an extra dimension is added that captures the excursion of the stock price beyond some predetermined threshold value B. With the inclusion of the path dependence of the stock price associated with the soft call requirement, Eq. (7b) is modified as follows:

$$\begin{aligned} V_{j,k}^{m+1} = \; & p_u V_{j+1,g(k,j+1)}^m + p_m V_{j,g(k,j)}^m \\ & + p_d V_{j-1,g(k,j-1)}^m - r(V_{j,g(k,j)}^m - \widetilde{V}_{j,g(k,j)}^m) \\ & - r_B \widetilde{V}_{j,g(k,j)}^m + c_i \mathbf{1}_{\{E_i\}}. \end{aligned} \quad (12)$$

3 SIGNIFICANCE OF COUPONS, CONVERSION RATIOS, SOFT CALL AND NOTICE PERIOD

Table 1 lists the parameter values that are used (unless otherwise specified) in the sample calculations of the convertible bond pricing model presented in this paper.

Table 1: List of parameter values used in the sample calculations of the convertible bond pricing model.

par value	100
annualized volatility	20%
dividend yield per year	1%
maturity date	5 year
coupon rate	4% per annum, paid semi-annually
conversion ratio	1
call period	starting 1.25 years from now till maturity
conversion period	throughout the life
call price	140
interest rate	flat at 5% per annum
credit spread	flat at 5% per annum

In Figure 1, we plot the convertible bond value against time corresponding to different stock price levels. At all levels of the stock price, the bond value exhibits a drop in value equals the size of the coupon payment across a coupon date. Within a coupon period (except for the last coupon period right before maturity), the bond value increases as time evolves forward, due to the effect of accrued interests. Within the last coupon period, the bond value may increase, decrease or stay at almost constant level, depending on the stock price level. The dotted curve shows the bond value against time corresponding to the stock price $S = 80$. At this low stock price level (20% below the conversion price), the value of the equity component is negligibly small. The bond value shows a general trend of increase with increasing time. In particular, the bond value increases within the last coupon period and matches the total value of par plus last coupon at maturity. At the stock price level $S = 100$ (same as conversion price), the convertible bond drops in value within the last coupon period (see solid curve). This may be attributed primarily to the higher rate of decrease in the value of the conversion provision close to maturity. At higher stock price level $S = 120$ (20% above the conversion price), the bond value shows a trend of slight decrease with increasing time (see dashed curve). However, the bond value stays almost at constant value within the last coupon period. This is due to the negligible time loss of the value of the conversion feature since the convertible bond will be almost sure to be converted into shares at maturity.

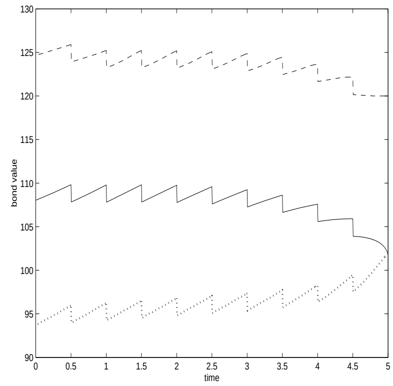

Figure 1: Plot of convertible bond value against time at different levels of stock price (dotted curve corresponds to $S = 80$, solid curve corresponds to $S = 100$ and dashed curve corresponds to $S = 120$)

In Table 2, we demonstrate the dependence of the bond value on the conversion ratio and stock price level (with the call provision excluded). At low stock price level, the bond value is not quite sensitive to the increase in conversion ratio. Similarly, the bond value is also insensitive to the increase in stock price when the conversion ratio is low. Both phenomena are due to the low value of the equity component of the convertible bond.

Table 2: The entries in the table are convertible bond values corresponding to different conversion ratios and stock price levels.

	conversion ratio				
stock price	0.8	0.9	1.0	1.1	1.2
50	77.07	77.94	79.23	80.87	82.96
100	95.07	102.62	111.04	119.78	128.94
120	107.60	117.93	128.91	140.07	151.46
130	114.33	126.10	138.15	150.50	162.99
140	121.61	134.44	147.68	161.00	174.44
150	128.98	142.99	157.16	171.54	185.98

Let $S^*(t)$ denote the time dependent critical stock price at which the convertible bond is terminated prematurely, due either to voluntary conversion or optimal calling. Figure 2 shows the plots of $S^*(t)$ against time t corresponding to the case with 30 days of notice period requirement (upper solid curve) and without notice period requirement (lower dashed curve). In the calculations, we have chosen a higher dividend yield of 3% per annum so as to better reveal the interaction of different features.

The cost borne on the issuer upon calling becomes higher when there is a notice period requirement since the bondholders are given an embedded option with life same as the length of the notice period. This option gives the bondholders the right to convert into shares or receive cash equals par plus accrued interest at the end of the notice period. As revealed from the comparison of the two curves in Figure 2, the impact of this embedded option on $S^*(t)$ can be quite significant. With the presence of the notice period requirement, the issuer exercises his calling privilege at a higher stock price level. In response to the delayed call by the issuer, the bondholders would also delay their conversion decision until the value of the conversion option is deeper-in-the-money.

The schedule of the coupon payments plays an important role in the determination of the conversion and calling policies. It is common in convertible bond indentures that the bondholders lose the accrued interests upon voluntary conversion (named as the screw clause). This clause may have significant impact on the critical stock price S^*_{conv} for optimal conversion. At times right after a coupon payment date, there is a higher incentive for the bondholders to exercise their conversion right since the accrued interest is small. As time evolves, S^*_{conv} increases since the cost incurred upon conversion becomes higher (due to accrued interest amount accumulated over time). On the other hand, the convertible bond may be terminated prematurely at stock price level S^*_{call} at which the issuer should optimally call the bond. Indeed, the critical stock price of termination is given by $S^* = \min(S^*_{call}, S^*_{conv})$. The plots of $S^*(t)$ against t in Figure 2 reveal that at times sufficiently distant from previous coupon date, the premature termination is caused by issuer's call since $S^*_{conv}(t)$ increases in time faster than $S^*_{call}(t)$.

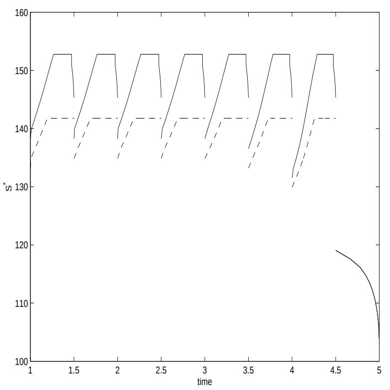

Figure 2: The upper solid curve shows the plot of the critical stock proce $S^*(t)$ against time with notice period requirement of 30 days. The lower dashed curve shows the plot of $S^*(t)$ against time corresponding to the same convertible bond but without notice period requirement. Within the time interval [4.5, 5.0], the two curves overlap.

With the presence of the notice period requirement, the issuer may optimally call the bond at a lower S^*_{call} when the time is approaching a coupon payment date (see upper solid curve in Figure 2). The issuer adopts such strategy since the coupon payment will be delayed until the end of the notice period upon calling.

Within the last coupon period, the rate of decrease of the value of the conversion privilege becomes significant so that the bondholders exercise conversion at lower value of S^*_{conv}. On the other hand, the issuer has little incentive to call the bond at times close to maturity. Hence, $S^*(t)$ is governed solely by the conversion strategy. Accordingly, there is no role played by the notice period requirement so the two curves in Figure 2 overlap at times within the last coupon period.

From our numerical experiments, we observe that S^*_{call} may change quite significantly with the length of the notice period. Also, S^*_{conv} is seen to depend sensibly on the coupon rate and dividend yield, also on other factors like the counting of time lapsed from previous coupon date.

Lastly, we use the one-factor model to examine the effects of the soft call requirement on the convertible bond value. In the two right columns in Table 3, we list the bond values with varying levels of trigger price and under different counting rules. In the calculations, the current stock price is taken to be 130. Here, we specify that the issuer can initiate the call only if the stock price stays above the trigger price consecutively (cumulatively) for 30 days. For the purpose of comparison, the convertible bond value is found to be equal to 138.24 if there is no call feature and equal to 133.68 if there is no soft protection requirement. These two values serve as the respective upper and lower bounds for the bond value subject to any soft call requirement.

Table 3: The entries in the right two columns are convertible bond values with varying level of trigger price and under the rules of consecutive counting and cumulative counting of the number of days of breaching the trigger price. The upper and lower bounds for the convertible bond value under any soft call requirement are found to be 138.24 and 133.68, respectively.

trigger price	consecutive counting	cumulative counting
120	134.10	134.08
130	134.41	134.24
140	135.23	134.98
150	135.84	135.67
160	136.39	136.24
170	136.79	136.65
180	137.06	136.94

By examining the results listed in Table 3, the following conclusions can be drawn:

1. The bond value increases with increasing trigger price. This is obvious since it becomes harder for the issuer to initiate the call with higher trigger price.

2. The bond value changes most significantly with respect to the change in the trigger price when the trigger price stays close to the call price of 140 (say, 130 to 150 in the sample calculations presented).

3. The bond value becomes higher when the soft call requirement is more stringent. This is because bondholders have better protection against calling by issuer. This explains why the convertible bond has higher value under the rule of consecutive counting compared to its counterpart under cumulative counting.

4 CONCLUSION

In this paper, we propose a more refined valuation algorithm for pricing one-factor contingent claims models for convertible bonds. Compared to earlier algorithms in the literature, our improved algorithm enables us to pursue more detailed investigation into the interaction of different features that affect the optimal conversion and call policies in convertible bonds. The time dependent behaviors of the critical stock price at which the convertible bond should be called by issuer or converted into shares by bondholders are seen to depend sensibly on various features in the bond indenture. In particular, we show that the notice period requirement and the coupon payment stream have profound impact on the value of the critical stock price. Also, we examine the effects of the conversion ratio and soft call requirement on the value of a convertible bond.

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REFERENCES

[1] K.G. Nyborg. The use and pricing of convertible bonds. *Applied Mathematical Finance*, 3:167-190, 1996.

[2] K. Tsiveriotis and C. Fernades. Valuing convertible bonds with credit risk. *Journal of Fixed Income*, 95-102, (Sept., 1998).

[3] Y.K. Kwok and K.W. Lau. Pricing algorithms for options with exotic path dependence. *Journal of Derivatives*, 9:28-38, 2001.