# Real options signaling game models for dynamic acquisition under information asymmetry

Chi Man Leung[1]
Department of Mathematics,
Hong Kong University of Science and Technology, Hong Kong, China

Yue Kuen Kwok
Department of Mathematics,
Hong Kong University of Science and Technology, Hong Kong, China

*Abstract*
We construct a real options signaling game model to analyze the impact of asymmetric information on the dynamic acquisition decision made by the aggressive acquirer firm and passive target firm in the takeover terms and timing. The target firm is assumed to have partial information on the synergy factor of the acquirer firm in generating the surplus value. Our dynamic acquisition game models are based on the market valuation of the surplus value of the acquirer and target firms, where the restructuring opportunities are modeled as exchange options. We analyze the various forms of equilibrium strategies on the deal and timing of takeover in the acquisition game and provide the mathematical characterization of the pooling and separating strategies adopted by the acquirer firm. We also determine the terms of takeover in the signaling game under varying levels of information asymmetry and synergy.

*Keywords: Decision analysis, real options signaling game, dynamic acquisition, information asymmetry, perfect Bayesian equilibrium*

# 1 Introduction

Firms may grow through internal investment or acquisitions. While the first choice generally takes a longer time to realize, the takeover of another firm provides revenue flows that are almost immediately available from the ongoing business of the target firm. There are numerous incentive factors that lead to acquisitions, like synergy and economy of scale, resource transfer and diversification, etc. In recent years, there have been a growing literature that uses real option models and signaling game theory analysis to analyze the occurrence and dynamics of returns of mergers and acquisitions (M&A). Lambrecht (2004) develops a dynamic model for the timing and terms of mergers motivated by economy of scale. He shows that market power strengthens the firms incentives to merger while higher product market uncertainty tends to delay mergers. He considers both friendly and hostile takeovers. Under friendly takeovers, both firms decide on the merger timing that maximizes the total net value of the merger. However, under hostile takeovers, the target firm presets a minimum

[1]Correspondence author; email: chimanleung@ust.hk

level of acceptable terms and the acquirer firm then decides on the timing of the takeover given those terms. Morellec and Zhdanov (2005) analyze dynamic takeover option game models based on the stock market valuations of the target firm and acquirer firms under information asymmetry on the synergy factor and competition between two acquirer firms. The restructuring opportunities under mergers are modeled as exchange options. The dynamic takeover models are later extended by Hackbarth and Morellec (2008) to analyze the dynamics of stock returns and firm level betas in acquisitions. The predictions on firm betas from their model agree well with empirical studies. Thijssen (2008) considers mergers and acquisitions that are motivated by both synergies and risk diversification. He compares the option values of the mergers under endogenously and exogenously determined roles of the target firm and acquirer firm. With strong benefit of diversification, he shows that mergers can be optimal even if synergies are negative. Borek *et al.* (2008) analyze a simple Bayesian merger game under two-sided asymmetric information on firms' types. They find that the usual prediction of adverse selection problem is likely to be misleading; that is, the low-type firms may not have strong incentive to enter into a merger deal. Stepanov (2014) consider takeovers via either a block trade or public tender offer under asymmetry of information about the acquirer's ability to generate revenue. He shows that high quality acquirer takes over a firm via a tender offer, intermediate quality acquirer negotiates a block trade while low quality acquirer does nothing. His result is consistent with the empirical observation that target firm's stock price reacts to tender offer more positively than to block trade.

There have been numerous empirical studies about the impact of information asymmetry on M&A activities. Officer *et al.* (2009) analyze a sample of 4801 M&A transactions from 1995-2004. Their studies reveal that when the value of (privately held) target firm is difficult to predict, the acquirer firm may adopt the stock-swap acquisition to reduce information asymmetry and increase its returns. Chae *et al.* (2014) study a sample of 443 post-M&A listed companies in the Korean stock market. They found that when a company faces high level of information asymmetry, it becomes more involved in M&A and acquire the target firm with lower level of information asymmetry in order to resolve information asymmetry. This phenomena is known as the information signaling hypothesis. The empirical studies performed by Cheng *et al.* (2008) show that there is a positive correlation between the takeover premium paid by the acquirer firm and information asymmetry of the value of the target firm. Draper and Paudyal (2008) analyze a sample of takeover announcements in UK. When information asymmetry is severe, their studies show that the undervalued firm can choose the timing of takeover earlier in order to attract the attention of market participants so that information asymmetry can be mitigated.

Our dynamic acquisition game models are based on the market valuations of the acquirer and target firms, similar to the exchange option formulation presented by Morellec and Zhdanov (2005) and Hackbath and Morellec (2008). Their acquisition models are complete information real game option models since they assume that all synergy parameters in the combined surplus value are known to both managers (decision makers in the acquisition game) of the acquirer and target firms. Instead of following their approach in the determination of the optimal deal and timing of takeover by the two firms, we enlarge the strategic space in the deal and timing of takeover available to the aggressive acquirer firm. Our model also extends their models by incorporating information asymmetry between acquirer firm and target firm in which the acquirer firm has full information on one of the synergy parameters while the target firm has only partial information of the synergy parameter. This new added feature allows us to examine the impact of information asymmetry to the takeover process, like the takeover timing and the premium paid to the target firm.

Using game theoretical analysis, we show that the acquirer firm of high type can reveal its private information to the target firm by offering a more attractive deal to the target firm and announcing the takeover earlier. Our analysis shows that the takeover timing and the size of the takeover deal are credible signals for the acquirer firm to reveal its private information to the target firm and resolve the problem of information asymmetry. Our results are consistent with the empirical results obtained by Draper and Paudyal (2008) and Chae *et al.* (2014). In addition, we have shown that the net surplus of the target firm is always higher under the equilibrium strategies proposed in our model, though the target firm is informationally disadvantaged. Also, we manage to provide a full characterization on the optimal takeover strategy adopted by the acquirer firm under various scenarios. Our theoretical results show that the acquirer firm of high type chooses to adopt separating strategy and differentiate from the low type counterpart only when the level of information asymmetry between the acquirer firm and the target firm is very high. This agrees with economic intuition since the acquirer firm should strike a balance between the signaling cost (by adopting separating strategy) and the loss in surplus value due to the existence of information asymmetry (by adopting pooling strategy) when choosing its optimal takeover strategy. Furthermore, our numerical results are consistent with economic intuition that it is optimal for the acquirer firm of high type to adopt separating strategy when there is a huge difference in the synergy factor between high type and low type acquirer firm. We also study the impact of information asymmetry on the takeover timing and takeover deal offered by the acquirer firm. Our numerical studies show that the target firm can always acquire higher share of the merged firm under the existence of information asymmetry. Also, the acquirer firm of high type chooses to initialize the takeover at later (earlier) time when the level of information asymmetry is low (high).

The paper is organized as follows. In the next section, we present the formulation of the dynamic model of takeover of a target firm by an acquirer firm. We analyze the deal and timing of takeover adopted by the aggressive acquirer firm that maximize the surplus value of the acquirer firm under complete information. In Section 3, we present the signaling game option model of dynamic acquisition where the synergy factor is the private information of the acquirer firm. The less informed target firm can update its belief system on the synergy factor by observing the takeover strategies of the acquirer firm. We analyze the nature of separating and pooling equilibriums. In Section 4, we present the numerical studies on the optimal takeover strategies under various information asymmetry and synergy factors. Summary and conclusive remarks are presented in the last section.

# 2 Model formulation

Our dynamic acquisition model considers restructuring opportunities as exchange options, similar to the mergers and acquisition models presented by Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008). We assume the continuous time framework and uncertainty is modeled by a complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Consider that there are two firms in the market, the passive target firm (Firm $T$) and strategic acquirer firm (Firm $A$), whose role is exogenously determined. Let $K_T$ and $K_A$ be the respective capital stock of the target firm and acquirer firm. Let $X_i$ denote the stochastic cash flow per unit of capital generated by Firm $i$, $i = T$ or $A$, whose dynamics under $(\Omega, \mathcal{F}, \mathbb{P})$ is governed by the following geometric Brownian motion:

$$dX_i(t) = \mu_i X_i(t)\, dt + \sigma_i X_i(t)\, dZ_i(t), \quad i = T \text{ or } A. \tag{2.1}$$

Here, $\mu_i > 0$ is the constant drift rate and $\sigma_i > 0$ is the constant volatility of Firm $i$, $i = T$ or $B$. Following the usual no bubble condition to avoid unbounded growth of $X_i(t)$ (Morellec and Zhdanov, 2005), we assume $\mu_i < r$, $i = T$ or $B$, where $r$ is the constant riskless interest rate. Also, $Z_T(t)$ and $Z_A(t)$ are a pair of correlated unit standard Brownian motions under $(\Omega, \mathcal{F}, \mathbb{P})$ with constant correlation coefficient $\rho$, where $\rho \in [-1, 1]$. For notational simplicity, we drop the time index $t$ by writing $X_i(t)$ as $X_i$, $Z_i(t)$ as $Z_i$ and in other continuous time stochastic processes.

We assume the continuous time acquisition game model to be perpetual where the more efficient acquirer firm can offer a takeover deal to the target firm at any time without any preset time limit. Following the same formulation on the additional surplus value generated by synergy through acquisition in Morellec and Zhdanov (2005), we assume that the post-acquisition combined surplus value is given by

$$V(X_A, X_T; \omega) = K_T X_T + K_A X_A + \alpha(K_A + K_T)(\omega X_A - X_T). \tag{2.2}$$

We assume positive synergy effect in acquisitions, so the synergy factors $\alpha$ and $\omega$ are positive real numbers. The last term in eq. (2.2) represents the additional surplus value proxied by the spread between the stochastic cash flows per unit capital and whose magnitude depends on the level of synergy between the two firms. This general specification assumes that the cash flow per unit capital generated by the target firm can be improved after acquisition (Shleifer and Vishny, 2003). More precisely, the gain in surplus is driven by the spread between $X_T$ and a multiple $\omega$ of $X_A$, where $\omega$ is expected to be more than one.

**Optimal takeover strategy under complete information**

First, we take the assumption of complete information, where the two synergy factors, $\alpha$ and $\omega$, are assumed to be known to both firms. The same assumption on $\alpha$ and $\omega$ is used in Morellec and Zhdanov (2005). This complete information case is used to serve for benchmark comparison in our later asymmetric information model. Let $\xi$ denote the fraction of the merged firm accrues to the acquirer firm, so the remaining fraction $1-\xi$ accrues to the target firm. The net surplus to the acquirer firm and target firm at the time of acquisition are given by

$$S_A(X_A, X_T; \omega, \xi) = \xi V(X_A, X_T; \omega) - K_A X_A, \tag{2.3a}$$

$$S_T(X_A, X_T; \omega, \xi) = (1 - \xi) V(X_A, X_T; \omega) - K_T X_T, \tag{2.3b}$$

respectively. The above payoff structures resemble that of an exchange option (Margrabe, 1978). Taking advantage of the homogeneity property of the net surplus functions $S_A$ and $S_T$ with respect to $X_A$ and $X_T$, we use $X_T$ as the numeraire and define the similarity variable to be the ratio $R = \frac{X_A}{X_T}$. Let $\mathbb{P}_T$ be the corresponding numeraire measure with $X_T$ as the numeraire. The dynamics of $R$ under $\mathbb{P}_T$ is given by

$$\frac{dR}{R} = \mu_R\, dt + \sigma_R\, dZ_R,$$

where $\mu_R = \mu_A - \mu_T$ and $\sigma_R\, dZ_R = \sigma_A\, dZ_A - \sigma_T\, dZ_T$, so that $\sigma_R^2 = \sigma_A^2 - 2\rho\sigma_A\sigma_T + \sigma_T^2$. Note that $Z_R$ is the unit standard Brownian motion under $\mathbb{P}_T$. In terms of $R$, the two net surplus functions when normalized by the numeraire $X_T$ take the form:

$$\begin{aligned} F_A(R;\omega,\xi) &= S_A(X_A, X_T;\omega,\xi)/X_T \\ &= [\xi\alpha(K_A+K_T)\omega - (1-\xi)K_A]\, R + \xi\, [K_T - \alpha(K_A+K_T)]\, ; \end{aligned} \tag{2.4a}$$

and

$$\begin{aligned} F_T(R;\omega,\xi) &= S_T(X_A, X_T;\omega,\xi)/X_T \\ &= (1-\xi)\, [K_A + \alpha(K_A+K_T)\omega]\, R - \xi K_T - (1-\xi)\alpha(K_A+K_T). \end{aligned} \tag{2.4b}$$

*Value functions and optimal thresholds with preset takeover deal*

For a given takeover deal $\xi$, we let $O_T(R;\omega,\xi)$ denote the option value function of the target firm's net surplus, and a similar definition for $O_A(R;\omega,\xi)$ of the acquirer firm. The solution of the two option value functions can be reduced to one-dimensional pricing problems with the single state variable $R$. The solution of $S_A(R;\omega,\xi)$ and $S_T(R;\omega,\xi)$ requires the determination of the optimal threshold values $R_A^*(\xi,\omega)$ and $R_T^*(\xi,\omega)$, respectively. According to the trigger strategy, the respective firm should optimally choose the merger decision when $R$ increases from below to the respective threshold value. Similar to the optimal stopping model in a perpetual American call option (Dixit and Pindyck, 1994), the solution to the two option value functions can be found to be

$$O_T(R;\omega,\xi) = \begin{cases} X_T F_T(R_T^*(\xi,\omega);\omega,\xi)\left[\frac{R}{R_T^*(\xi,\omega)}\right]^\beta & \text{if } R < R_T^*(\xi,\omega) \\ X_T F_T(R;\omega,\xi) & \text{if } R \geq R_T^*(\xi,\omega) \end{cases}, \tag{2.5$a$}$$

$$O_A(R;\omega,\xi) = \begin{cases} X_T F_A(R_A^*(\xi,\omega);\omega,\xi)\left[\frac{R}{R_A^*(\xi,\omega)}\right]^\beta & \text{if } R < R_A^*(\xi,\omega) \\ X_T F_A(R;\omega,\xi) & \text{if } R \geq R_A^*(\xi,\omega) \end{cases}, \tag{2.5$b$}$$

where $\beta$ is the positive root of the quadratic equation

$$\frac{\sigma_R^2}{2}x(x-1) + (\mu_A - \mu_T)x - r = 0.$$

The corresponding optimal threshold values of the target firm and acquirer firm for a given takeover deal $\xi$ are found to be

$$R_T^*(\xi,\omega) = \frac{\beta}{\beta-1}\frac{\xi K_T + (1-\xi)\alpha(K_A+K_T)}{(1-\xi)[\alpha\omega(K_A+K_T)+K_A]}, \tag{2.6a}$$

$$R_A^*(\xi,\omega) = \frac{\beta}{\beta-1}\frac{\xi[\alpha(K_A+K_T)-K_T]}{\xi[K_A+\alpha(K_A+K_T)\omega]-K_A}, \tag{2.6b}$$

respectively. To ensure that the threshold $R_A^*(\xi,\omega)$ is positive so that the acquirer firm has the incentive to participate in the takeover process, we assume that $\alpha(K_A+K_T)-K_T>0$ (Morellec and Zhdanov, 2005). This assumption is not too restrictive since $\alpha$ is usually closed to 1. For a fixed value of $\omega$, it is easily seen that $R_T^*(\xi,\omega)$ is an increasing function of $\xi$ since the target firm's optimal threshold should be higher when the takeover deal becomes less favorable at a higher value of $\xi$. Reversing the argument for the acquirer firm, we observe that $R_A^*(\xi,\omega)$ is a decreasing function of $\xi$. From eq. (2.6a), we observe the one-to-one correspondence between $\xi$ and $R_T^*(\xi,\omega)$. We write $\xi(R)$ as the takeover deal of the acquirer firm as a function of the threshold level $R$. By inverting the relation in eq. (2.6a), we obtain

$$\xi(R)=1-\frac{\beta K_T}{(\beta-1)[K_A+\alpha\omega(K_A+K_T)]R+\beta K_T-\beta\alpha(K_A+K_T)}. \tag{2.7}$$

*Acquirer's value function and optimal strategy under strategic takeover*

The specific challenge in our dynamic acquisition model is the determination of $\xi$ such that the target firm will agree with the takeover deal when the acquirer firm initiates the acquisition. The acquirer firm determines $\xi$ optimally together with the optimal threshold level $R$ at which the takeover deal is accepted by the target firm. Lambrecht (2004) and Morellec and Zhdanov (2005) argue that the negotiated outcome of the merger of two firms in equilibrium should satisfy $R_A^*(\xi,\omega)=R_T^*(\xi,\omega)$; that is, at the intersection points $(\bar{\xi},\bar{R})$ of the two plots of $R_A^*(\xi,\omega)$ and $R_T^*(\xi,\omega)$ (see Figure 1). In this paper, we assume that the acquirer firm takes a more aggressive role in merger that it sets the takeover deal at its optimal choice. The target firm is relatively passive in the sense that it does not negotiate the takeover deal $\xi$. However, the target firm would come into terms with the merger at its optimal threshold $R_T^*(\xi,\omega)$. In other words, the target firm would accept the takeover deal at $R_T^*(\xi,\omega)$, where its net surplus value is maximized at a given takeover deal $\xi$. In response, the acquirer firm has to delay the initiation of takeover until the optimal threshold $R_T^*(\xi,\omega)$ of the target firm in order to guarantee that the target firm will take the deal $\xi$ once takeover is initiated by the acquirer firm. We argue that the choice of $\xi$ should not be limited to $\bar{\xi}$ at which $R_A^*(\bar{\xi},\omega)=R_T^*(\bar{\xi},\omega)$, the choice concluded by Lambrecht (2004) and Morellec and Zhdanov (2005). Instead, the acquirer would choose $\xi\geq\bar{\xi}$ such that its net surplus value is maximized and initiates takeover at $R_T^*(\xi,\omega)$ with the proposed takeover deal $\xi$. The strategy space for the acquirer firm on choosing $\xi$ is enlarged when compared with the single choice $\bar{\xi}$. The thickened portion of the plot of $R_T^*(\xi,\omega)$, $\xi\geq\bar{\xi}$, in Figure 1 represents the optimal strategic set of $(\xi,R)$ that can be taken by the acquirer for maximizing its net surplus value. Note that the initiation of takeover at a threshold above the acquirer's own optimal threshold $R_A^*(\xi,\omega)$ would undermine the value of the net surplus. For the acquirer firm, there is a tradeoff between choosing a higher $\xi$ while facing a widening gap in $R_A^*(\xi,\omega)$ and $R_T^*(\xi,\omega)$.

Under complete information on the synergy factors, how would the aggressive acquirer firm determines the optimal pair $(\xi,R_T^*)$? We let $O_A^c(R;\omega,\xi)$ denote the acquirer's option value of net surplus under *complete information*, where the acquirer strategically delay the trigger threshold of takeover to be $R_T^*$ (instead of $R_A^*$). At $R\leq R_T^*$, $O_A^c(R;\omega,\xi(R_T^*))$ assumes the solution form as that of $O_A(R;\omega,\xi)$ in the continuation region [see eq. (2.5b)] except

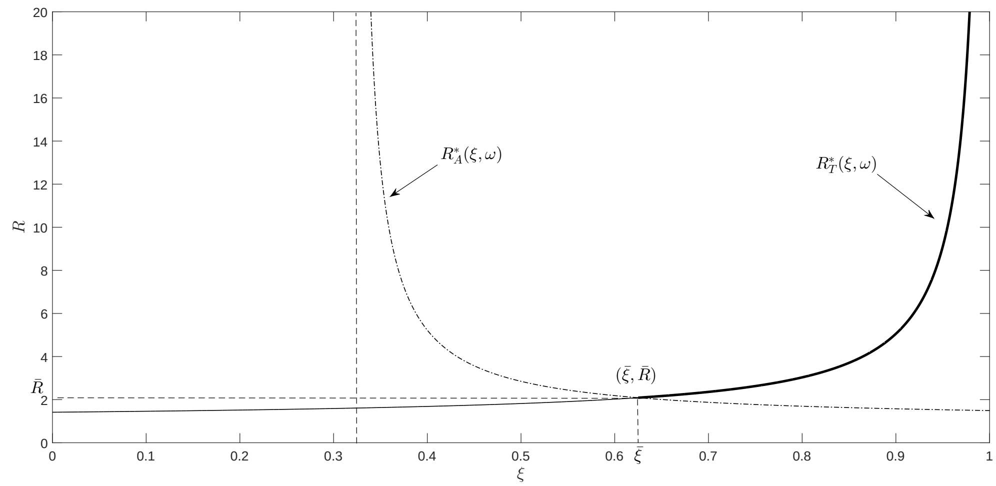


Figure 1: Plot of the takeover threshold values $R_T^*(\xi,\omega)$ and $R_A^*(\xi,\omega)$ against $\xi$ under complete information. The intersection point of the two curves is $(\bar{\xi},\bar{R})$. The portion of the thickened curve of $R_T^*(\xi,\omega)$, $\xi \geq \bar{\xi}$, represents the optimal strategic set of $(\xi, R)$ that can be taken by the acquirer firm.

that $R_A^*$ is replaced by $R_T^*$. We then have

$$O_A^c(R;\omega,\xi(R_T^*)) = X_T\left\{[\xi(R_T^*)\alpha(K_A+K_T)\omega - [1-\xi(R_T^*)]K_A]R_T^* \right. \\ \left. + \xi(R_T^*)[K_T - \alpha(K_A+K_T)]\right\}\left(\frac{R}{R_T^*}\right)^\beta, \tag{2.8}$$

where the solution is obtained by applying the value matching condition but not the smooth pasting condition at $R = R_T^*$. The acquirer firm chooses its optimal takeover deal $\xi^*$ such that the above net surplus value is maximized. By virtue of eq. (2.7), $O_A^c(R;\omega,\xi(R_T^*))$ can be visualized as a function of $R_T^*$. By applying the first order condition of setting the derivative of $O_A^c(R;\omega,\xi(R_T^*))$ with respect to $R_T^*$ to be zero, we obtain the following cubic equation for $R_T^*$:

$$[(\beta-1)^3\alpha(K_A+K_T)\omega A_\omega^2](R_T^*)^3 \\ - [2\beta(\beta-1)^2\alpha(K_A+K_T)\omega A_\omega B + \beta^2(\beta-1)K_T A_\omega^2 + \beta(\beta-1)^2 A_\omega^2 B](R_T^*)^2 \\ + [\beta^2(\beta-1)\alpha(K_A+K_T)\omega B^2 + 2\beta^2(\beta-1)\alpha(K_A+K_T)A_\omega B + \beta(\beta-1)K_T A_\omega B]R_T^* \\ - \beta^3\alpha(K_A+K_T)B^2 = 0, \tag{2.9}$$

where

$$A_\omega = K_A + \alpha(K_A+K_T)\omega \quad \text{and} \quad B = \alpha(K_A+K_T) - K_T > 0.$$

Given that $B > 0$ [equivalent to positivity of $R_A^*(\xi,\omega)$], one can show that the cubic equation always give a unique real positive root. Once the optimal threshold $R_T^*$ is found, by virtue of eq. (2.7), we can determine the corresponding takeover deal $\xi^*(R_T^*)$. Furthermore, it can be shown that the optimal pair $(\xi^*, R_T^*)$ lies in the thickened portion of the plot of

$R_T^*(\xi)$ (see Figure 1). As a numerical verification, we obtain $(\bar{\xi}, \bar{R}) = (0.6250, 2.0900)$ and $(\xi^*, R_T^*) = (0.8710, 4.1466)$ using the following set of parameter values: $r = 0.05$, $\mu_B = 0.03$, $\mu_T = 0.01$, $\sigma_B = 0.2$, $\sigma_T = 0.2$, $\rho = 0.25$, $K_A = 5$, $K_T = 2$, $\alpha = 1$, $\omega = 1.5$.

As the aggressive acquirer firm maximizes the value of $O_A^c(R; \omega, \xi)$ among the optimal strategic set $\xi \geq \bar{\xi}$, so the acquirer's option value of net surplus would be higher or at least equal to that attained at $\bar{\xi}$. On the other hand, the passive target firm may have lower option value of net surplus compared to that attained at $\bar{\xi}$. However, the target firm would accept the takeover deal $\xi$ proposed by the acquirer firm at its optimal threshold $R_T^*(\xi)$.

# 3 Equilibrium strategies under information asymmetry

Our framework of analyzing the signaling game model is similar to the approach used by Morellec and Schürhoff (2011) in corporate investment and financing under asymmetric information. In our information asymmetry formulation of the signaling game, we assume that while the precise value of the synergy factor $\omega$ is the private information held by the acquirer firm, the information on $\omega$ that is known to the target firm at initiation observes the following Bernoulli distribution: $\omega$ may assume the high value $\omega_H$ with probability $p$ and the low value $\omega_L$ with probability $1 - p$, where $\omega_H > \omega_L$ and $p \in (0, 1)$. As in most signaling games, the informationally disadvantaged target firm can improve its estimation on $\omega$ by observing the timing and terms of takeover made by the acquirer firm. The optimal timing is translated into optimal threshold under trigger strategy assumed in our model.

*Belief functions*

More specifically, we define the initial belief function $D_0$ to be

$$D_0 = (P[\omega = \omega_H], P[\omega = \omega_L]) = (p, 1 - p). \tag{3.1$a$}$$

We write $M = (\xi, R)$ to be the takeover package offered by the acquirer firm. Under our signaling game model framework, the target firm may be able to deduce the updated information on the type (high type or low type) of the acquirer firm based on $M$. More precisely, we define the updated belief function $D_M$ by

$$D_M = (P[\omega = \omega_H | M], P[\omega = \omega_L | M]) = (p_M, 1 - p_M). \tag{3.1$b$}$$

Since the acquirer firm is assumed to adopt pure strategy (single takeover deal and takeover threshold), we consider the following two scenarios:

(a) If both types of the acquirer firm adopt a common strategy (pooling), the target firm cannot identify the true type so that the belief system remains to be $(p, 1 - p)$.

(b) If the types of the acquirer firm adopt different strategies (separating), the target firm is able to identify the true type, so the updated belief is either $(1, 0)$ (high type) or $(0, 1)$ (low type).

As a summary, the updated belief function $D_M$ can take the following three possible values:

(i) $D_M = (p, 1-p)$
Here, $M$ fails to reveal the information on the type of the acquirer firm.

(ii) $D_M = (1, 0)$
The true "high" type of the acquirer firm is revealed to the target firm through $M$.

(iii) $D_M = (0, 1)$
The true "low" type of the acquirer firm is revealed to the target firm through $M$.

In other words, $p_M$ may assume the value $p$, 1, or 0, corresponding to the above three scenarios, respectively.

*Expected surplus value of the merged firm*
Given a takeover package $M = (\xi, R)$ offered by the acquirer firm, the target firm's valuation of the merged firm is the expected surplus value of the merged firm given $M$. We define

$$\omega_M = p_M \omega_H + (1 - p_M)\omega_L. \tag{3.2}$$

In other words, $\omega_M$ may assume the value $p\omega_H + (1-p)\omega_L$, $\omega_H$ or $\omega_L$, corresponding to $p_M$ equals $p$, 1 or 0, respectively. Since the post-acquisition combined surplus value $V(X_A, X_T; \omega)$ [see eq. (2.2)] is a linear function in $\omega$, we have

$$\begin{aligned} V(X_A, X_T; \omega_M) &= \mathbb{E}[V(X_A, X_T; \omega)|M] \\ &= p_M V(X_A, X_T; \omega_H) + (1 - p_M) V(X_A, X_T; \omega_L). \end{aligned} \tag{3.3}$$

*Surplus value functions of the target and acquirer firms*
Let $R_T^*(\xi, \omega_M)$ be the optimal threshold value above which the target firm would take the takeover deal based on the updated belief of $\omega_M$. By following a similar derivation as in the complete information case, we obtain the target firm's surplus function based on $\omega_M$ as follows:

$$O_T^M(R; \omega_M, \xi) = \begin{cases} X_T F_T(R_T^*(\xi, \omega_M); \omega_M, \xi) \left[\frac{R}{R_T^*(\xi, \omega_M)}\right]^\beta & \text{if } R < R_T^*(\xi, \omega_M) \\ X_T F_T(R; \omega_M, \xi) & \text{if } R \geq R_T^*(\xi, \omega_M) \end{cases}. \tag{3.4a}$$

Since the target firm is passive, the target firm's optimal threshold for a given $\xi$ and $\omega_M$ takes the same form as that of the complete information case [see eq. (2.6a)], where

$$R_T^*(\xi, \omega_M) = \frac{\beta}{\beta - 1} \frac{\xi K_T + (1-\xi)\alpha(K_B + K_T)}{(1-\xi)[\alpha\omega_M(K_B + K_T) + K_B]}. \tag{3.4b}$$

On the other hand, under the signaling game model that exhibits separating and pooling equilibrium, the optimal threshold of the aggressive acquirer firm would not take the same form as that in the complete information case. However, the acquirer's threshold must be higher than or equal to $R_T^*(\xi, \omega_M)$ in order to guarantee that the deal $\xi$ will be taken up by the target firm. The acquirer firm's surplus value function $O_A^M(R; \omega, \xi)$ based on $\omega = \omega_L$ or $\omega = \omega_H$ takes the same form as that shown in eq. (2.5b).

Next, we examine how $R_A^*$ and $\xi$ are determined based on various takeover strategies of the acquirer firm under either the separating or pooling equilibrium. In particular, we consider the least-cost separating equilibrium where the acquirer firm of high type chooses the separating equilibrium that minimizes the signaling cost. Here, the signaling cost is defined to be the option value of the acquirer's net surplus under complete information minus that under separating equilibrium. This is the cost borne on the acquirer of high type for signaling its type to the target firm under separating equilibrium.

## 3.1 Characterization of the separating equilibrium

Under separating equilibrium, the acquirer firm of high type or low type would choose different takeover strategies and its true synergy factor $\omega$ can be revealed to the target firm. This is in contrast with the pooling equilibrium, where the acquirer firm of the low type adopts the mimicking strategy under which it uses the same takeover deal and takeover strategy as those of the high type firm. We write $M_H^* = (\xi_H, R_H^*)$ and $M_L^* = (\xi_L, R_L^*)$ as the optimal takeover package under information asymmetry for the acquirer firm of high type and low type, respectively. We also write $\xi_L^c$ and $R_L^c$ as the optimal takeover deal and threshold when $\omega = \omega_L$ under the complete information case; and similar notation for $\xi_H^c$ and $R_H^c$ when $\omega = \omega_H$.

Under separating equilibrium, the acquirer firm of low type cannot take advantage of the information asymmetry since it cannot mimic the acquirer firm of high type. In this case, the optimal takeover package $(\xi_L, R_L^*)$ would be identical to that under complete information of $\omega_L$, so

$$\xi_L = \xi_L^c \quad \text{and} \quad R_L^* = R_L^c. \tag{3.5a}$$

On the other hand, when the acquirer firm is of high type, we have $\omega_M = \omega_H$ under separating equilibrium. To guarantee that the deal is taken by the target firm, the optimal threshold adopted by the acquirer firm satisfies

$$R_H^*(\xi_H) \geq R_T^*(\xi_H, \omega_H). \tag{3.5b}$$

Next, we discuss the mimicking strategy that can be adopted by the acquirer firm of low type. We then consider the strategy adopted by the high type counterpart in order to signal its true type to the target firm. We present the systematic procedure to determine $M_H^* = (\xi_H^*, R_H^*)$ of the acquirer firm of high type using the argument of the least-cost separating equilibrium.

**Strategic space of mimicking**

Suppose the acquirer firm is low type with $\omega = \omega_L$, we examine the condition under which it is better off for the low type acquirer to choose the mimicking strategy. Suppose the low type acquirer mimics the takeover strategy as if it is high type, the surplus value function is given by $X_T F_A(R^*; \omega_L, \xi) \left(\frac{R}{R^*}\right)^\beta$, where $R^*$ is the threshold at which the acquisition decision is exercised and the value funtion $F_A$ is defined in eq. (2.4a). Note that $R^*$ would not be the same as $R_T^*(\xi, \omega_L)$ [see eq. (2.6a)]. On the other hand, suppose the mimicking strategy is not adopted, then the corresponding surplus function would become $X_T F_A(R_L^c; \omega_L, \xi_L^c) \left(\frac{R}{R_L^c}\right)^\beta$. One may characterize the mimicking region in the $\xi$-$R^*$ plane inside which the set of points

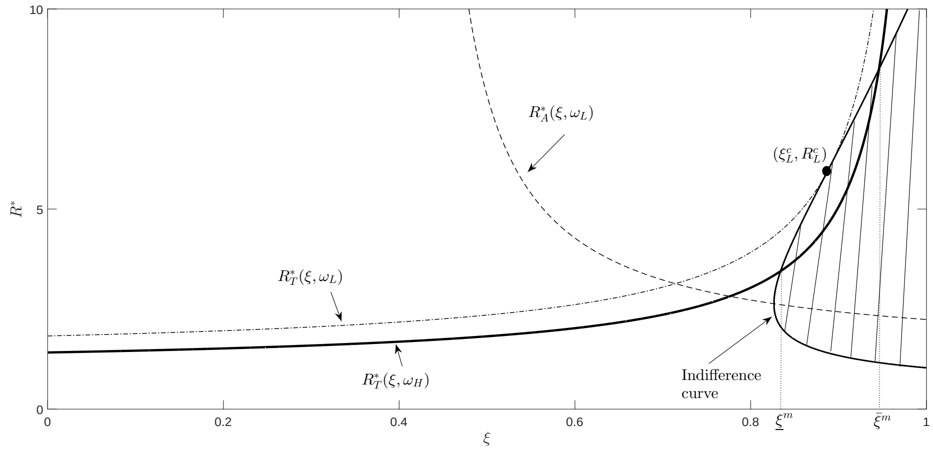


Figure 2: Plot of the optimal takeover threshold values $R_T^*(\xi,\omega_H)$, $R_T^*(\xi,\omega_L)$, $R_A^*(\xi,\omega_L)$ at varying values of $\xi$ and the indifference curve as defined by eq. (3.7). The feasible region of adopting the mimicking strategy lies on the right hand side of the indifference curve (shaded region). The indifference curve intersects the curve of $R_T^*(\xi,\omega_H)$ at $\underline{\xi}^m$ and $\bar{\xi}^m$, $\underline{\xi}^m < \bar{\xi}^m$. The curve of $R_T^*(\xi,\omega_L)$ is tangential to the indifference curve at $(\xi_L^c, R_L^c)$. The left most point of the mimicking region lies on the curve of $R_A^*(\xi,\omega_L)$. The parameter values used in generating the curves are the same as those for plotting Figure 1, except that $\omega_H$ and $\omega_L$ are taken to be 1.5 and 1, respectively.

satisfy

$$F_A(R^*;\omega_L,\xi)\left(\frac{R}{R^*}\right)^\beta > F_A(R_L^c;\omega_L,\xi_L^c)\left(\frac{R}{R_L^c}\right)^\beta. \tag{3.6}$$

Inside the mimicking region, the low type acquirer is better off by adopting the mimicking strategy. The boundary of the mimicking region is defined by the indifference curve at which we observe equality of the above two surplus functions under mimicking or otherwise. The set of points $(\xi, R^*)$ on the indifference curve in the $\xi$-$R^*$ plane satisfy

$$F_A(R^*;\omega_L,\xi)\left(\frac{R}{R^*}\right)^\beta = F_A(R_L^c;\omega_L,\xi_L^c)\left(\frac{R}{R_L^c}\right)^\beta. \tag{3.7}$$

On the indifference curve, the low type acquirer is indifferent between mimicking as high type or not mimicking.

Using the same set of parameter values for plotting Figure 1, except that we take $\omega_H = 1.5$ and $\omega_L = 1$, we generate the plots of the optimal threshold values $R_T^*(\xi,\omega_H)$, $R_T^*(\xi,\omega_L)$ and $R_A^*(\xi,\omega_L)$ at varying values of $\xi$ in Figure 2. To generate the plot of the indifference curve using eq. (3.7), we cancel the common factor $R^\beta$, visualize $R_L^c$ and $\xi_L^c$ as known quantites, then determine $R^*$ for varying values of $\xi$. The shaded region to the right hand side of the indifference curve is the mimicking region, inside which ineq. (3.6) is satisfied. It is seen that the mimicking region lies below the curve of $R_T^*(\xi,\omega_L)$ and the curve touches the region tangentially at $(\xi_L^c, R_L^c)$. This is consistent with the following intuition. In order that the low type acquirer firm is profitable to mimic as high type, it would offer a takeover deal $\xi$ at a threshold that should not go above $R_T^*(\xi,\omega_L)$. The point $(\xi_L^c, R_L^c)$, where $R_L^c = R_T^*(\xi_L^c, \omega_L)$, satisfies eq. (3.7) identically and it also lies on the curve $R_T^*(\xi,\omega_L)$ under complete information, so this explains the tangency property. From financial intuition, the mimicking strategy adopted by the acquirer firm can take the least value of $\xi$ only at the optimal threshold $R_A^*(\xi,\omega_L)$, so the left most point of the mimicking region lies on the curve of $R_A^*(\xi,\omega_L)$. For a given takeover deal $\xi$ that lies inside the mimicking region, the threshold $R^*$ falls within the interval $(R_l^*(\xi), R_u^*(\xi))$, where $R_l^*(\xi)$ and $R_u^*(\xi)$ are the respective smaller and larger root of eq. (3.7). In addition, the curve $R_T^*(\xi,\omega_H)$ intersects the indifference curve at $\xi = \underline{\xi}^m$ and $\bar{\xi}^m$, where $\underline{\xi}^m < \bar{\xi}^m$. We observe that $R_T^*(\xi,\omega_H)$ lies inside the mimicking region when $\xi \in (\underline{\xi}^m, \bar{\xi}^m)$. This would indicate that the high type acquirer firm would fail to signal its type to the target firm when it offers a deal $\xi \in (\underline{\xi}^m, \bar{\xi}^m)$ at the threshold $R_T^*(\xi,\omega_H)$ since the takeover pair $(\xi, R_T^*(\xi,\omega_H))$ lies inside the mimicking region. In this case, the high type acquirer firm has to give up the option value and choose to delay the takeover until $R$ reaches a higher threshold $R_u^*(\xi)$ such that the acquirer firm of low type would be indifferent between mimicking and revealing its true type to the target firm. In other words, the strategy space of the takeover threshold $R_H^*(\xi)$ of the high type acquirer firm is given by

$$R_H^*(\xi) = \begin{cases} R_u^*(\xi) & \text{if } \xi \in (\underline{\xi}^m, \bar{\xi}^m) \\ R_T^*(\xi,\omega_H) & \text{if otherwise} \end{cases}. \tag{3.8}$$

The technical result stated in Lemma 1 relates the relative magnitude of $R_T^*(\xi,\omega_j)$ and $R_A^*(\xi,\omega_j)$, $j = L, H$, which is relevant to our later analysis of equilibrium strategies of takeover.

**Lemma 1** *We let $(\xi^\ell, R^\ell)$ denote the left most point of the indifference curve in the $\xi$-$R$ plane (see Figure 2). For any $\xi \geq \xi^\ell$, we have*

$$R_T^*(\xi,\omega_j) > R_A^*(\xi,\omega_j), \quad j = L, H. \tag{3.9}$$

The proof of Lemma 1 requires the determination of the intersection point of the curves $R_T^*(\xi,\omega_j)$ and $R_A^*(\xi,\omega_j)$, $j = L, H$, with the indifference curve, and applying the monotonicity properties of these curves with respect to $\xi$.

Suppose the takeover package of the high type acquirer firm under complete information $(\xi_H^c, R_H^c)$, where $R_H^c = R_T^*(\xi_H^c, \omega_H)$, lies outside the mimicking region, then it can signal its type without signaling cost by adopting the takeover package $(\xi_H^c, R_H^c)$. If otherwise, one needs to refine the choice of the takeover package of high type acquirer firm via the notion of least-cost separating equilibrium.

**Least-cost separating equilibrium**

We would like to establish that when $(\xi_H^c, R_H^c)$ lies inside the mimicking region, it is always suboptimal for the acquirer firm of high type to offer a takeover package $(\xi, R_H^*)$, where $\xi \in (\underline{\xi}^m, \bar{\xi}^m)$ and $R_H^*(\xi) = R_u^*(\xi)$ [see eq. (3.8)]. The dominance property of the acquirer firm's surplus functions associated with these takeover strategies is summarized in Lemma 2:

**Lemma 2** *The takeover package* $(\xi, R_u^*(\xi))$*, where* $\xi \in (\underline{\xi}^m, \bar{\xi}^m)$*, is dominated by the takeover package* $(\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ *since the associated normalized acquirer's surplus functions observe*

$$F_A(R_u^*(\xi); \omega_H, \xi) \left[\frac{R}{R_u^*(\xi)}\right]^\beta < F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[\frac{R}{R_T^*(\underline{\xi}^m, \omega_H)}\right]^\beta, \quad \xi \in (\underline{\xi}^m, \bar{\xi}^m). \tag{3.10}$$

The proof of Lemma 2 is relegated to Appendix A.

Furthermore, we would like to establish that it is sub-optimal for the acquirer firm of high type to offer the takeover package $(\xi, R_T^*(\xi, \omega_H))$, where $\xi \geq \bar{\xi}^m$. Such takeover strategy is strictly dominated by $(\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$. To show the claim, we recall that the takeover package of high type acquirer firm under complete information $(\xi_H^c, R_H^c)$ maximizes the net surplus of the high type acquirer firm. That is,

$$\xi_H^c = \underset{\xi}{\operatorname{argmax}}\, F_A(R_T^*(\xi, \omega_H); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_H)}\right]^\beta, \qquad R_H^c = R_T^*(\xi_H^c, \omega_H).$$

Since $(\xi_H^c, R_H^c)$ satisfies eqs. (2.7) and (2.9) and eq. (2.9) has a unique real positive root (namely, $R_H^c$), we deduce that $F_A(R_T^*(\xi, \omega_H); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_H)}\right]^\beta$ is strictly increasing when $\xi < \xi_H^c$ and strictly decreasing when $\xi > \xi_H^c$. Together with the assumption that $\xi_H^c \in (\underline{\xi}^m, \bar{\xi}^m)$, we can deduce that for any $\xi > \bar{\xi}^m$ and $R_H^* = R_T^*(\xi, \omega_H)$, we have

$$\begin{aligned} F_A(R_T^*(\xi, \omega_H); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_H)}\right]^\beta &< F_A(R_T^*(\bar{\xi}^m, \omega_H); \omega_H, \bar{\xi}^m) \left[\frac{R}{R_T^*(\bar{\xi}^m, \omega_H)}\right]^\beta \\ &< F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[\frac{R}{R_T^*(\underline{\xi}^m, \omega_H)}\right]^\beta. \end{aligned} \tag{3.11a}$$

The last inequality is obtained by virtue of ineq. (3.10).

Ineqs. (3.10) and (3.11a) imply that the takeover strategy $M_H^s = (\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ maximizes the net surplus of the acquirer firm of high type. To argue that this takeover strategy is the least-cost separating equilibrium, it remains to show that it is suboptimal for the acquirer firm of high type to offer any takeover package $M = (\xi, R_H^*)$ that lies inside the mimicking region, where $(\underline{\xi}^m, \bar{\xi}^m)$ and $R_H^* \in (R_T^*(\underline{\xi}, \omega_H), R_u^*(\xi))$. To show the claim, we observe that the acquirer firm's type cannot be revealed to the target firm when the takeover package $M$ lies inside the mimicking region. Since the updated belief $D_M$ cannot be $(1, 0)$, so we would assume that the off-equilibrium belief of offering such takeover package $M$ is taken to be $D_M = (0, 1)$, the most pessimistic belief. In other words, the target firm accepts the deal only when $R_H^* \geq R_T^*(\xi, \omega_L)$. Since the entire mimicking region lies below the curve $R^* = R_T^*(\xi, \omega_L)$ (see Figure 2), so the target firm accepts the offer only when the acquirer firm offers the takeover package $M_L^c = (\xi_L^c, R_L^c)$, which is the only intersection point between the curve $R^* = R_T^*(\xi, \omega_L)$ and the indifference curve. However, such takeover package $M_L^c = (\xi_L^c, R_L^c)$ is strictly dominated by $M_H^s = (\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ since

$$F_A(R_L^c; \omega_H, \xi_L^c)\left(\frac{R}{R_L^c}\right)^\beta = F_A(R_u(\xi_L^c); \omega_H, \xi_L^c)\left[\frac{R}{R_u^*(\xi_L^c)}\right]^\beta$$
$$< F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m)\left[\frac{R}{R_T^*(\underline{\xi}^m, \omega_H)}\right]^\beta. \tag{3.11b}$$

The last inequality is obtained again by virtue of ineq. (3.10). Hence, it is sub-optimal for the high type acquirer firm to offer any takeover package $M$ that lies inside the mimicking region. We then conclude that the takeover strategy $(\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ is the desired least-cost separating equilibrium for the acquirer firm of high type.

The perfect Bayesian equilibrium of the takeover strategy taken by the acquirer firm under separating equilibrium can be summaried in Proposition 1.

**Proposition 1** *The separating equilibrium can be characterized as follows:*

*(i) When $(\xi_H^c, R_H^c)$ lies outside the mimicking region, the acquirer firm of high type offers a takeover package $M_H^c = (\xi_H^c, R_H^c)$ and the acquirer firm of low type offers a takeover package $M_L^c = (\xi_L^c, R_L^c)$ under separating equilibrium. In other words, the takeover strategy is the same as that under complete information.*

*(ii) When $(\xi_H^c, R_H^c)$ lies inside the mimicking region, the acquirer firm of high type offers a takeover package $M_H^s = (\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ and the acquirer firm of low type offers a takeover package $M_L^c = (\xi_L^c, R_L^c)$ under least-cost separating equilibrium.*

One can derive several important insights from the results in Proposition 1.

1. Under least-cost separating equilibrium, we observe that the acquirer firm of high type initializes the takeover at a lower threshold since $R_T^*(\underline{\xi}^m, \omega_H) < R_T^*(\xi_H^c, \omega_H) = R_H^c$, where $\underline{\xi}^m < \xi_H^c$. On the other hand, the acquirer firm offers more attractive deal to the target firm under this equilibrium. The takeover package (timing and deal) serves as a credible signal for the acquirer firm of high type to reveal its high type status to the target firm. This result agrees with the empirical studies done by Draper and Paudyal (2008), which state that the undervalued firm can announce the timing of takeover earlier to mitigate information asymmetry.

2. Although the target firm is informationally disadvantaged, Proposition 1 reveals that the target firm can receive a larger proportion of the merged firm under least-cost separating equilibrium since $\underline{\xi}^m < \xi_H^c$. Furthermore, one can show that the net surplus of the target firm is increased when we compare the respective net surplus value under least-cost separating equilibrium and complete information (as shown by the following inequalities):

$$F_T(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m)\left(\frac{R}{R_T^*(\underline{\xi}^m, \omega_H)}\right)^\beta > F_T(R_H^c; \omega_H, \underline{\xi}^m)\left(\frac{R}{R_H^c}\right)^\beta$$
$$> F_T(R_H^c; \omega_H, \xi_H^c)\left(\frac{R}{R_H^c}\right)^\beta.$$

Here, the first inequality follows from the definition of $R_T^*(\underline{\xi}^m, \omega_H)$ and the second inequality follows from $\underline{\xi}^m < \xi_H^c$ and eq. (2.4b).

## 3.2 Characterization of the pooling equilibrium

Since there is a signaling cost for the acquirer firm of high type to adopt the separating strategy, the firm may be better off to choose the pooling strategy if the signaling cost is too substantial. We would like to characterize the pooling equilibrium in our dynamic acquisition model and examine the conditions under which the acquirer firm of high type may prefer the pooling strategy to separating strategy.

We write $M_p = (\xi_p, R_p^*)$ as the takeover pair under pooling equilibrium. Since the acquirer firm's quality type cannot be revealed to the target firm under pooling equilibrium, the target firm's belief system remains to be $D_{M_p} = (p, 1-p)$ as there is no information updating. In order to guarantee that the target firm would accept the deal $\xi_p$, the takeover threshold under pooling equilibrium $R_p^*$ should observe

$$R_p^* \geq R_T^*(\xi_p, \omega_p), \tag{3.12}$$

where $\omega_p = p\omega_H + (1-p)\omega_L$. To the acquirer firm of low type, it is obvious to adopt the pooling strategy since its net surplus value under pooling strategy is always higher than that under complete information. In other words, we deduce that $M_p^* = (\xi_p, R_p^*)$ should lie inside the mimicking region since the normalized low type acquirer's surplus function satisfies

$$F_A(R_p^*; \omega_L, \xi_p) \left(\frac{R}{R_p^*}\right)^\beta > F_A(R_L^c; \omega_L, \xi_L^c) \left(\frac{R}{R_L^c}\right)^\beta. \tag{3.13}$$

We plot the optimal threshold values $R_T^*(\xi, \omega_p)$, $R_T^*(\xi, \omega_L)$, $R_A^*(\xi, \omega_H)$, $R_A^*(\xi, \omega_L)$ against varying values of $\xi$ and the indifference curve in Figure 3. We use the same set of parameter values used in plotting Figure 2 while the probability value $p$ is set to be 0.5. The feasible pooling region is seen to be bounded by the indifference curve in the above and the curve of $R_T^*(\xi, \omega_p)$ at below. The curve $R_T^*(\xi, \omega_p)$ cuts the indifference curve at two points: $\xi = \underline{\xi}^p$ and $\xi = \bar{\xi}^p$, where $\underline{\xi}^p < \bar{\xi}^p$. We deduce that the takeover deal $\xi_p$ lies within the interval $[\underline{\xi}^p, \bar{\xi}^p]$, where the pooling region is defined. In order to minimize the loss of net surplus value, the acquirer firm of high type should choose the optimal threshold of acquisition to be $R_T^*(\xi, \omega_p)$ since $R_p^* \geq R_T^*(\xi, \omega_p) > R_T^*(\xi, \omega_H) > R_A^*(\xi, \omega_H)$ [see ineqs. (3.9) and (3.12)]. In other words, the strategic space of pooling equilibrium is seen to be

$$\left\{M_p^* = (\xi, R_T^*(\xi, \omega_p)) : \xi \in [\underline{\xi}^p, \bar{\xi}^p]\right\}. \tag{3.14}$$

Suppose pooling equilibrium is adopted, then the acquirer firm of high type chooses $\xi_p \in [\underline{\xi}^p, \bar{\xi}^p]$ such that

$$\xi_p = \underset{\xi}{\operatorname{argmax}}\, F_A(R_T^*(\xi, \omega_p); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi_p, \omega_p)}\right]^\beta. \tag{3.15}$$

When $M_H^c = (\xi_H^c, R_H^c)$ lies inside the mimicking region, according to Proposition 1, the acquirer firm of high type would choose the takeover deal $M_H^s = (\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ under

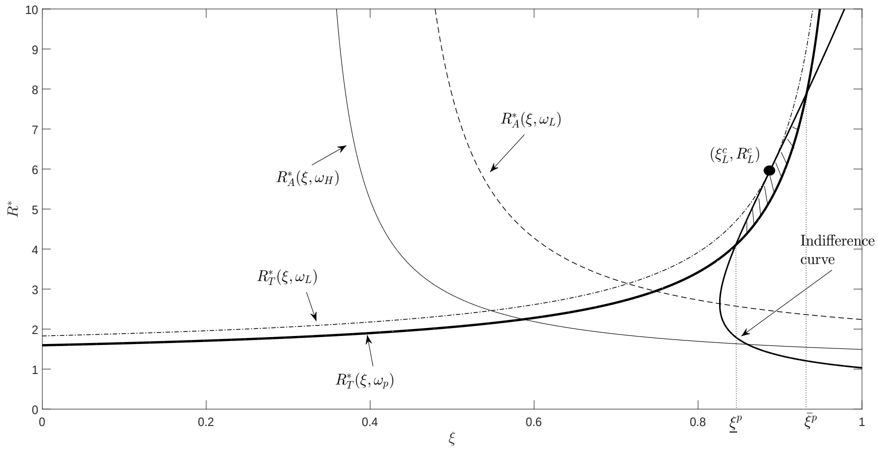


Figure 3: Plot of the optimal threshold values $R^*_T(\xi,\omega_p)$, $R^*_T(\xi,\omega_L)$, $R^*_A(\xi,\omega_H)$, $R^*_A(\xi,\omega_L)$ against varying values of $\xi$ and the indifference curve defined by eq. (3.7). The feasible region of adopting the pooling strategy (shaded region) is bounded by the indifference curve and the curve of $R^*_T(\xi,\omega_p)$. The indifference curve intersects the curve of $R^*_T(\xi,\omega_p)$ at $\underline{\xi}^p$ and $\bar{\xi}^p$, $\underline{\xi}^p < \bar{\xi}^p$. The curve of $R^*_T(\xi,\omega_L)$ is tangential to the indifference curve at $(\xi^c_L, R^c_L)$. The parameter values used in generating the curves are same as those for plotting Figure 2, except that $p$ is taken to be 0.5.

separating equilibrium. For further consideration, under the condition that $\xi^c_H \in [\underline{\xi}^m, \bar{\xi}^m]$, the adoption of either pooling strategy or separating strategy by the acquirer firm of high type depends on the relative magnitude of the surplus value functions under these two respective strategies. More precisely, the acquirer firm of high type would adopt pooling strategy if

$$F_A(R^*_T(\xi_p,\omega_p);\omega_H,\xi_p))\left[\frac{R}{R^*_T(\xi_p,\omega_p)}\right]^\beta > F_A(R^*_T(\underline{\xi}^m,\omega_H);\omega_H,\underline{\xi}^m))\left[\frac{R}{R^*_T(\underline{\xi}^m,\omega_H)}\right]^\beta. \quad (3.16)$$

As a summary, provided that ineq. (3.16) holds, then the acquirer firm of either type would adopt the pooling strategy $\xi_p$ as defined in eq. (3.15). The target firm's belief system remains to be $D_M = (p, 1-p)$.

Intuitively, we expect that the acquirer firm of high type prefers pooling strategy to separating strategy if the level of information asymmetry is relatively mild so that the loss in surplus value due to information asymmetry is smaller than the signaling cost incurred for adopting the separating strategy. We show in Proposition 2 that ineq. (3.16) holds when the probability $p$ is sufficiently large so that the acquirer firm adopts the pooling strategy under equilibrium.

The perfect Bayesian equilibrium of the takeover strategy taken by the acquirer firm under pooling equilibrium is summarized in Proposition 2.

**Proposition 2** *Suppose $\xi^c_H \in [\underline{\xi}^m, \bar{\xi}^m]$, there exists $p_0 \in [0,1]$ such that when $p > p_0$, then ineq. (3.16) holds. In this case, there exists a pooling equilibrium in which the acquirer firm of either type would adopt the pooling strategy with the choice of $\xi_p$ as determined by eq. (3.15) at the takeover threshold $R^*_T(\xi_p,\omega_p)$. This pooling equilibrium strictly dominates the least-cost separating equilibrium in Pareto sense.*

The proof of Proposition 2 is presented in Appendix B.

In summary, the optimal takeover strategy adopted by the acquirer firm of high type or low type under equilibrium can be summarized as follows:

1. When $M_H^c = (\xi_H^c, R_H^c)$ lies *outside* the mimicking region, then the separating equilibrium prevails. The acquirer firm of high type offers a takeover package $M_H^c = (\xi_H^c, R_H^c)$ and the low type offers a takeover package $M_L^c = (\xi_L^c, R_L^c)$. In other words, the acquirer firm adopts the same strategy as that under complete information.

2. When $M_H^c = (\xi_H^c, R_H^c)$ lies *inside* the mimicking region, we consider the following separate cases:

   (a) If $p \leq p_0$, then least-cost separating equilibrium prevails. The acquirer firm of high type offers the takeover package $M_H^* = (\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ and the low type offers $M_L^c = (\xi_L^c, R_L^c)$.

   (b) If $p > p_0$, then pooling equilibrium prevails. The acquirer firm of either type offers the same takeover package $M_p^* = (\xi_p, R_T^*(\xi_p, \omega_p))$.

# 4 Numerical studies of optimal takeover deals and thresholds

In this section, we would like to examine the properties of the optimal takeover deal $\xi$ and threshold $R_T^*$ adopted by the acquirer form of either high type or low type with respect to varying values of $p$ and synergy ratio $\omega_R = \omega_H/\omega_L$. In all our calculations of the optimal takeover deals and threshold values, we use the same set of parameter values that are used in generating Figure 2.

**Optimal takeover deals**

In Figure 5a, we show the plot of the optimal takeover deal $\xi_H^*$ adopted by the acquirer firm of high type against probability $p$. Note that the takeover deal $\xi_H^p$ under pooling equilibrium is decreasing in $p$ for low value of $p$ and increasing in $p$ for high value of $p$. Since the acquirer of high type always demand a more favorable deal under pooling equilibrium than that under least-cost separating equilibrium, so $\xi_H^p > \underline{\xi}^m$ (note that $\underline{\xi}^m$ is independent of $p$). By Proposition 2, there exists a threshold value $p_0$ such that pooling equilibrium prevails when $p > p_0$. In our calculations, $p_0$ is found to be 0.78. We then have

$$\xi_H^* = \begin{cases} \underline{\xi}^m & \text{when } p < p_0 \text{ (least-cost separating equilibrium)} \\ \xi_H^p & \text{when } p > p_0 \text{ (pooling equilibrium)} \end{cases}. \tag{4.1}$$

We observe $\xi_H^* \leq \xi_H^c$, where $\xi_H^c = \xi_H^p$ at the value $p = 1$.

In a similar manner, we show the plot of the optimal takeover deal $\xi_L^*$ adopted by the acquirer firm of low type against $p$ in Figure 5b. By Proposition 2, when $p < p_0 = 0.78$, least-cost separating equilibrium prevails. Hence, we have $\xi_L^* = \xi_L^c$, where $\xi_L^c = \xi_L^p$ at the value $p = 0$ (complete information). When $p > p_0 = 0.78$, pooling equilibrium prevails, so $\xi_L^* = \xi_L^p$.

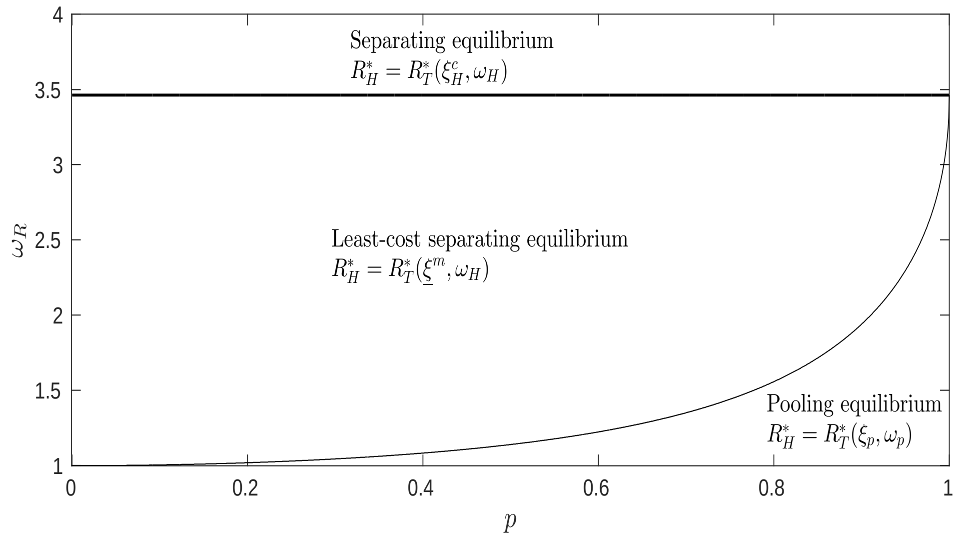


Figure 4: Characterization of the three types of strategic takeover equilibriums in the $\omega_R$-$p$ plane. The acquirer firm of high type can reveal its type to the target firm at zero signaling cost when the synergy ratio $\omega_R = \omega_H/\omega_L$ is sufficiently high. In this case of separating equilibrium, we have $R_H^* = R_H^c$. When $\omega_R$ falls below certain threshold $\omega_R^*$, pooling equilibrium prevails when the probability $p$ of being the high type is sufficiently large. The optimal threshold $R_p^*$ is equal to $R_T^*(\xi_p, \omega_p)$ [see eq. (3.12)]. At low values of $p$, the acquirer firm of high type adopts the least-cost separating equilibrium strategy $M_H^s = (\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$.

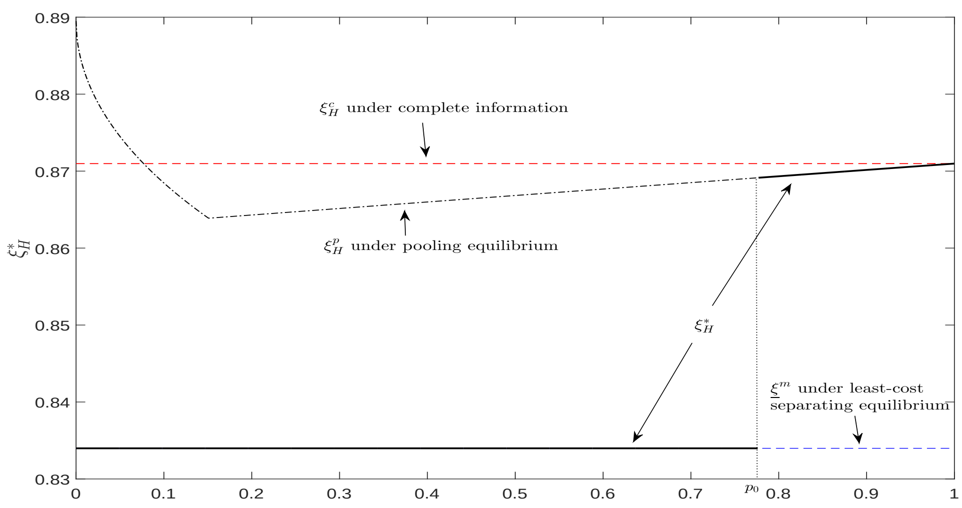


Figure 5a: Plot of the optimal takeover deal $\xi_H^*$ against $p$ for the acquirer firm of high type. We observe (i) when $p < p_0$, $\xi_H^* = \underline{\xi}^m$ under least-cost separating equilibrium; (ii) when $p > p_0$, $\xi_H^* = \xi_H^p$ under pooling equilibrium. The parameters used are the same as those used in plotting Figure 2. Here, $p_0 = 0.78$.

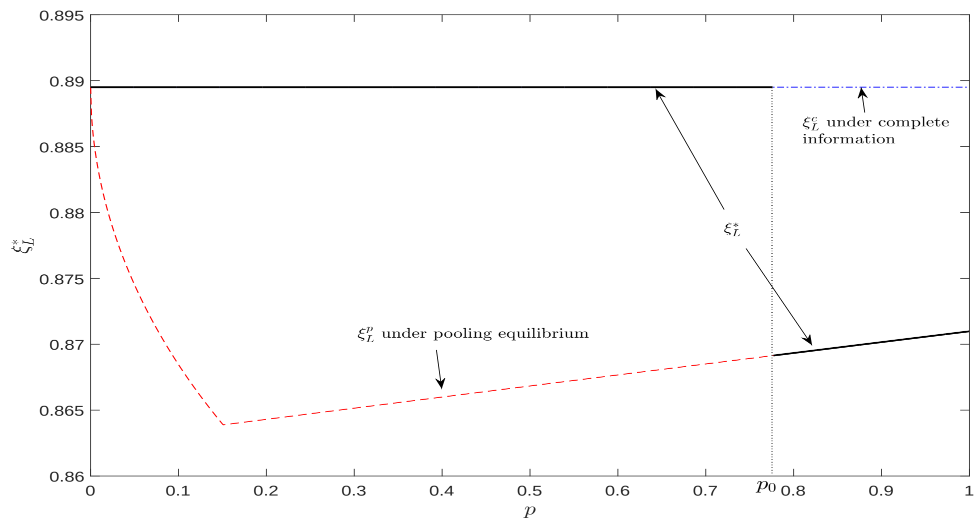


Figure 5b: Plot of the optimal takeover deal $\xi_L^*$ against $p$ for the acquirer firm of low type. We observe (i) when $p < p_0 = 0.78$, $\xi_L^* = \xi_L^c$ under complete information, (ii) when $p > p_0 = 0.78$, $\xi_L^* = \xi_L^p$ under pooling equilibrium. The parameters used are the same as those used in plotting Figure 5a.

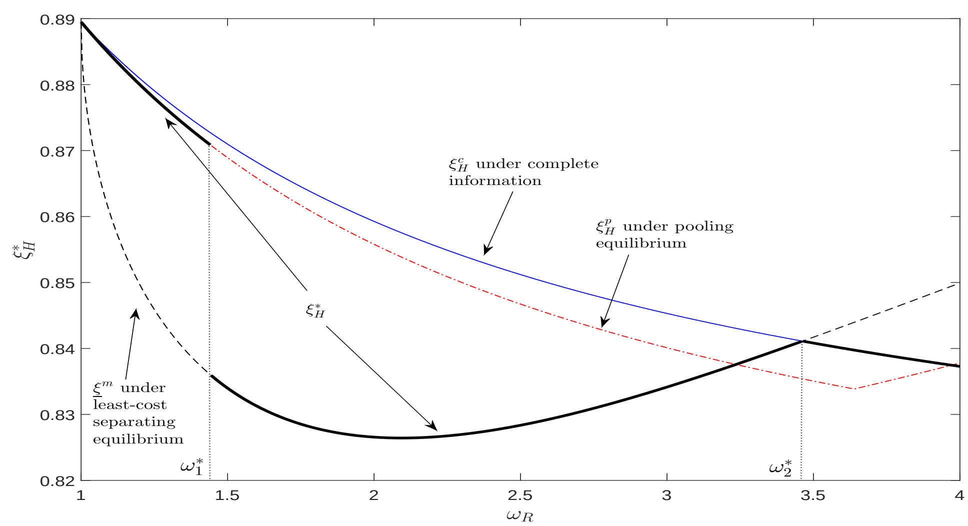


Figure 6a: Plot of the optimal takeover deal $\xi_H^*$ against $\omega_R = \omega_H/\omega_L$ for the acquirer firm of high type. We observe (i) when $\omega_R < \omega_1^*$, we have $\xi_H^* = \xi_H^p$ under pooling equilibrium; (ii) when $\omega_1^* < \omega_R < \omega_2^*$, we have $\xi_H^* = \underline{\xi}^m$ under least-cost separating equilibrium; (iii) when $\omega_R > \omega_2^*$, we have $\xi_H^* = \xi_H^c$ under complete information. The parameters used are the same as those in plotting Figure 2 except that $p$ is taken to be 0.75. Here, $\omega_1^* = 1.44$ and $\omega_2^* = 3.46$.

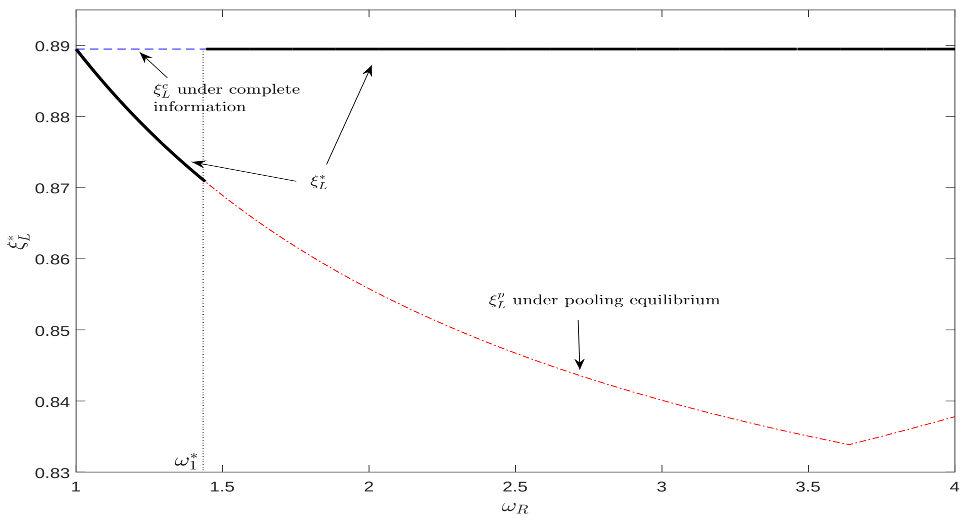


Figure 6b: Plot of the optimal takeover deal $\xi_L^*$ against $\omega_R$ for the acquirer firm of low type. We observe (i) when $\omega_R < \omega_1^*$, we have $\xi_L^* = \xi_L^p$ under pooling equilibrium; (ii) when $\omega_R > \omega_1^*$, we have $\xi_L^* = \xi_L^c$ under complete information. The parameters used are the same as those in plotting Figure 6a. Here, $\omega_1^* = 1.44$.

In Figure 6a, we show the plot of the optimal takeover deal $\xi_H^*$ against the synergy ratio $\omega_R = \omega_H/\omega_L$, with $p$ taken to be 0.75. We deduce from Figure 4 that when $\omega_R$ is sufficiently high, separating equilibrium without signaling cost prevails. Accordingly, we have $\xi_H^* = \xi_H^c$ (complete information). In this calculation example, the corresponding threshold for $\omega_R$ is found to be $\omega_2^* = 3.46$. On the other hand, when $\omega_R < \omega_1^* = 1.44$, pooling equilibrium prevails so that $\xi_H^* = \xi_H^p$. Lastly, least-cost separating equilibrium prevails when $\omega_1^* < \omega_R < \omega_2^*$ so that $\xi_H^* = \underline{\xi}^m$. As a summary, we have

$$\xi_H^* = \begin{cases} \xi_H^p & \text{when } \omega_R < \omega_1^* \text{ (pooling equilibrium)} \\ \underline{\xi}^m & \text{when } \omega_1^* < \omega_R < \omega_2^* \text{ (least-cost separating equilibrium)} \\ \xi_H^c & \text{when } \omega_R > \omega_2^* \text{ (complete information)} \end{cases}. \tag{4.2}$$

For the transition from pooling equilibrium to least-cost separating equilibrium, one observes a downwards jump in $\xi_H^*$.

In a similar manner, we show the plot of $\xi_L^*$ against $\omega_R$ in Figure 6b. The acquirer firm of low type can adopt two equilibrium takeover strategies: $\xi_L^* = \xi_L^p$ under pooling equilibrium when $\omega_R < \omega_1^*$ and $\xi_L^* = \xi_L^c$ under complete information when $\omega_R > \omega_1^*$. There is an upward jump in $\xi_L^*$ when pooling equilibrium transits to separating equilibrium (complete information).

We observe from the figures that the informationally disadvantaged target firm can always acquire a higher proportion (or at least the same level as that under complete information) of the merged firm under the existence of information asymmetry, regardless of the true type of the acquirer firm and the type of takeover strategy (separating or pooling) chosen by the acquirer firm. To explain this apparently counter-intuitive result, we argue that the target

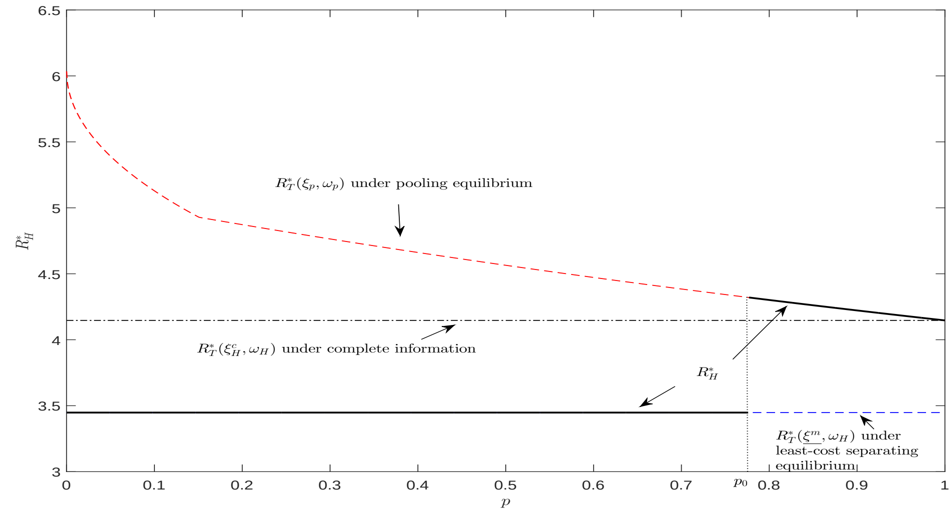


Figure 7a: Plot of the optimal threshold $R_H^*$ against $p$ for the acquirer firm of high type. We observe (i) when $p < p_0$, we have $R_H^* = R_T^*(\underline{\xi}^m, \omega_H)$ under least-cost separating equilibrium; (ii) when $p > p_0$, we have $R_H^* = R_T^*(\xi_p, \omega_p)$ under pooling equilibrium. The parameters used are the same as those used in plotting Figure 5a. Here, $p_0 = 0.78$.

firm can extract the signaling cost of the high type acquirer firm and receives better takeover deals.

**Optimal threshold values at takeover**

In Figure 7a, we show the plot of the optimal takeover threshold $R_H^*$ of the acquirer firm of high type against $p$. Similar to Figure 5a, pooling equilibrium prevails when $p > p_0$ and $R_H^* = R_T^*(\xi_p, \omega_p)$; least-cost separating equilibrium prevails when $p < p_0$ and $R_H^* = R_T^*(\underline{\xi}^m, \omega_H)$ (independent of $p$). At high value of $p$, the loss of surplus value is less significant due to information asymmetry, so pooling equilibrium prevails. The acquirer firm of high type chooses to delay takeover at higher optimal threshold of $R_H^*$ instead of signaling the synergy type by choosing lower optimal threshold at $R_T^*(\underline{\xi}^m, \omega_H)$ (least-cost separating strategy).

The properties of $R_L^*$ against $p$ are revealed in Figure 7b. For the low cost acquirer firm, it chooses the optimal threshold $R_L^* = R_T^*(\xi_L^c, \omega_L)$ under complete information at $p < p_0$ since least-cost separating equilibrium prevails. At high value of $p$, the low cost acquirer firm would take advantage of pooling equilibrium to enhance surplus value by choosing lower value of the optimal threshold $R_L^* = R_T^*(\xi_p, \omega_p)$.

In Figure 8a, we show the plot of the optimal takeover threshold $R_H^*$ of the acquirer firm of high type against $\omega_R$. Similar to Figure 6a, we have

$$R_H^* = \begin{cases} R_T^*(\xi_p, \omega_p) & \text{when } \omega_R < \omega_1^* \text{ (pooling equilibrium)} \\ R_T^*(\underline{\xi}^m, \omega_H) & \text{when } \omega_1^* < \omega_R < \omega_2^* \text{ (least-cost separating equilibrium)} \\ R_T^*(\xi_H^c, \omega_H) & \text{when } \omega_R > \omega_2^* \text{ (complete information)} \end{cases}. \quad (4.3)$$

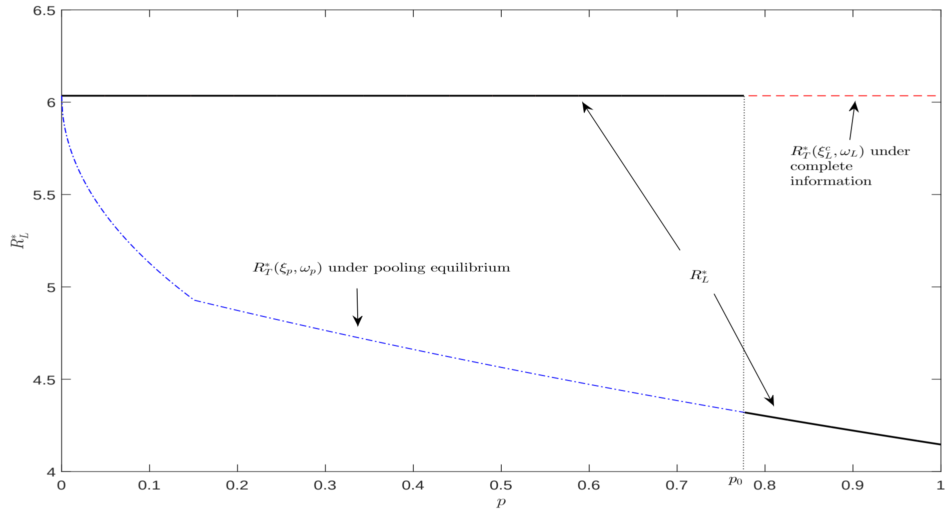


Figure 7b: Plot of the optimal threshold $R_L^*$ against $p$ for the acquirer firm of low type. We observe (i) when $p < p_0$, we have $R_L^* = R_T^*(\xi_L^c, \omega_L)$ under complete information; (ii) when $p > p_0$, we have $R_L^* = R_T^*(\xi_p, \omega_p)$ under pooling equilibrium. The parameters used are the same as those used in plotting Figure 5a.

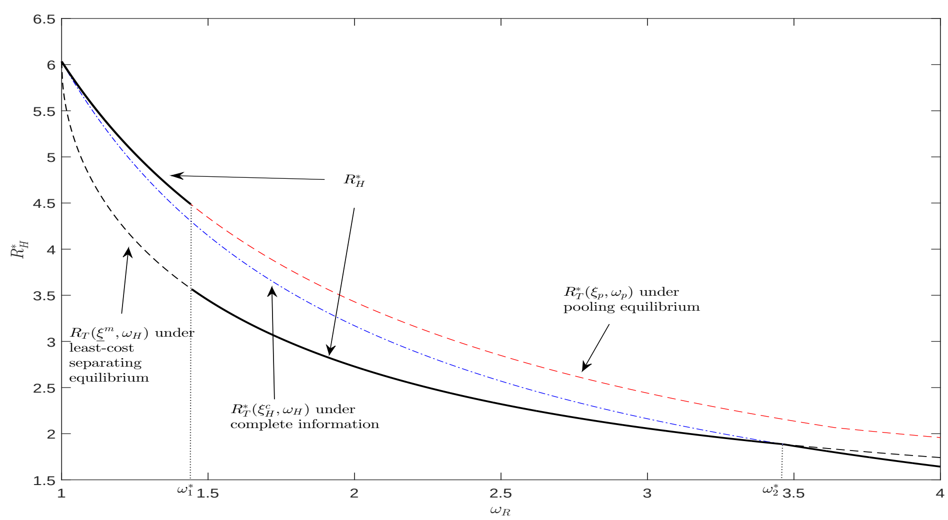


Figure 8a: Plot of the optimal threshold $R_H^*$ against $\omega_R = \omega_H/\omega_L$ for the acquirer firm of high type. We observe (i) when $\omega_R < \omega_1^*$, we have $R_H^* = R_T^*(\xi_p, \omega_p)$ under pooling equilibrium; (ii) when $\omega_1^* < \omega_R < \omega_2^*$, we have $R_H^* = R_T^*(\underline{\xi}^m, \omega_H)$ under least-cost separating equilibrium; (iii) when $\omega_R > \omega_2^*$, we have $R_H^* = R_T^*(\xi_H^c, \omega_H)$ under complete information. The parameters used are same as those used in plotting Figure 6a. Here, $\omega_1^* = 1.44$ and $\omega_2^* = 3.46$.

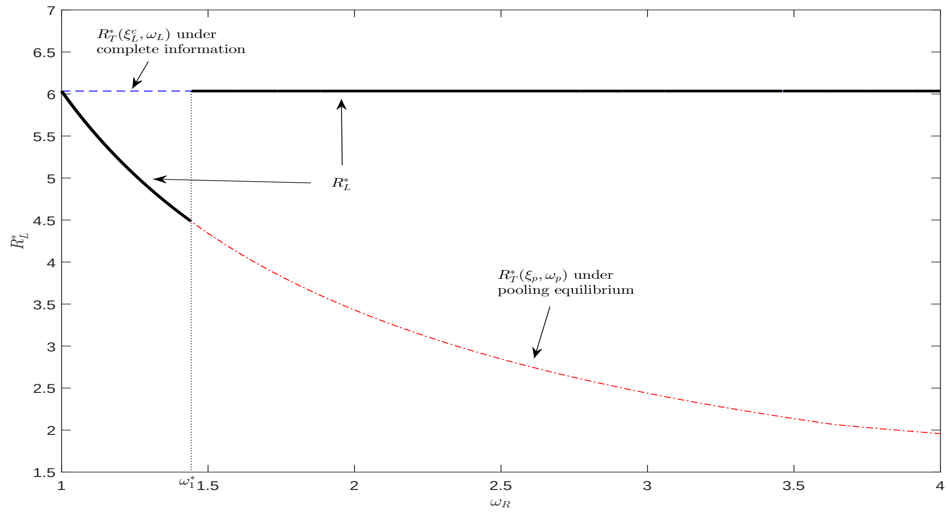


Figure 8b: Plot of the optimal takeover deal $R_L^*$ against $\omega_R$ for the acquirer firm of low type. We observe (i) when $\omega_R < \omega_1^*$, we have $R_L^* = R_T^*(\xi_p, \omega_p)$ under pooling equilibrium; (ii) when $\omega_R > \omega_1^*$, we have $R_L^* = R_T^*(\xi_L^c, \omega_L)$ under complete information. The parameters used are the same as those in plotting Figure 6a. Here, $\omega_1^* = 1.44$.

At low value of synergy ratio $\omega_R$, pooling equilibrium prevails since the loss of surplus value of the acquirer firm of high type due to information asymmetry is less significant. The acquirer firm of high type would choose high value of optimal takeover threshold. When the synergy ratio increases, the high type acquirer firm chooses low value of optimal takeover threshold to signal its type under least-cost separating equilibrium. With further increase in $\omega_R$, where $\omega_R > \omega_2^*$, separating equilibrium prevails so that $R_H^* = R_T^*(\xi_H^c, \omega_H)$ under complete information.

Lastly, we show the properties of $R_L^*$ against $\omega_R$ in Figure 8b. For the low cost acquirer firm, when $\omega_R < \omega_1^*$, it chooses lower value of the optimal threshold $R_L^* = R_T^*(\xi_p, \omega_p)$ to take advantage of pooling equilibrium. On the other hand, when $\omega_R > \omega_1^*$, separating equilibrium prevails so the low cost acquirer firm chooses the optimal threshold $R_L^* = R_T^*(\xi_L^c, \omega_L)$ under complete information.

In summary, we observe that information asymmetry speeds up the takeover process when either the synergy ratio $\omega_R$ is sufficiently large ($\omega_R \geq \omega_1^*$) or the probability $p$ is sufficiently small so that the acquirer firm of high type adopts separating strategy. On the other hand, if $\omega_R$ is sufficiently small or $p$ is sufficiently large, the acquirer firm of high type chooses to slow down the takeover process while the acquirer firm of low type chooses to speed up the takeover process under pooling equilibrium.

# 5 Conclusion

This paper analyzes a dynamic acquisition game model that is based on the market valuation of the surplus values of the acquirer and target firms. In our real signaling game model of acquisition, the two firms are assumed to have information asymmetry on the synergy factor of the acquirer firm. We discuss the characterization of the perfect Bayesian equilibrium of the takeover strategies on the timing and terms in the acquisition. We discuss the strategic spaces of the optimal takeover deals chosen by the acquirer firm under separating and pooling equilibriums. We derive the conditions on the choices of least-cost separating strategy and pooling strategy by the acquirer firm under varying model parameters. We show how the takeover deals and optimal takeover threshold would depend on the probability of the acquirer firm being the high type and the ratio of the synergy factor of high type to that of low type. Our theoretical analyzes agree with the economic intuition that pooling equilibrium prevails when the loss in surplus value of the high type acquirer firm is less than the signaling cost of separating. This occurs when the ratio of the synergy factor is below certain threshold and the probability of being high type is sufficiently high.

# 6 References


Borek, T., Buehler, S., Schmutzler, A., 2008. Mergers under asymmetric information: A simple reduced-form approach. *Working Paper of University of St. Gallen.*

Chae, J., Chung, J.Y., Yang, C.W., 2014. Does information asymmetry affect merger and acquisitions in an emerging market?. *Working paper of Seoul National University.*

Cheng, P., Li, J., Tong, W.H.S., 2008. Information asymmetry in the takeover market. *Working paper of Hong Kong Polytechnic University.*

Dixit, A., Pindyck, R., 1994. Investment under uncertainty. Princeton University Press, Princeton, New Jersey.

Draper, P., Paudyal, K., 2008. Information asymmetry and bidders' gains. *Journal of Business Finance and Accounting*, vol. 35 (3)&(4), 376-405.

Lambrecht, B.M., 2004. The timing and terms of mergers motivated by economics of scale. *Journal of Financial Economics*, vol. 72, 41-62.

Margrabe, W., 1978. The value of the option to exchange one asset for another. *Journal of Finance*, vol. 33, 177-186.

Morellec, E., Zhdanov, A., 2005. The dynamics of mergers and acquisitions. *Journal of Financial Economics*, vol. 77, 649-672.

Morellec, E., Zhdanov, A., 2008. Financing and takeovers. *Journal of Financial Economics*, vol. 87, 556-581.

Morellec, E., Schürhoff, N., 2011. Corporate investment and financing under asymmetric information. *Journal of Financial Economics*, vol. 99, 262-288.

Officer, M.S., Poulsen, A.B. and Stegemoller, M., 2009. Target-firm information asymmetry and acquirer returns. *Review of Finance* vol. 13, 467-493.

Shleifer, A., Vishny, R., 2003. Stock market driven acquisitions. *Journal of Financial Economics*, vol. 70, 295-311.

Stepanov, S., 2014. Takeovers under asymmetric information: Block Trades and Tender Offers in Equilibrium. *Working Paper of International College of Economics and Finance, Moscow*.

Thijssen, J.J., 2008. Optimal and strategic timing of mergers and acquisitions motivated by synergies and risk diversification. *Journal of Economic Dynamics and Control*, vol. 32, 1701-1720.

**Appendix A - Proof of Lemma 2**

Recall from eq. (2.4a) that $F_A(R;\omega,\xi)$ can be expressed as a bilinear function in $R$ and $\xi$, where

$$F_A(R;\omega,\xi) = \xi(A_\omega R - B) - K_A R,$$

with $A_\omega = K_A + \alpha(K_A + K_T)\omega > 0$ and $B = \alpha(K_A + K_T) - K_T > 0$. Note that for $\xi \in (\underline{\xi}^m, \bar{\xi}^m)$, the optimal takeover threshold $R_u^*(\xi)$ lies on the indifference curve and satisfies

$$F_A(R_u^*(\xi);\omega_L,\xi)\left[\frac{R}{R_u^*(\xi)}\right]^\beta = F_A(R_L^c;\omega_L,\xi_L^c)\left(\frac{R}{R_L^c}\right)^\beta.$$

Suppose we write $F_A(R_u^*(\xi);\omega_L,\xi) = \xi[A_{\omega_L}R_u^*(\xi) - B] - K_A R_u^*(\xi)$, the above equation can be rewritten as

$$\xi = \frac{F_A(R_L^c;\omega_L,\xi_L^c)\left[\frac{R_u^*(\xi)}{R_L^c}\right]^\beta + K_A R_u^*(\xi)}{A_{\omega_L}R_u^*(\xi) - B}.$$

By combining the above relations and observing the following properties: (i) $\dfrac{A_{\omega_H}R - B}{A_{\omega_L}R - B}$ is a decreasing function with respect to $R$, (ii) $R_u^*(\xi) > R_T^*(\xi,\omega_H) > R_T^*(\underline{\xi}^m,\omega_H)$ for $\xi \in (\underline{\xi}^m, \bar{\xi}^m)$ , we obtain

$$\begin{aligned}
&F_A(R_u^*(\xi);\omega_H,\xi)\left[\frac{R}{R_u^*(\xi)}\right]^\beta \\
=&\frac{A_{\omega_H}R_u^*(\xi) - B}{A_{\omega_L}R_u^*(\xi) - B}\left\{F_A(R_L^c;\omega_L,\xi_L^c)\left[\frac{R_u^*(\xi)}{R_L^c}\right]^\beta + K_A R_u^*(\xi)\right\}\left[\frac{R}{R_u^*(\xi)}\right]^\beta - K_A R_u^*(\xi)\left[\frac{R}{R_u^*(\xi)}\right]^\beta \\
<&\frac{A_{\omega_H}R_T^*(\underline{\xi}^m,\omega_H) - B}{A_{\omega_L}R_T^*(\underline{\xi}^m,\omega_H) - B}\left\{F_A(R_L^c;\omega_L,\xi_L^c)\left[\frac{R_u^*(\xi)}{R_L^c}\right]^\beta + K_A R_u^*(\xi)\right\}\left[\frac{R}{R_u^*(\xi)}\right]^\beta \\
&- K_A R_u^*(\xi)\left[\frac{R}{R_u^*(\xi)}\right]^\beta.
\end{aligned}$$

Furthermore, by noting $\beta > 1$ and $R_u^*(\xi) > R_T^*(\underline{\xi}^m,\omega_H)$, we deduce that

$$\begin{aligned}
&F_A(R_u^*(\xi);\omega_H,\xi)\left[\frac{R}{R_u^*(\xi)}\right]^\beta \\
<&\frac{A_{\omega_H}R_T^*(\underline{\xi}^m,\omega_H) - B}{A_{\omega_L}R_T^*(\underline{\xi}^m,\omega_H) - B}\left[F_A(R_L^c;\omega_L,\xi_L^c)\left(\frac{R}{R_L^c}\right)^\beta\right] \\
&+ \left[\frac{A_{\omega_H}R_T^*(\underline{\xi}^m,\omega_H) - B}{A_{\omega_L}R_T^*(\underline{\xi}^m,\omega_H) - B} - 1\right]K_A R_u^*(\xi)\left[\frac{R}{R_u^*(\xi)}\right]^\beta \\
<&\frac{A_{\omega_H}R_T^*(\underline{\xi}^m,\omega_H) - B}{A_{\omega_L}R_T^*(\underline{\xi}^m,\omega_H) - B}\left[F_A(R_L^c;\omega_L,\xi_L^c)\left(\frac{R}{R_L^c}\right)^\beta\right] \\
&+ \left[\frac{A_{\omega_H}R_T^*(\underline{\xi}^m,\omega_H) - B}{A_{\omega_L}R_T^*(\underline{\xi}^m,\omega_H) - B} - 1\right]K_A R_T^*(\underline{\xi}^m,\omega_H)\left[\frac{R}{R_T^*(\underline{\xi}^m,\omega_H)}\right]^\beta
\end{aligned}$$

$$= \frac{A_{\omega_H} R_T^*(\underline{\xi}^m, \omega_H) - B}{A_{\omega_L} R_T^*(\underline{\xi}^m, \omega_H) - B} \left[ F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_L, \underline{\xi}^m) \left( \frac{R}{R_T^*(\underline{\xi}^m, \omega_H)} \right)^\beta \right]$$
$$+ \left[ \frac{A_{\omega_H} R_T^*(\underline{\xi}^m, \omega_H) - B}{A_{\omega_L} R_T^*(\underline{\xi}^m, \omega_H) - B} - 1 \right] K_A R_T^*(\underline{\xi}^m, \omega_H) \left[ \frac{R}{R_T^*(\underline{\xi}^m, \omega_H)} \right]^\beta.$$

The last equality is established by observing that $(\underline{\xi}, R_T^*(\underline{\xi}^m, \omega_H))$ lies in the intersection between the indifference curve and the curve of $R_T^*(\xi, \omega_H)$.

Lastly, by expressing $F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H :, \underline{\xi}^m)$ as a bilinear form of $\xi$ and $R$, we finally obtain

$$F_A(R_u^*(\xi); \omega_H, \xi) \left[ \frac{R}{R_u^*(\xi)} \right]^\beta$$
$$< \left\{ \underline{\xi}^m \left[ A_{\omega_H} R_T^*(\underline{\xi}^m, \omega_H) - B \right] - K_A R_T^*(\underline{\xi}^m, \omega_H) \right\} \left[ \frac{R}{R_T^*(\underline{\xi}^m, \omega_H)} \right]^\beta$$
$$= F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[ \frac{R}{R_T(\underline{\xi}^m, \omega_H)} \right]^\beta.$$

Hence, we establish ineq. (3.10) and so the takeover package $(\xi, R_u^*(\xi))$, where $\xi \in (\underline{\xi}^m, \bar{\xi}^m)$, is dominated by the takeover package $(\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$.

## Appendix B - Proof of Proposition 2

To establish the proof, it is necessary to use the following equivalent condition under which ineq. (3.16) holds. Suppose that $\xi_H^c \in (\underline{\xi}^m, \bar{\xi}^m)$, ineq. (3.16) holds if and only if

$$\max_{\xi \in [\max(\underline{\xi}^m, \underline{\xi}^p), \bar{\xi}^p]} F_A(R_T^*(\xi, \omega_p); \omega_H, \xi) \left[ \frac{R}{R_T^*(\xi, \omega_p)} \right]^\beta > F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[ \frac{R}{R_T^*(\underline{\xi}^m, \omega_H)} \right]^\beta. \tag{B.1}$$

The proof of ineq. (B.1) is presented at the end of this Appendix.

We write the left hand side of ineq. (B.1) as a function of $p$, where

$$f(p) = \max_{\xi \in [\max(\underline{\xi}^m, \underline{\xi}^p), \bar{\xi}^p]} F_A(R_T^*(\xi, \omega_p); \omega_H, \xi) \left[ \frac{R}{R_T^*(\xi, \omega_p)} \right]^\beta.$$

The right hand side of ineq. (B.1) is independent of $p$, which is written as

$$F_A^s = F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[ \frac{R}{R_T^*(\underline{\xi}^m, \omega_H)} \right]^\beta.$$

In order to show that ineq. (B.1) holds when $p$ is sufficiently large, it suffices to show that

(a) $f(p)$ is strictly increasing with respect to $p$;

(b) $f(0) < F_A^s$;

(c) $f(1) > F_A^s$.

To show property (a), we use the following technical results: (i) The interval $[\max(\underline{\xi}^m, \underline{\xi}^p), \bar{\xi}^p]$ becomes widened with an increasing value of $p$; (ii) For $0 < p_2 < p_1 \leq 1$ and $\xi > \xi^\ell$, by virtue of Lemma 1, we have $R_T^*(\xi, \omega_{p_2}) > R_T^*(\xi, \omega_{p_1}) \geq R_T^*(\xi, \omega_H) > R_A^*(\xi, \omega_H)$. Using these properties, we deduce that for $p_1 > p_2$,

$$\begin{aligned} f(p_1) &= \max_{\xi \in [\max(\underline{\xi}^m, \underline{\xi}^{p_1}), \bar{\xi}^{p_1}]} F_A(R_T^*(\xi, \omega_{p_1}); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_{p_1})}\right]^\beta \\ &\geq \max_{\xi \in [\max(\underline{\xi}^m, \underline{\xi}^{p_2}), \bar{\xi}^{p_2}]} F_A(R_T^*(\xi, \omega_{p_1}); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_{p_1})}\right]^\beta \\ &> \max_{\xi \in [\max(\underline{\xi}^m, \underline{\xi}^{p_2}), \bar{\xi}^{p_2}]} F_A(R_T^*(\xi, \omega_{p_2}); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_{p_2})}\right]^\beta > f(p_2). \end{aligned}$$

To show property (b), we observe that when $p = 0$, the curve $R^* = R_T^*(\xi, \omega_p) = R_T^*(\xi, \omega_L)$ touches the indifference curve at $(\xi_L^c, R_L^c)$. Since $\xi_L^c \in (\underline{\xi}^m, \bar{\xi}^m)$, we deduce that

$$\begin{aligned} f(0) = F_A(R_L^c; \omega_H, \xi_L^c) \left(\frac{R}{R_L^c}\right)^\beta &= F_A(R_u^*(\xi_L^c); \omega_H, \xi_L^c) \left[\frac{R}{R_u^*(\xi_L^c)}\right]^\beta \\ &< F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[\frac{R}{R_T^*(\underline{\xi}^m, \omega_H)}\right]^\beta, \end{aligned}$$

by virtue of ineq. (3.10). To show property (c), we observe that when $p = 1$, the curve $R^* = R_T^*(\xi, \omega_p) = R_T^*(\xi, \omega_H)$ touches the indifference curve at $(\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ and $(\bar{\xi}^m, R_T^*(\bar{\xi}^m, \omega_H))$. Since $\xi_H^c \in (\underline{\xi}^m, \bar{\xi}^m)$, we deduce that

$$\begin{aligned} f(1) &= \max_{\xi \in [\underline{\xi}^m, \bar{\xi}^m]} F_A(R_T^*(\xi, \omega_H); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_H)}\right]^\beta \\ &= F_A(R_T^*(\xi_H^c, \omega_H); \omega_H, \xi_H^c) \left[\frac{R}{R_T^*(\xi_H^c, \omega_H)}\right]^\beta > F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[\frac{R}{R_T^*(\underline{\xi}^m, \omega_H)}\right]^\beta. \end{aligned}$$

Once properties (a), (b) and (c) have been established, we can deduce that there exists an unique $p_0 \in (0, 1)$ such that $f(p_0) = F_A^s$ and $f(p) > F_A^s$ if and only if $p > p_0$. Hence, Proposition 2 is established.

*Proof of ineq. (B.1)*

The equivalence property between ineqs. (3.16) and (B.1) is trivial when $\underline{\xi}^m \leq \underline{\xi}^p$. When $\underline{\xi}^m > \underline{\xi}^p$, by virtue of Lemma 1, we have $R_T^*(\xi, \omega_p) \geq R_T^*(\xi, \omega_H) > R_A^*(\xi, \omega_H)$ for any $\xi \in [\underline{\xi}^p, \underline{\xi}^m]$. Using this fact, for any $\xi \in [\underline{\xi}^p, \underline{\xi}^m]$, we have

$$\begin{aligned} F_A(R_T^*(\xi, \omega_p); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_p)}\right]^\beta &< F_A(R_T^*(\xi, \omega_H); \omega_H, \xi) \left[\frac{R}{R_T^*(\xi, \omega_H)}\right]^\beta \\ &< F_A(R_T^*(\underline{\xi}^m, \omega_H); \omega_H, \underline{\xi}^m) \left[\frac{R}{R_T^*(\underline{\xi}^m, \omega_H)}\right]^\beta. \qquad \text{(B.2)} \end{aligned}$$

The last inequality is established by observing that $M_H^s = (\underline{\xi}^m, R_T^*(\underline{\xi}^m, \omega_H))$ is the least-cost separating equilibrium. Based on ineq. (B.2), we deduce that ineq. (B.1) holds if ineq. (3.16) holds. Conversely, it is quite trivial to show that if ineq. (B.1) holds, then ineq. (3.16) holds.