Applied Complex Variables for Scientists and Engineers

Second Edition

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521701389

First published 2010

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Kwok, Y. K. (Yue-Kuen), 1957–
Applied complex variables for scientists and engineers / Yue Kuen Kwok. – 2nd ed.
p. cm.
Includes index.
ISBN 978-0-521-70138-9 (pbk.)
1. Functions of complex variables – Textbooks. I. Title.

QA331.7.K88 2010
515′.9–dc22 2010008597

ISBN 978-0-521-70138-9 Paperback

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Preface

This textbook is intended to be an introduction to complex variables for mathematics, science and engineering undergraduate students. The prerequisites are some knowledge of calculus (up to line integrals and Green's Theorem), though basic familiarity with differential equations would also be useful.

Complex function theory is an elegant mathematical structure on its own. On the other hand, many of its theoretical results provide powerful and versatile tools for solving problems in physical sciences and other branches of mathematics. The book presents the important analytical concepts and techniques in deriving most of the standard theoretical results in introductory complex function theory. I have included the proofs of most of the important theorems, except for a few that are highly technical. This book distinguishes itself from other texts in complex variables by emphasizing how to use complex variable methods. Throughout the text, many of the important theoretical results in complex function theory are followed by relevant and vivid examples in physical sciences. These examples serve to illustrate the uses and implications of complex function theory. They are drawn from a wide range of physical and engineering applications, like potential theory, steady state temperature problems, hydrodynamics, seepage flows, electrostatics and gravitation. For example, after discussing the mathematical foundations of the Laplace transform and Fourier transform, I show how to use the transform methods to solve initial-boundary problems arising from heat conduction and wave propagation problems. The materials covered in the book equip students with the analytical concepts of complex function theory together with the technical skills to apply complex variable methods to physical problems.

Throughout the whole textbook, both algebraic and geometric tools are employed to provide the greatest understanding, with many diagrams illustrating the concepts introduced. The book contains some 340 stimulating exercises, with solutions given to most of them. They are intended to aid students to grasp the concepts covered in the text and foster the skills in applying complex variable techniques to solve physical problems. Students are strongly advised to work through as many exercises as possible since mathematical knowledge can only be gained through active participation in the thinking and learning process.

The book begins by carefully exploring the algebraic, geometric and topological structures of the complex number field. In order to visualize the complex infinity, the Riemann sphere and the corresponding stereographic projection are introduced. Applications of complex numbers in electrical circuits are included.

Analytic functions are introduced in Chapter 2. The highlights of the chapter are the Cauchy–Riemann relations and harmonicity. The uses of complex functions in describing fluid flows and steady state heat distributions are illustrated.

In Chapter 3, the complex exponential function is introduced as an entire function which is equal to its derivative. The description of steady state temperature distributions by complex logarithm functions is illustrated. The mapping properties of complex trigonometric functions are examined. The notion of Riemann surfaces is introduced to help visualize multi-valued complex functions.

Complex integration forms the cornerstone of complex variable theory. The key results in Chapter 4 are the Cauchy–Goursat theorem and the Cauchy integral formulas. Other interesting results include Gauss' mean value theorem, Liouville's theorem and the maximum modulus theorem. The link of analytic functions and complex integration with the study of conservative fields is considered. Complex variable methods are seen to be effective analytical tools to solve conservation field models in potential flows, gravitational potentials and electrostatics.

Complex power series are the main themes in Chapter 5. We introduce different types of convergence of series of complex functions. The various tests that examine the convergence of complex series are discussed. The Taylor series theorem and Laurent series theorem show that a convergent power series is an analytic function within its disk or annulus of convergence, respectively. The notion of analytic continuation of a complex function is discussed. As an application, the solution to the potential flow over a perturbed circle is obtained as a power series in a perturbation parameter.

In Chapter 6, we start with the discussion of the classification of isolated singularities by examining the Laurent series expansion in a deleted neighborhood of the singularity. We then examine the theory of residues and illustrate the applications of the calculus of residues in the evaluation of complex integrals. The concept of the Cauchy principal value of an improper integral is introduced. Fourier transforms and Fourier integrals are considered. The residue calculus method is applied to compute the hydrodynamic lift and moment of an immersed obstacle.

The solutions of boundary value problems and initial-boundary value problems are considered in Chapter 7. The Poisson integral formula and the Schwarz integral formula for Dirichlet problems are derived. The inversion of the Laplace transform via the Bromwich contour integral is discussed. The Laplace transform techniques are applied to obtain the solutions of initial-boundary value problems arising from heat conduction and wave propagation models.

In the last chapter, we explore the rich geometric structure of complex variable theory. The geometric properties associated with mappings represented by complex functions are examined. The link between analyticity and conformality is derived. Various types of transformations that perform the mappings of regions are introduced. The bilinear and Schwarz–Christoffel transformations are discussed in full context. A wide range of physical examples are included to illustrate how to use these transformations to transform conservative field problems with complicated configurations into those with simple geometries. We also show how to use the hodograph transformations to solve seepage flow problems.

I would like to thank Ms Odissa Wong for her careful typing and editing of the manuscript, and her patience in entertaining the seemingly endless changes in the process. Also, I would like to thank the staff of Cambridge University Press for their editorial assistance in the production of this book. Last but not least, special thanks go to my wife Oi Chun and our two daughters, Grace and Joyce, for their forbearance while this book was written. Their love and care have been my main source of support in everyday life and work.