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VALUATION OF EMPLOYEE RELOAD OPTIONS USING UTILITY MAXIMIZATION APPROACH

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The reload provision in an employee stock option is an option enhancement that allows the employee to pay the strike upon exercising the stock option using his owned stocks and to receive new "reload" stock options. The usual Black-Scholes risk neutral valuation approach may not be appropriate to be adopted as the pricing vehicle for employee stock options, due to the non-transferability of the ownership of the options and the restriction on short selling of the firm's stocks as hedging strategy. In this paper, we present a general utility maximization framework to price non-tradeable employee stock options with reload provision. The risk aversion of the employee enters into the pricing model through the choice of the utility function. We examine how the value of the reload option to the employee is affected by the number of reloads outstanding, the risk aversion level and personal wealth. In particular, we explore how the reload provision may lower the difference between the cost of granting the option and the private option value and improve the compensation incentive of the option award.

Keywords: Employee stock options; utility maximization; reload provision; compensation incentives; dead-weight loss.

1. Introduction

Employee stock options have emerged as the most important compensation vehicle used by companies to retain and motivate their employees. The increased popularity may have been attributed to the perceived alignment of employees' interests with those of the shareholders, favorable accounting and tax treatments, and others. In order to provide better compensation incentives for the employees, employee stock

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options contain specific provisions that are not commonly found in conventional exchange-traded or over-the-counter options. For instance, these options usually carry a longer term and they can only be exercised after a vesting period (the vesting may last for a duration of 2 to 5 years). During the vesting period, the option is forfeited if the employee leaves the firm. The stock options may be performance vested, that is, they are exercisable only if the stock price breaches a prescribed level. The employees are not allowed to sell their employee stock options and not to short sell the firm's stocks.

The reload provision in an employee stock option is an option enhancement. In a typical reload design, the exercise of a reload option automatically triggers the awarding of new options. Assuming that the strike price of the option is paid using the firm's stocks, the employee receives fresh at-the-money options equal in number to the number of shares tendered in the exercise process. The expiration date may remain the same as that of the original option or extend beyond the original expiration date. Option grants may specify the maximum number of times the option may be "reloaded" before it reverts to being the standard American call option. The reload feature enables employees to lock in the gain on the part that represents the in-the-moneyness of the stock option, while continues to retain the potential upside growth of the stock price through the holding of the "reloaded" options. More details of the reload provision can be found in a paper of Frederic W. Cook and Company [8].

Johnson and Tian [13] examine the value and incentive effects of six types of non-traditional executive stock option plans, including performance-vested options, repriceable options, purchased options, reload options, and others. They show that the change of various option specific parameters might have profound impact on incentive strengths. Brenner *et al.* [1] examine the rescission feature in executive stock options, where the option holders are allowed to rescind an exercise decision, returning the shares acquired to the company and obtaining a refund of the exercise price. In these papers, the pricing models follow the Black-Scholes paradigm so that only the market value of "tradeable" employee stock option is obtained. However, the analysis of the incentive effects on employees should look into the private value of the stock option to the employee.

The early exercise behaviors of employee stock options are quite different from those of tradeable American options. Hemmer *et al.* [10] document a positive relation between the variance of returns on the employee stock options and the remaining life of the option at exercise. In simple terms, employees in general exercise option at a lower critical stock price in order to diversify their risk. Carr and Linetsky [2] model the forfeiture or early exercise of an executive stock option as a point process. The intensity of the point process is the sum of two parameters: a constant parameter due to voluntary or involuntary employment termination before option's maturity, and another non-constant parameter due to the executive's desire for liquidity or diversification. The non-constant parameter is assumed to be stock price dependent, and its value depends on the occupation time of the stock price staying in the in-the-money region. For employee reload options, Dybvig and Loewenstein [7] show by dominance argument that there exists a simple exercise policy when the number of reloads is infinite. The holder should exercise whenever a new maximum of the stock price is realized, and such optimal policy is independent of the risk preference of the holder. Under such scenario, one can then argue that the cost to the issuer of the option is equal to the market value of the option. Dai and Kwok [3] demonstrate that the reload feature is closely related to the shout feature in shout call options. Also, they obtain the market value of an infinite-reload option in terms of the price function of a lookback option. On the development of numerical valuation algorithms, Hemmer *et al*. [11] and Saly *et al*. [18] propose the recursive binomial schemes for the numerical valuation of the Black-Scholes value of multi-reload options. Later, Dai and Kwok [4] construct the trinomial scheme that prices reload options under time vesting requirement. They also explore the characterization of the optimal exercise policies of the reload rights.

Tian [19] considers the optimal contracting and incentive effects of non-traded employee stock options using a single-period principal-agent model. The employee is the risk averse agent and the firm acts as the risk neutral or risk averse principal, and there is an information asymmetry regarding the agent's effort. He finds that when the employee is modestly risk averse, restricted stocks are the more preferred form of incentive pay compared to the stock options. The value to the employee of non-traded stock option depends significantly on the employee's risk aversion level, investment opportunities and compensation package. In a similar study by Henderson [12] on the optimal form of compensation using a continuous time utility maximization model, her analysis suggests that companies with low stock price volatilities should compensate employees with stock rather than options. Also, she concludes that optimal incentives decrease with firm-specific risk but may increase or decrease with market risk.

Meulbroek [17] observes that the value of equity-linked compensation to undiversified managers would be much less than the cost of providing the compensation. Such "dead-weight costs" arise since the exposure to firm-specific risk that aligns incentives is costly to the employees. Her analysis reveals a striking gap between firm cost and employee benefit of option awards. Duan and Wei [6] consider the risk incentives of different types of employee stock options. They use the GARCH option valuation framework to show how the option's value depends on the systematic risk. For non-indexed stock options, they find that the executives have the incentive to increase the systematic risk, while the opposite risk incentives are seen in indexed options.

Under the trading and hedging constraints imposed on employee stock options, the preference free pricing paradigm based on the Black-Scholes framework should not be adopted to find the private value of the option to the holder. Lambert *et al*. [16] and Kulatilaka and Marcus [14] are among the pioneers who advocate the use of the expected utility maximization approach to find the holder's value. Detemple and Sundaresan [5] develop an efficient binomial pricing framework to price an employee stock option subject to trading restrictions using the utility maximization approach. The exercise decision will be made according to the standard rules of dynamic programming approach: comparing utility of final wealth given early exercise with the expected utility of final wealth with no early exercise. Hall and Murphy [9] perform a detailed analysis on the pay / performance sensitivity of employee stock options. Using the certainty-equivalent framework, they explain why premium options (strike price is higher than the stock price at grant date) are rarely granted in practice and why companies often reprice the exercise price on underwater options (prevailing stock price falls below the strike price). In a more recent work by Tian [20], the author extends the certainty equivalent valuation framework by allowing the executive to invest his unrestricted wealth freely between the market portfolio and the risk-free asset. He shows that the incentive to increase the stock price would not always increase as more options are granted. Also, stock options may create incentive to increase systematic risk, and these incentive effects are sensitive to the choice of exercise price.

In this paper, we construct a pricing model for valuation of the holder's value and the issuer's cost of the employee reload options using the utility maximization approach. The risk aversion, personal wealth and number of reload rights outstanding all enter into the pricing formulation. The pricing model is then applied to examine the impact of compensation incentive of the reload feature to different types of employees. In our calculations, we measure the compensation incentive by the ratio of percentage increase of the employee's private value of the option to the percentage increase of the stock price. In particular, this paper investigates how the reload provision affects the exercise behaviors of the employees and examines whether it helps narrow the "dead-weight cost". We adopt the stochastic maximization techniques to determine the critical stock price at which the option holder should exercise the reload provision. This is done by applying the dynamic programming procedure of comparing the expected utility value upon exercise and expected utility value of continuation at each node in the trinomial tree that simulates the stochastic movement of the stock price. Once the optimal exercise policy and expected utility value have been determined, the value of the option to the employee is then obtained using the certainty-equivalent approach. The cost to the employer can be computed once the exercise behaviors of the employee are known.

We apply our pricing model to examine the following issues:

- Can the reload provision reduce the "dead-weight" cost?
- How does the cost of granting option award to the employer increase with the number of reloads?
- How do the number of reload rights outstanding, moneyness of the option and the risk aversion level of employee influence the compensation incentive and the cost ratio?
- How the exercise behaviors of the employees are affected by the reload feature?

- Is the reload feature more desirable to retain the higher-level employees or lower-level employees?

This paper is organized as follows. In the next section, we consider the pricing model of employee stock options with reload feature under the utility maximization framework. The details of the design of the numerical algorithm for computing the private option value to the employees will be discussed. Analysis of the employee's exercise behaviors, cost to the employer, compensation incentives and dead-weight losses will be presented in Section 3. We conclude the paper with summary of results and conclusive remarks in the last section.

2. Numerical scheme for valuing reload options under utility maximization framework

Due to non-transferability and hedging constraints on the employee reload options, we employ the utility maximization and certainty equivalent framework to find the private value to the holder of the options. We assume that in the employee's portfolio, he holds n_s units of stock, n_o units of reload option with maturity date T and has non-firm-related wealth c. The stock price is assumed to follow the lognormal diffusion process with actual drift rate μ and volatility σ. The stock pays continuous dividend yield q and the dividends are added to the non-firm-related wealth. We assume that the non-firm-related wealth is invested at the riskless interest rate r. By adopting the Capital Asset Pricing Model, the stock's drift rate μ is given by $r+\beta(r_m-r)$, where β is the firm's systematic risk and r_m is the return on the market portfolio. There is no sale or purchase of additional units of stocks. The number of units of stock held is altered only upon exercising of the reload rights. Let X denote the strike price of the reload option and S^* denote the stock price at the exercise moment. The number of units of stock required to pay for the strike is $n_o\dfrac{X}{S^*}$ and new units of stock received is n_o, so that there is a net increase of $n_o\left(1-\dfrac{X}{S^*}\right)$ shares upon exercising. After each exercise, the number of reload rights outstanding is reduced by one. The new strike price of the fresh "reloaded" option is set equal S^* and the number of units of new reload option is $n_o\dfrac{X}{S^*}$. All fresh "reloaded" option received after each exercise have the same expiration date as that of the original option. Upon the exercise of the last reload, no new reload option will be granted to the employee.

2.1. *Maximization of expected utility of terminal wealth*

Let W_T denote the total terminal wealth of the employee, which is the sum of the non-firm-related wealth, the value of firm's stocks and the value of options in his portfolio at maturity time T. The employee is assumed to have constant relative risk aversion γ so that the utility function $\widetilde{U}(W_T)$ takes the form

$$\widetilde{U}(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}, \quad \gamma \neq 1. \tag{2.1}$$

Subject to uncertainty of the stock price dynamics and risk preference, we investigate the optimal policy adopted by the employee to exercise the reload rights such that the expected utility of terminal wealth is maximized.

2.2. *Certainty-equivalent approach*

Let $U(S,t;k,X,n_s,n_o,c)$ denote the solution to the maximization of expected terminal utility value based on the portfolio composition of n_s units of stocks, n_o units of reload options and non-firm-related wealth c at the stock price level S and time t. The strike price of the reload option is X and there are k reload rights outstanding. The certainty equivalent E_c of the reload option is defined to be the amount of cash paid to the employee to replace the award of the reload option such that the employee is indifference between the cash award and option award with reference to expected terminal utility. Like the non-firm-reload wealth, the cash earns the riskless interest rate r over time. The certainty equivalent E_c is determined by the following relation

$$U(S,t;k,X,n_s,n_o,c) = U(S,t;k,X,n_s,0,c+E_c). \tag{2.2}$$

We take E_c as the private value of reload options to the employee.

2.3. *Construction of the numerical scheme*

We use the trinomial tree to simulate the stochastic movement of the stock price. The stock price S will become either uS, S or dS after one time period Δt, where $u = 1/d = e^{\lambda\sigma\sqrt{\Delta t}}$. Here, λ is a parameter whose value is commonly chosen to be $\sqrt{3}$. The probability values of up-move, zero-move and down-move are given by

$$\begin{aligned} p_u &= \frac{1}{2\lambda^2} + \frac{\left(\mu - \frac{\sigma^2}{2}\right)\sqrt{\Delta t}}{2\lambda\sigma} \\ p_m &= 1 - \frac{1}{\lambda^2} \\ p_d &= \frac{1}{2\lambda^2} - \frac{\left(\mu - \frac{\sigma^2}{2}\right)\sqrt{\Delta t}}{2\lambda\sigma}. \end{aligned} \tag{2.3}$$

Let N denote the total number of time steps in the trinomial tree. Let S_j denote the stock price with j net upward jumps, $j = -N, -N+1, \cdots, 0, \cdots, N-1, N$, where $S_j = S_0 u^j$ and S_0 is the stock price at the tip of the trinomial tree. The numerical scheme for computing the expected terminal utility U is complicated by the presence of 5 additional path dependent state variables: k, X, n_s, n_o and c, besides the state variable S and time variable t.

When we compute the expected terminal utility, the expectation is taken over all scenarios of stock price movement and possible inter-temporal exercise of the reload rights. The strike price of the "reloaded" option will be increased upward to the prevailing stock price at the exercise moment. In addition, the number of units of options and the amount of shares held will be adjusted according to the rules of reload upon exercise. Also, the non-firm-related wealth grows at the riskless interest rate, plus new additions due to dividends received. Since the expected terminal utility exhibits complicated path dependence, we construct the numerical algorithm for computing the expected terminal utility via the forward shooting grid (FSG) approach (Kwok and Lau, [15]). The FSG approach is characterized by the augmentation of an auxiliary path dependent state vector at each lattice node of the trinomial tree that simulates the discrete stock price process.

Let $\mathbf{F} = (F_1, \cdots, F_n)$ be a vector of path dependent state variables associated with the utility function. The path evolution function describes the correlated evolution of $\mathbf{F}$ with the stock price S over time. In the discrete world of trinomial tree, we construct the discrete path evolution function that relates $\mathbf{F}$ at time $t + \Delta t$ to $\mathbf{F}$ and S at time t. The key step in the construction of the FSG algorithm is to find the evolution function f such that

$$\mathbf{F}_{t+\Delta t} = f(\mathbf{F}_t, S_t). \tag{2.4}$$

Let $\widehat{X}, \widehat{n_s}$ and $\widehat{n_o}$ be the strike price, number of shares and options held at initiation $t = 0$, respectively, and let c_0 denote the initial personal wealth. The future strike price, number of shares and options held will depend on $\widehat{X}, \widehat{n_s}$ and $\widehat{n_o}$ and the prevailing stock price S^* at the last exercise moment. In the discrete world of trinomial stock price tree, the value assumed by S^* will be one of the nodal value of stock price on the tree. Like the use of the integer index "j" for identifying the discrete value of stock price, we use the integer index "m" for identifying the discrete value of strike price. While j lies between $-N$ and N, the strike price index m lies between j_0 and N, where $S_{j_0} < \widehat{X} < S_{j_0+1}$. Let $X(m)$ denote the strike price corresponding to the index m, then

$$X(m) = \begin{cases} \widehat{X} & \text{if } m = j_0 \\ S_m = S_0 u^m & \text{if } j_0 + 1 \le m \le N \end{cases}. \tag{2.5}$$

It can be shown that after any number of reloads, the number of units of stocks and options held are given by

$$\widehat{n_s} + \widehat{n_o}\left(1 - \frac{\widehat{X}}{S^*}\right) \quad \text{and} \quad \widehat{n_o}\frac{\widehat{X}}{S^*},$$

respectively, where S^* is the prevailing stock price at the last exercise moment. The dependence of n_s and n_o on the prevailing strike price $X(m)$ (as indexed by m) is given by

$$n_s(m) = \begin{cases} \widehat{n_s} & \text{if } m = j_0 \\ \widehat{n_s} + \widehat{n_o}\left(1 - \frac{\widehat{X}}{X(m)}\right) & \text{if } j_0 + 1 \le m \le N \end{cases}, \tag{2.6a}$$

$$n_o(m) = \begin{cases} \widehat{n_o} & \text{if } m = j_0 \\ \widehat{n_o}\left(\frac{\widehat{X}}{X(m)}\right) & \text{if } j_0 + 1 \le m \le N \end{cases}. \tag{2.6b}$$

When reloading has not occurred so far, the index m stays at the value j_0. The order of computational complexity of the numerical scheme is reduced by two since the three path dependent state variables $X(m), n_s(m)$ and $n_o(m)$ depend only on one single index m.

The non-option-wealth c increases in value at the rate of return equals r, plus addition of dividends received from holding the stocks. Again, we discretize the state variable c so that the values assumed by c are $c_\ell = c_0 + \ell\ \Delta c, \ell = 0, 1, \cdots L$. Knowing the non-option wealth c_ℓ^n at time $n\ \Delta t$, the non-firm-wealth c^{n+1} at the next time step is given by the evolution function

$$s^n(\ell, m, j) = c_\ell^n(1 + r\ \Delta t) + n_s(m)S_j^n q\ \Delta t. \tag{2.7}$$

In general, c^{n+1} does not fall into one of the pre-set grid values c_ℓ. Suppose the quadratic interpolation rule is adopted, we express c^{n+1} in terms of a weighted average of neighboring nodal values. Given the index values j, ℓ, m, n, we find the three neighboring nodes $\ell' - 1, \ell'$ and $\ell' + 1$ and the corresponding weighing coefficients $\alpha_{-1}^{\ell'}, \alpha_0^{\ell'}$ and $\alpha_1^{\ell'}$ such that

$$c^{n+1} = \alpha_{-1}^{\ell'} c_{\ell'-1}^{n+1} + \alpha_0^{\ell'} c_{\ell'}^{n+1} + \alpha_1^{\ell'} c_{\ell'+1}^{n+1}. \tag{2.8}$$

Figure 1 shows the pictorial representation of the trinomial tree calculations with the inclusion of the forward shooting grid procedure coupled with quadratic interpolation of neighboring nodal values.

2.4. *Dynamic programming procedure*

Let $U_{j,\ell,m}^{k,n}$ denote the numerical approximation to the expected terminal utility at time $t = n\ \Delta t$, stock price $S = S_0 u^j, c = c_0 + \ell\ \Delta c$, strike price index m and reload rights outstanding k. We compute $U_{j,\ell,m}^{k,n}$ using the usual backward induction in trinomial scheme and the dynamic programming procedure of choosing the maximum of the continuation value and exercise value at each node. The numerical scheme of the backward induction procedure can be represented by

$$\begin{aligned} U_{j,\ell,m}^{k,n} = \max \Big(& p_u \left(\alpha_{-1}^{\ell'} U_{j+1,\ell'-1,m}^{k,n+1} + \alpha_0^{\ell'} U_{j+1,\ell',m}^{k,n+1} + \alpha_1^{\ell'} U_{j+1,\ell'+1,m}^{k,n+1} \right) \\ & + p_m \left(\alpha_{-1}^{\ell'} U_{j,\ell'-1,m}^{k,n+1} + \alpha_0^{\ell'} U_{j,\ell',m}^{k,n+1} + \alpha_1^{\ell'} U_{j,\ell'+1,m}^{k,n+1} \right) \\ & + p_d \left(\alpha_{-1}^{\ell'} U_{j-1,\ell'-1,m}^{k,n+1} + \alpha_0^{\ell'} U_{j-1,\ell',m}^{k,n+1} + \alpha_1^{\ell'} U_{j-1,\ell'+1,m}^{k,n+1} \right), \\ & U_{j,\ell,j}^{k-1,n} \Big), \qquad k \ge 1. \end{aligned} \tag{2.9}$$

The first term is the continuation value, which is given by the expected utility calculated based on the trinomial stock price movement and quadratic interpolation procedure [see Eq. (2.8)]. Note that when there is no exercise, m and k stay at the same value. Upon exercising the reload, the cash value stays the same so that the cash index ℓ is unchanged, k decreases by one and m becomes j so that the exercise payoff is $U^{k-1,n}_{j,\ell,j}$. At maturity, the terminal payoff of the option is known so that we can evaluate $U^{k,N}_{j,\ell,m}$ for all possible values of j, ℓ, m. We start from $k = 0$, then proceed to $k = 1, 2, \cdots$ successively. When $k = 0, U^{0,n}_{j,\ell,m}$ is simply given by the continuation value. This is because the holder has no exercise right, so it is not necessary to apply the dynamic programming procedure.

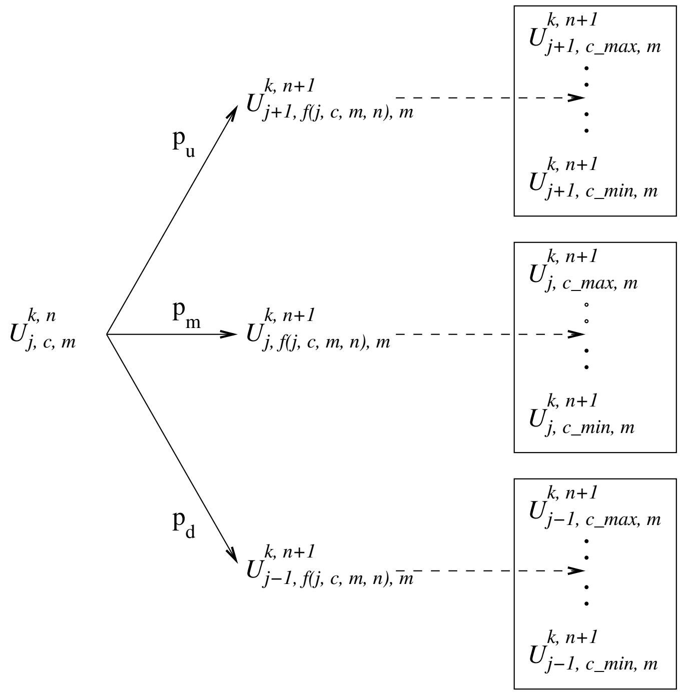

Fig. 1. The evolution function f in the forward shooting grid method has dependence on the stock price index j, non-option wealth c, strike price index m and time level n. The values of $U^{k,n+1}_{j+1,f(j,c,m,n),m}, U^{k,n+1}_{j,f(j,c,m,n),m}$ and $U^{k,n+1}_{j-1,f(j,c,m,n),m}$ are interpolated from the neighboring nodal values at time step $n + 1$ [see Eq. (2.9)].

2.5. *Cost to the employer*

The cost to the employer is computed based on the Black-Scholes risk neutral valuation framework except that the exercise policy is not determined endogenously by the optimality condition. This is not an optimal stopping problem, but rather it resembles a barrier option pricing problem. The exercise policy of the reload right is dictated by the holder, and this has been solved as part of the solution in the above dynamic programming procedure. To compute the employer's cost, the exercise boundary is treated as a barrier and the exercise payoff as the rebate.

3. Influence of the reload feature on incentive effects and optimal contracting

Under the utility maximization pricing framework, we applied the numerical scheme described in Section 2 to perform a series of numerical experiments to examine the influence of the reload feature on the incentive effects and optimal contracting. In our calculations, we use the following parameter values: $r = 6\%$, $\beta = 1, r_m = 12.5\%, \sigma = 30\%, q = 2\%$, and initial stock price $S_0 = 0.1$. All options have the time to maturity of 5 years. We take 50 time steps per year and the number of nodes in the discretization of the non-firm-related wealth is 20. Unless otherwise stated, the risk aversion coefficient ρ is set to be 2, the initial non-firm-related wealth is 75, the initial number of units of stock is 750 and the total cost to the employer of issuing the options is fixed at 50. The numerical values of the stock price and wealth are taken to be relatively small in order to avoid instability in the interpolation step in the numerical calculations. Such choices are justified since one can always rescale the stock price and wealth by an appropriate choice of unit of currency.

The percentage of dead-weight loss is defined by

$$\frac{\text{cost to employer} - \text{value to employee}}{\text{cost to employer}} \times 100\%.$$

In Figure 2, we show the plot of the percentage of dead-weight loss against the number of reloads outstanding at risk aversion coefficient $\rho = 2.0$ and $\rho = 2.5$. The percentage of dead-weight loss shows a general trend of decrease with respect to increasing number of reload rights. This is because the reload provision can lock in some of the gain through the process of stock price appreciation thus reducing the dead-weight loss. Also, a higher risk aversion would lower the private value to the employee, thus leading to a higher percentage of dead-weight loss. The sensitivity of the dead-weight loss to the risk aversion level can be quite significant.

We also examine the impact of the moneyness (ratio of the stock price to the strike price) of the employee option on the percentage of dead-weight loss. From the two curves in Figure 3, we observe that the dead-weight loss decreases as the moneyness increases. This would mean the gap between cost and value widens when the option becomes deeper out-of-the-money. Such observation may partly explain the repricing practices where the employer resets the strike price downward on outstanding options when the firm's stock declines in value (option becomes out-of-the-money). Hall and Murphy [9] have similar findings on the decrease of dead-weight loss with an increase of moneyness for European style options. Figure 3 again confirms that the dead-weight loss decreases with more reload rights outstanding.

We define the cost ratio to be the ratio of the cost of issuing one unit of the employee reload option to the market value of the one American option with the same strike and maturity. Since the employee exercises his option farther from optimality

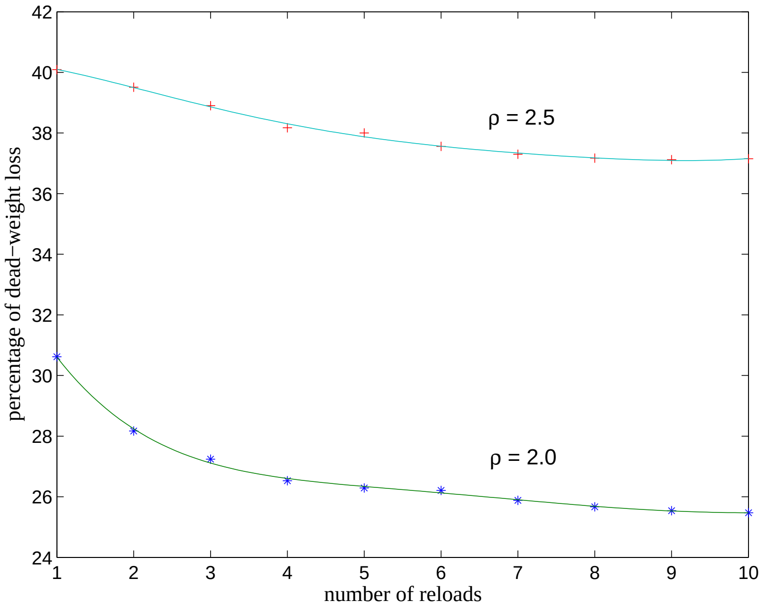

Fig. 2. Plot of the percentage of dead-weight loss against the number of reloads outstanding at varying values of risk aversion level of the employee.

at a higher risk aversion level, the cost ratio decreases as the risk aversion coefficient increases. The cost to the employer should increase monotonically as the number of reload rights increases. Also, the cost ratio becomes less than one when there is only one reload right since the holder's value of an one-reload option is less than the market value of the American call counterpart. The above intuitive arguments are confirmed by the plots of cost ratio against number of reloads in Figure 4. The impact of the employee's risk aversion on the cost ratio is less significant compared to that of the dead-weight loss.

To measure the compensation incentive of an employee reload option, we define "leverage" to be the corresponding percentage increase in the private value of the option to the employee for one percentage point increase in the stock price. In Figure 5, we plot the leverage against the number of reloads with varying values of risk aversion level. The leverage increases with more reload rights outstanding showing that the reload provision does provide a better alignment of the employee's interest with the interest of the shareholder. The leverage has a higher value when ρ is lower, indicating that the compensation incentive works better for less risk averse employees. We also analyze the impact of moneyness of the option on the incentive strength as measured by leverage (see Figure 6). The leverage is maximized when the option is slightly in-the-money, which is in agreement with the results of Hall and Murphy [9]. With reload provision, the influence of moneyness on leverage is

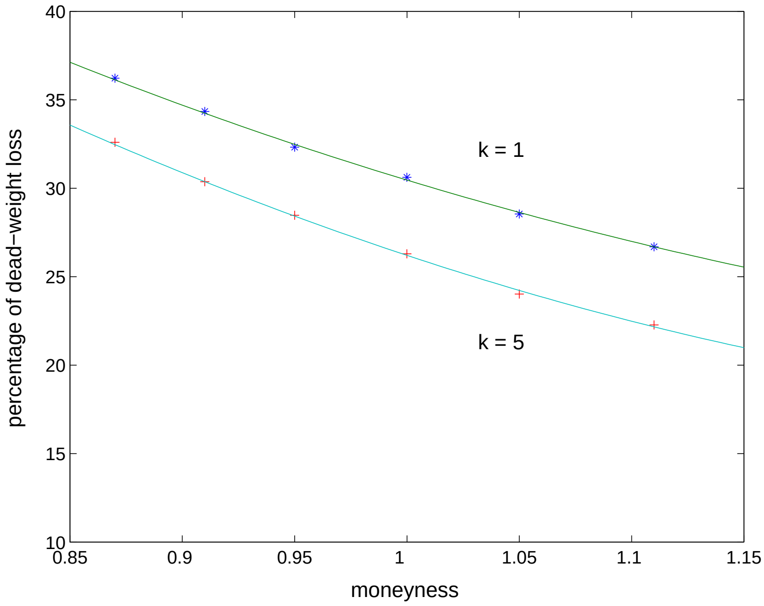

Fig. 3. Plot of the percentage of dead-weight loss against the moneyness of reload option with varying number of reloads outstanding.

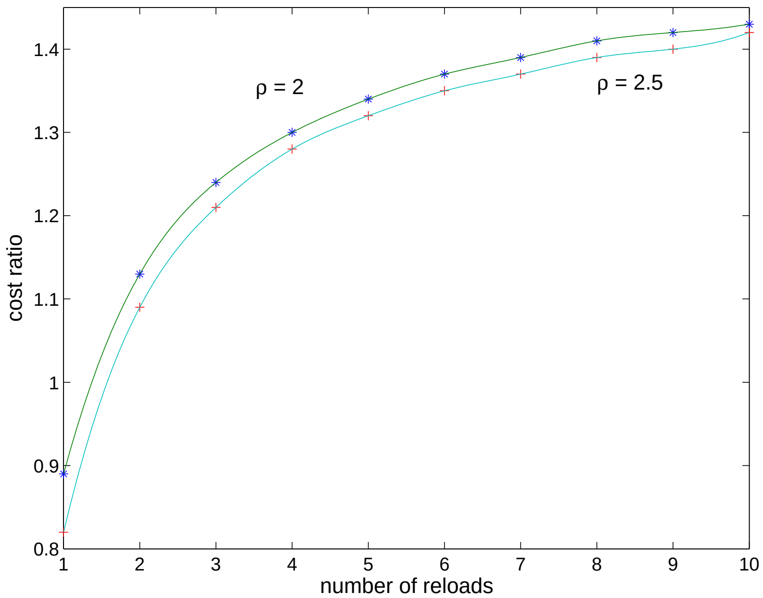

Fig. 4. Plot of the cost ratio against the number of reloads outstanding with varying level of risk aversion of the employee.

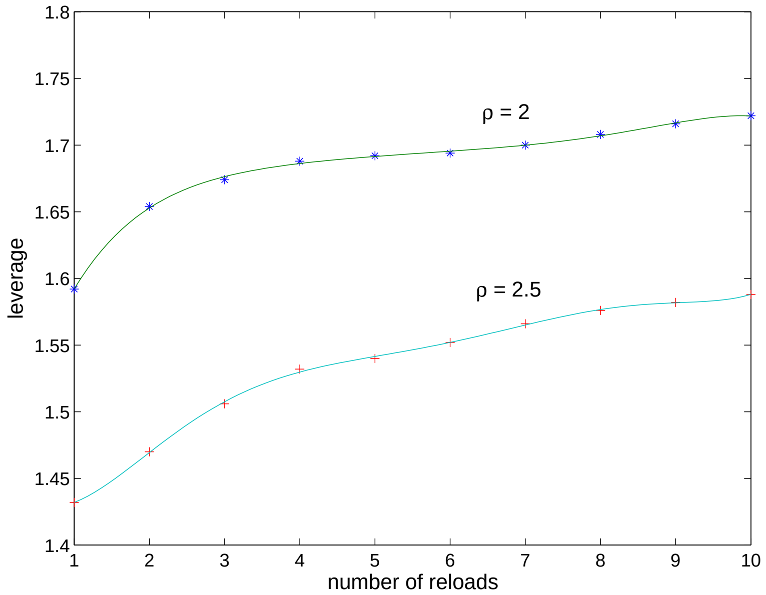

Fig. 5. Plot of the leverage (measure of compensation incentive) against the number of reloads outstanding with varying level of risk aversion of the employee.

less significant. This is not surprising since an out-of-the-money reload option can be reset to become an at-the-money reload option upon exercising the reload right.

In Figure 7, we show the time dependence of the critical stock price at which the holder should exercise the reload right with varying number of reloads outstanding. The critical stock price increases with increasing time to expiry, like usual American call options. Also, the holder chooses to exercise at a lower critical stock price when there are more reload rights outstanding.

Lastly, we examine the influence of employee's initial wealth and proportion of wealth (split between stock and non-firm-related wealth) on the dead-weight loss and compensation incentive (as measured by leverage). The employee reload options are assumed to have 5 reloads outstanding. In Table 1, we list the percentage of dead-weight loss (value in the first entry) and leverage (value in the second entry) with varying employee's initial wealth and percentage of portfolio wealth in stocks. The dead-weight loss is seen to decrease with increasing employee's initial wealth indicating that option awards are more desirable for higher-level employees. As expected, the dead-weight loss becomes lower when the employees hold less portion of their wealth in firm's stocks. Unlike the dead-weight loss, the compensation incentives do not vary significantly with the employee's initial wealth and portion of wealth in stocks.

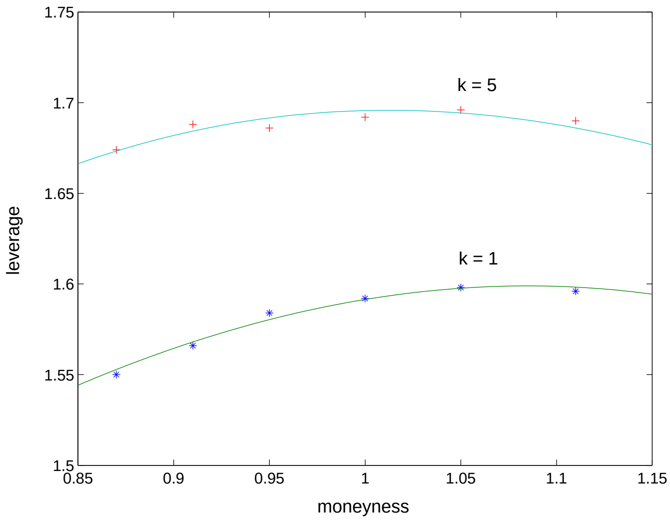

Fig. 6. Plot of the leverage against moneyness with varying number of reloads outstanding.

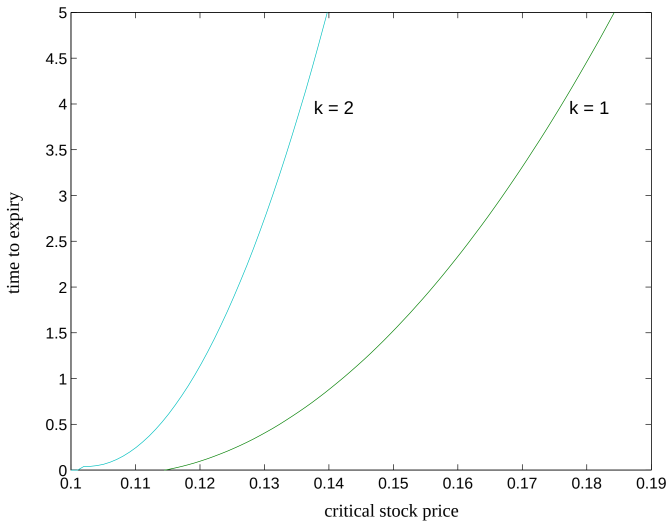

Fig. 7. Plot of the time dependence of the critical stock price with varying number of reloads outstanding.

4. Summary and Conclusion

We have constructed the lattice tree algorithm for pricing non-tradeable employee stock options with reload provision under the framework of maximizing the expected

initial wealth \ % of wealth in stocks	67%	50%	33%
50	43.89%; 1.73	35.69%; 1.76	27.97%; 1.81
100	39.40%; 1.61	29.45%; 1.70	18.85%; 1.81
200	36.03%; 1.58	24.46%; 1.69	11.72%; 1.82

Table 1. We list the percentage of dead-weight loss (first entry) and leverage (second entry) of an employee option with 5 reloads outstanding and varying employee's initial wealth and percentage of portfolio wealth in stocks.

utility value of the terminal wealth. The pricing model exhibits a high level of path dependence of the stock price and reloading history. We demonstrate how the forward shooting grid procedure can resolve the complicated path dependence in the pricing algorithm. The private value of the reload option to the employee is obtained via the certainty equivalent approach. The cost to the employer is computed via the risk neutral valuation framework, where the critical stock price at which reload commences is determined by the optimal decision of the option holder.

With the total cost of option awards to the employer being fixed, our calculations indicate that more reload rights can reduce the dead-weight loss. The dead-weight loss decreases with the moneyness of the employee options. Also, it decreases with increasing employee's initial wealth and smaller portion of employees wealth in stocks. The compensation incentives (as measured by the leverage of the private option value) are seen to increase with more reload rights and lower employee's risk aversion. Incentive effects are maximized when the option is slightly in-the-money, but they do not vary significantly with employees initial wealth and portion of wealth in stocks. The employer's cost increases monotonically with the number of reload rights and decreases with increasing employee's risk aversion. Similar to usual American call options, the critical stock price at which the reload is exercised increases with longer time to expiry of the option. With less number of reload rights outstanding, the holder should use each reload right more cautiously so that the critical stock price increases.

In this paper, we have addressed the question: "What is the best way for the employer to give certain dollars of compensation?" One potential future work is to address an alternative issue: "What is the optimal compensation composition that maximizes firm value?"

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