Hybrid Equity-Credit Modeling for Contingent Convertibles

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Agenda

1. Product nature of CoCo bonds
   - Trigger mechanisms: Mechanical (book-value or market-value) and/or regulatory
   - Loss absorption mechanisms: Conversion to equity and/or principal writedown
   - Market development
2. Enhanced equity-credit models
   - Jump-to-non-viability feature
   - Numerical algorithms
3. Sensitivity analysis of model parameters.

Product nature

- A contingent convertible (CoCo) is a high-yield debt instrument that automatically converts into equity and/or suffers a write down when the issuing bank gets into a state of a possible nonviability. This is like a reverse convertible.

- It provides deliveraging of debts and boosts up equity-debt ratio in times of nonviability of the issuing bank.

- The CoCo bonds are qualified as Additional Tier 1 (part of the Core Tier 1 capital that is beyond the Common Equity Tier 1* capital). The new Basel Accords require the Core Tier 1 capital to be at least 6.0% of risk-weighted assets. The Additional Tier 1 category consists of instruments that are not CET 1 but are still regarded as safe enough to be Tier 1 eligible. Almost all recent CoCo issues in Europe have been AT 1 eligible.

* Common Equity Tier 1 ratio is the book value of common equity divided by the amount of risk-weight assets.

Triggers can be based on a mechanical rule or regulators' discretion.

1. *Accounting trigger*

   The Lloyds and Credit Suisse CoCos have been structured with the Core Tier 1 ratio (CT1) as an indicator of the health of the bank. The accounting trigger level may be 5% or 7%.

   - Accounting triggers may not be activated in a timely fashion, depending crucially on the frequency at which the ratio is calculated as well as the rigor and consistency of internal risk models.

2. *Regulatory trigger (nonviability trigger)*

   It is a discretionary choice in the hands of the bank's national regulator. This type of trigger may reduce the marketability of a CoCo bond due to uncertainty of trigger.

Market-value trigger

This could address the shortcoming of inconsistent accounting valuations, and reduce the scope for balance sheet manipulation and regulatory forbearance. Share prices or CDS spreads (forward looking parameters) could be used. For example, when the share price breaches a well-defined barrier level, this will trigger the conversion into shares.

- Creation of incentives for stock price manipulation.

The design for the trigger mechanism has to be robust to price manipulation and speculative attacks.

Market development

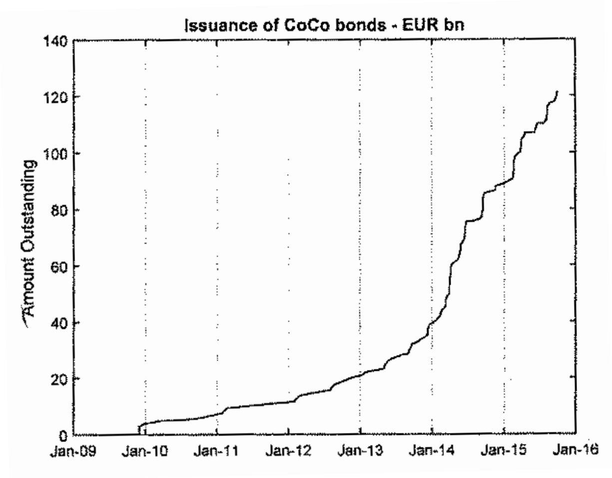
As banks felt more pressure from markets and regulators to boost their Tier 1 capital, they started to issue CoCo with trigger levels at or above the preset minimum for satisfying the going-concern contingent capital requirement.

Banks have issued close to $200 billion worth of CoCos since 2009 to late 2016. China has issued close to 500 billion RMB of CoCos since the first launch in early 2014.

Retail investors and private banks in Asia and Europe are attracted by the relatively high nominal yield that CoCos offer in the current low interest rate environment.

Most CoCo bonds issued in China are rated the same credit rating levels as those of the issuing banks, neglecting the potential loss absorption upon equity conversion and/or principal writedown.

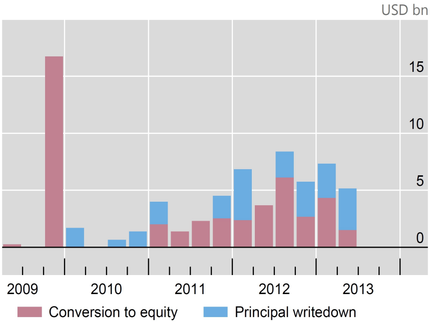
Market development



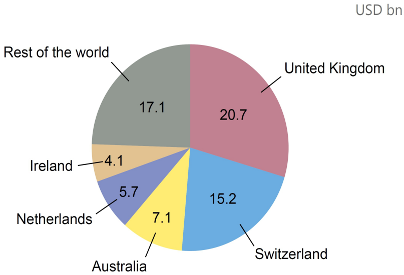
Sale of CoCo bonds since the first launch in December, 2009

Mostly by European banks as a way to repair their balance sheets...

By type of loss absorption mechanism



By nationality of issuing bank



Sources: Bloomberg;

Examples

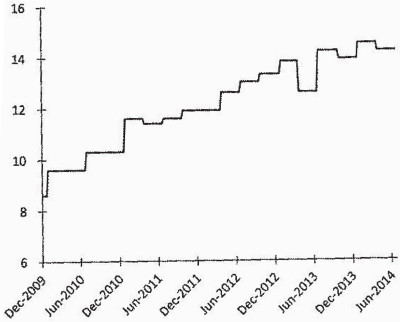
Contingent Convertibles Examples

Issuer	Conversion in Shares: Lloyds	Conversion in Shares: Credit Suisse	Write Down: Rabobank	Write Down: UBS
Issue size	GBP 7 bn (32 series)	USD 2 bn	EUR 1.25 bn	USD 2 bn
Rating (Fitch)	BB	BBB	-	BBB-
Issue date	December 1, 2009	February 17, 2011	March 12, 2010	February 22, 2010
Maturity	10–20 year	30 year—callable 5.5 years	10 year	10 year—callable 5 years
Coupon	1.5–2.5% increase of the coupon of the hybrid bond exchanged for the ECN	7.875%	Libor+3.5	7.25%
Write down			75%	100%
Conversion Price	59 Pence	max (USD20, CHF20, $\bar{S}$)		
Accounting Trigger	Core Tier 1 Ratio	Core Tier 1 Ratio	Equity Capital/RWA	Core Tier 1 Ratio
Accounting Trigger Level	5%	7%	7%	5%
Regulatory Trigger	No	Yes	No	Yes

More than half of all CoCos are currently unrated. The existence of discretionary triggers creates uncertainty in credit rating. Coupon rates are typically very high, like Libor+3.5% or 1.5-2.5% above that of ECN (Equity Commitment Note).

CoCo of Lloyds

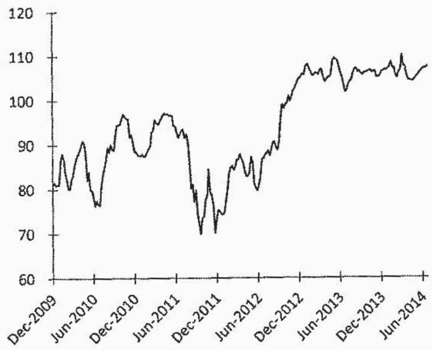
Historical time-series of the tier-1 capital ratio, CoCo price, stock price and CDS spread for the Lloyds Banking Group.



(a) Tier-1 capital ratio (in %).

The Tier-1 capital ratio increased steadily from Dec 2010 to Dec 2012 and had a small dip in earlier 2013 (after a period of dip in the stock price in late 2011 until Dec 2012). It then moved up to the steady level of 14%.

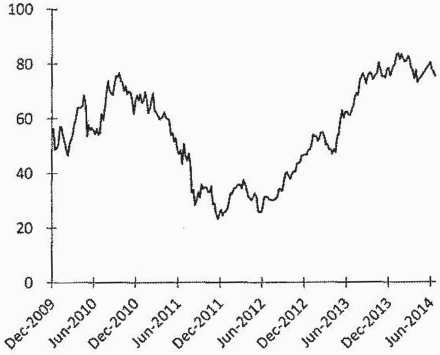
Bond price



(b) CoCo bond price.

At the beginning of 2013, with the ending of the distressed state in the previous one and half year, the CoCo bond had been traded like a high yield corporate bond with low credit risk.

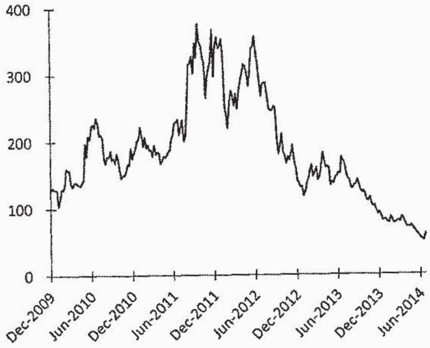
Stock price



(c) Stock price (in GBP pence).

The stock price suffered a great dip starting Jun 2011 and recovered in mid 2013.

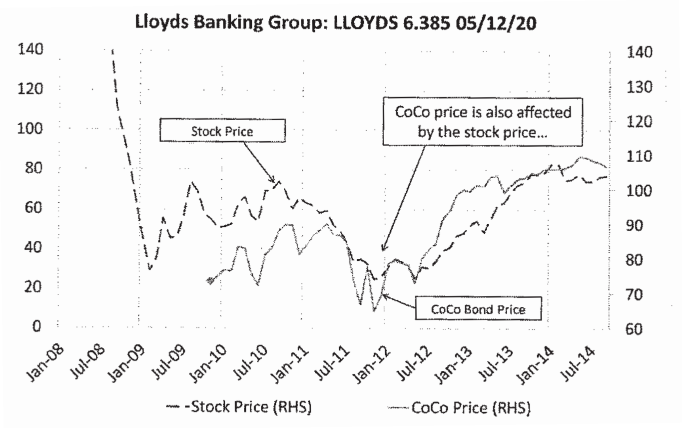
CDS spread



(d) CDS spread (in bps).

The CDS spread (reflecting the credit risk of the bank) peaked in 2011 and declined steadily to a level well beyond 100 (highly rated bank) since mid 2013. The stock price remained stable within the strong period during which the CDS spread declined quite drastically.

Correlated moves of stock price and CoCo bond price



Investors in the CoCos Market

- Investors demand higher coupons to compensate the potential loss at a conversion.
- Yield enhancement under the low interest rate environment since 2008.

General Views

- The fair valuation is difficult given its hybrid nature and the infrequent observation of the capital ratio (asymmetric information).
- Lack of complete and consistent credit rating.
- High uncertainty on what may happen at a trigger since there has been no past record. For example, it is not certain whether the stock price would always jump down upon regulatory trigger.

Structural Approach (Brigo et al., 2015)

- Starts with the modeling of the balance sheet dynamics such as the firm asset value and bank deposits.
- Allows one to analyze the impact of CoCos on the bank's capital structure.
- A natural starting point since the capital ratio (tier one capital / risk-weighted asset) is a balance sheet quantity.

Drawbacks

- Not flexible enough for calibration to traded security prices.
- Equity is priced as a contingent claim and hence its dynamics is not easily tractable.
- Not straightforward to incorporate a jump in the stock price to capture a potential write down.

Reduced Form Approach (Cheridito and Xu, 2015)

- Better flexibility to perform calibration of model parameters using the market prices of traded derivatives, like the credit default swap (CDS) spreads.
- Efficient for pricing CoCos: only requires the specification of the conversion intensity and the jump magnitude of the stock price at the conversion time.
- Feasible to capture the possibility of sudden trigger even when the capital ratio is far from the triggering threshold.

Drawbacks

- It completely ignores the contractual feature of an accounting trigger (not incorporating the contractual specification on the capital ratio).
- It stays silent on the interaction between stock price and capital ratio, and does not provide a structural interpretation of the triggering events.

Equity Derivative Approach (De Spiegeleer and Schoutens, 2012)

- Approximating the accounting trigger by the first passage time of the stock price process hitting an implied barrier level.
- CoCo bond can be loosely replicated by using barrier-type options.
- Extension to stochastic volatility or jump diffusion process.

Drawback

- Empirical evidence shows that the capital ratio does not always show positively correlated move with the stock price.

Hybrid equity-credit modeling

- Bivariate equity-credit modeling of the stock price and the capital ratio together with a jump-to-non-viability (PONV) feature.
- Structural modeling on the capital ratio hitting the triggering threshold and the stock price at conversion.
- Reduced form is added to capture a PONV trigger that is related to a sudden insolvency of the bank leading to a trigger.
- Integrating the reduced form approach and structural approach is natural for the pricing of a CoCo bond that has both regulatory and accounting triggers.

- The Common Equity Tier 1 capital ratio process can only be observed infrequently. Several works (Ritzema, 2015; Cheridito and Xu, 2016) illustrate how to use the CDS rates and other tradeable instruments to calibrate the CET 1 process. This provides the justification for the use of risk neutral valuation under a risk neutral measure Q.

CoCo Bond Structure

- Stock price process by $S = (S_t)_{t \geq 0}$ with constant interest rate r.
- The no-arbitrage price of a CoCo can be decomposed into three components:

$$P_{CoCo} = \sum_{i=1}^{n} c_i e^{-rt_i} Q(\tau > t_i) + F e^{-rT} Q(\tau > T) + \mathbb{E}^Q \left[e^{-r\tau} G S_\tau \mathbf{1}_{\{\tau \leq T\}} \right],$$

  where c_i is the coupon, F is the principal, τ is the random conversion time, G is the number of shares received upon conversion and Q stands for the risk neutral measure.
- The coupons and principal payment (first two terms) form a defaultable coupon-bearing bond.
- The challenge is the computation of the conversion value P_E (last term):

$$P_E = \mathbb{E}^Q \left[e^{-r\tau} G S_\tau \mathbf{1}_{\{\tau \leq T\}} \right].$$

The key challenge is the joint modeling of the conversion time τ and stock price S_τ.

Joint process of stock price and capital ratio

The state variables are the stock price process $S_t = \exp(x_t)$ and capital ratio process $H_t = \exp(y_t)$.

- Jump diffusion stock price process and a mean-reverting capital ratio.

We assume $X_t = (x_t, y_t)$ follows the bivariate process as follows:

$$dx_t = \left(r - q - \frac{\sigma^2}{2}\right) dt + \sigma\ dW_t^1 + \gamma\left[dN_t - \lambda(x_t, y_t)dt\right], \qquad x_0 = x,$$
$$dy_t = \kappa(\theta - y_t)\,dt + \eta\left(\rho\ dW_t^1 + \sqrt{1-\rho^2}\ dW_t^2\right), \qquad y_0 = y.$$

- N_t denotes the Poisson process that models the arrival of the PONV trigger and $\lambda(x_t, y_t)$ is the state dependent intensity of N_t.

Random time of equity conversion

- Accounting trigger: the first passage time of the log capital ratio y_t to a lower threshold y_B as
$$\tau_B = \inf\{t \geq 0;\ y_t = y_B\}.$$
- The random time of PONV trigger is modeled by the first jump of the Poisson process N_t as
$$\tau_R = \inf\{t \geq 0;\ N_t = 1\}.$$
- The random time of equity conversion is the earlier of τ_B and τ_R
$$\tau = \tau_B \wedge \tau_R.$$
- It is assumed that the two random times do not occur at the same time almost surely. The calculation involves the convolution of the two random times.

Two technical identities

Let $\mathcal{H}(\xi)$ be the payoff at the corresponding random time $\xi = \tau_R$ or $\xi = \tau_B$.

Consider the convolution of the two random times, τ_B and τ_R and the two building blocks:

$$\begin{aligned}
\mathbb{E}^Q\left[\mathcal{H}(\tau_B)\mathbf{1}_{\{\tau_R > \tau_B\}}\right] &= \mathbb{E}^Q\left[e^{-\int_0^{\tau_B} \lambda_u \, du}\mathcal{H}(\tau_B)\right], \\
\mathbb{E}^Q\left[\mathcal{H}(\tau_R)\mathbf{1}_{\{\tau_R < \tau_B\}}\right] &= \mathbb{E}^Q\left[\int_0^{\tau_B} \lambda_u e^{-\int_0^u \lambda_s \, ds}\mathcal{H}(u) \, du\right].
\end{aligned}$$

The state dependent intensity $\lambda_t = \lambda(x_t, y_t)$ introduces dependence among τ_R and τ_B, which adds further complexity to the convolution.

Conversion Probability

We consider a general stochastic intensity $\lambda_t = \lambda(X_t)$ with Markov process $X_t \in \mathbb{R}^2$. Since $\tau = \tau_B \wedge \tau_R$, the decomposition into the two events $\{\tau_B > \tau_R\}$ and $\{\tau_R > \tau_B\}$ gives

$$Q(\tau \leq t) = \mathbb{E}^Q\left[\mathbf{1}_{\{\tau_B \wedge \tau_R \leq t\}}\right] = \mathbb{E}^Q\left[\mathbf{1}_{\{\tau_R \leq t\}}\mathbf{1}_{\{\tau_B > \tau_R\}}\right] + \mathbb{E}^Q\left[\mathbf{1}_{\{\tau_B \leq t\}}\mathbf{1}_{\{\tau_R > \tau_B\}}\right].$$

Lemma (Conversion Probability)

For a fixed $t > 0$, the probability of equity conversion is given by

$$Q(\tau \leq t) = \int_0^t \mathbb{E}^Q\left[\lambda_u e^{-\int_0^u \lambda_s \, ds}\mathbf{1}_{\{\tau_B > u\}}\right] du + \mathbb{E}^Q\left[e^{-\int_0^{\tau_B} \lambda_u \, du}\mathbf{1}_{\{\tau_B \leq t\}}\right].$$

Proof.

The first term corresponds to the scenario where the PONV trigger occurs prior to the accounting trigger. By virtue of iterated expectation, we obtain

$$\begin{aligned} \mathbb{E}^Q\left[\mathbf{1}_{\{\tau_R \le t\}}\mathbf{1}_{\{\tau_B > \tau_R\}}\right] &= \mathbb{E}^Q\left[\mathbb{E}^Q\left[\mathbf{1}_{\{\tau_R \le t\}}\mathbf{1}_{\{\tau_B > \tau_R\}}\middle|\, \tau_R = u\right]\right] \\ &= \int_0^t \mathbb{E}^Q\left[\lambda(X_u)\, e^{-\int_0^u \lambda(X_s)\, ds}\mathbf{1}_{\{\tau_B > u\}}\right] du. \end{aligned}$$

The second term gives the conversion probability conditional on accounting trigger occurring prior to the PONV trigger. It is straightforward to obtain

$$\mathbb{E}^Q\left[\mathbf{1}_{\{\tau_B \le t\}}\mathbf{1}_{\{\tau_R > \tau_B\}}\right] = \mathbb{E}^Q\left[e^{-\int_0^{\tau_B} \lambda(X_u)\, du}\mathbf{1}_{\{\tau_B \le t\}}\right].$$

□

Constant Intensity

- The conversion value is

$$P_E = G\mathbb{E}^Q \left[e^{-r\tau} S_\tau \mathbf{1}_{\{\tau \leq T\}} \right], \qquad \tau = \tau_B \wedge \tau_R.$$

- When the intensity λ is constant, we have

$$Q(\tau \leq t) = \int_0^t \lambda e^{-\lambda u} [1 - Q(\tau_B \leq u)] \, du + \mathbb{E}^Q \left[e^{-\lambda \tau_B} \mathbf{1}_{\{\tau_B \leq t\}} \right].$$

- The density function of the random conversion time can be expressed as

$$Q(\tau \in dt) = \lambda e^{-\lambda t} [1 - Q(\tau_B \leq t)] + e^{-\lambda t} Q(\tau_B \in dt).$$

The random times τ_B and τ_R become uncorrelated under constant intensity.

Stock price measure

We consider the adjusted stock price process

$$\widetilde{S}_t = S_0 \exp\left(\int_0^t \sigma \; dW_s^1 - \int_0^t \frac{\sigma^2}{2} \; ds - \gamma\lambda t\right) (1+\gamma)^{N_t}, t \geq 0.$$

We can construct a change-of-measure as

$$Z_t = \left.\frac{dQ^*}{dQ}\right|_{\mathcal{F}_t} = \frac{\widetilde{S}_t}{\widetilde{S}_0} = \frac{e^{-(r-q)t} S_t}{S_0},$$

where Q^* is interpreted as the *stock price measure*.

Under the assumption of constant intensity, we can use the stock price measure Q^* to reduce the dimensionality of the pricing problem by one.

Lemma (Change-of-Measure)

Under the stock price measure Q^*, the bivariate process of (x_t, y_t) evolves as

$$dx_t = \left(r - q - \frac{\sigma^2}{2} - \gamma\lambda\right) dt + \sigma\ dB_t^1 + \gamma\ dN_t, \qquad x_0 = x,$$

$$dy_t = [\kappa(\theta - y_t) + \rho\sigma\eta]\, dt + \eta\ dB_t^2, \qquad y_0 = y,$$

where $\langle dB_t^1, dB_t^2 \rangle = \rho\ dt$. The intensity of the Poisson process N_t becomes

$$\lambda^* = (1 + \gamma)\,\lambda.$$

Constant Intensity

Applying the change-of-measure formula, we have

$$\begin{aligned}\mathbb{E}^Q\left[e^{-r\tau}S_\tau\mathbf{1}_{\{\tau\le T\}}\right] &= S_0\mathbb{E}^Q\left[e^{-q\tau}\frac{e^{-(r-q)\tau}S_\tau}{S_0}\mathbf{1}_{\{\tau\le T\}}\right]\\ &= S_0\mathbb{E}^{Q^*}\left[e^{-q\tau}\mathbf{1}_{\{\tau\le T\}}\right],\end{aligned}$$

where $\tau=\tau_B\wedge\tau_R$.

Hence, we only need to compute a truncated Laplace transform on τ under the stock price measure Q^*.

Proposition (Conversion Value)

Denote $\lambda^* = (1+\gamma)\lambda$. The conversion value under constant intensity λ is given by

$$P_E = GS_0 \left\{ \int_0^T \lambda^* e^{-(\lambda^*+q)u} \left[1 - Q^*(\tau_B \le u)\right] du + \mathbb{E}^{Q^*}\left[e^{-(\lambda^*+q)\tau_B} \mathbf{1}_{\{\tau_B \le T\}} \right] \right\}.$$

When $\lambda^* + q > 0$, the conversion value can be expressed as

$$P_E = GS_0 \left\{ \frac{\lambda^*}{\lambda^* + q} \left[1 - e^{-(\lambda^*+q)T} Q^*(\tau_B > T) \right] + \frac{q}{\lambda^* + q} \mathbb{E}^{Q^*}\left[e^{-(\lambda^*+q)\tau_B} \mathbf{1}_{\{\tau_B \le T\}} \right] \right\}.$$

Proof.

It suffices to replace λ by $\lambda^* = (1+\gamma)\lambda$ and take the expectation under the stock price measure Q^*. The remaining procedure is similar to that of Lemma 1. We then apply integration by parts and note that $\frac{\partial}{\partial t} Q^*(\tau_B \le t)$ gives the density of τ_B, we obtain the result.

First passage time

It is necessary to compute the first passage time distribution

$$Q^* \left(\tau_B \leq T \right)$$

and the associated truncated Laplace transform

$$\mathbb{E}^{Q^*} \left[e^{-\left(\lambda^* + q\right)\tau_B} \mathbf{1}_{\{\tau_B \leq T\}} \right].$$

We construct the Fortet algorithm to solve the first passage time problem for the mean-reverting process. The determination of the density function of the first passage time to a barrier is resorted to an effective recursive algorithm for solving an integral equation derived based on the strong Markov property of the underlying asset price process.

Strong Markov property: For $t > \tau$, $x_t - x_\tau$ depends only on x_τ, where τ is a stopping time.

Fortet Scheme

Let τ_B denote the first passage time of y_t hitting the threshold y_B and $q(t)$ be the corresponding density function defined by $\Pr\{\tau_B \in dt\} = q(t)\,dt$. The transition probability density for the process y_t conditional on $s < t$ is defined by

$$\Pr\left\{ y_t \in dy \middle| y_s \in dy' \right\} = f\left(y, t; y', s\right) dy.$$

Suppose y_B is a barrier that lies between y_0 and y_1, by the continuity and strong Markov property of the process, we obtain the following integral equation

$$f\left(y, t; y_0, 0\right) = \int_0^t q(s)\, f\left(y, t; y_B, s\right) ds.$$

Integrating y on both sides over $(-\infty, y_B]$, we obtain the Fortet equation

$$N\left[\frac{y_B - \mu(t,0)}{\Sigma(t,0)}\right] = \int_0^t q(s)\; N\left[\frac{y_B - \mu(t,s)}{\Sigma(t,s)}\right]\Bigg|_{y_s = y_B} ds,$$

where $\mu(t,s)$ and $\Sigma^2(t,s)$ are in closed-form since y_t is a Gaussian process.

Special Case 1: Equity Derivative Approach

- When $\kappa = 0$ and $\rho = 1$, the capital ratio process becomes a lognormal process that is perfectly correlated to the stock price process.
- In this case, the event "capital ratio hitting a lower threshold" is equivalent to the event "the stock prices hitting a down barrier":

$$\tau_B \sim \tau_S, \qquad \tau_S = \inf \{t \geq 0; S_t = S_B\},$$

for a certain implied barrier S_B.
- This reduces to the Black-Cox formula

$$\begin{aligned} Q(\tau_S < t) &= N\left[\frac{\ln(S_B/S_0) - \left(r - q - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right] \\ &+ \left(\frac{S_B}{S_0}\right)^{\frac{2(r-q)}{\sigma^2}-1} N\left[\frac{\ln(S_B/S_0) + \left(r - q - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right] \end{aligned}$$

where $N(\cdot)$ is the standard cumulative normal distribution.
- The conversion probability under Q^* can be obtained similarly.

Special Case 2: Reduced Form Approach

- Suppose that the accounting trigger is never activated (say, $y_t >> y_B$ or $\eta \to 0$), then

$$P_{CoCo} = \sum_{i=1}^{n} c_i e^{-(r+\lambda)t_i} + Fe^{-(r+\lambda)T} + GS_0 \left[1 - e^{-(1+\gamma)\lambda T}\right].$$

- Easy-to-implement: the model parameters are the conversion intensity λ and the jump size of the stock price upon conversion γ.
- One can estimate the implied default intensity from the CDS spread using the rule-of-thumb as

$$\lambda_{CDS} = \frac{c}{1 - R}$$

where R is the recovery fraction.
- As the conversion always occurs before default (as loss absorption mechanism), we can interpret the CDS implied intensity as a *lower bound* estimate of λ.
- Alternatively, one may estimate the conversion intensity from deep OTM put options based on a jump-to-default diffusion model.

State Dependent Intensity

When the intensity is state dependent as $\lambda(x, y)$, we cannot retain the nice analytical tractability of our model.

Further interaction between the stock price and capital ratio is embedded besides the correlation through the Brownian motions.

The conversion value can be expressed as

$$P_E = \mathbb{E}^Q \left[e^{-\int_0^{\tau_B} (r+\lambda_u)\ du} GS_{\tau_B} \mathbf{1}_{\{\tau_B \leq T\}} + \int_0^{\tau_B \wedge T} e^{-\int_0^u (r+\lambda_s)\ ds} \lambda_u (1+\gamma) GS_u du \right],$$

which is a two-dimensional pricing problem.

Numerical solution of a two-dimensional partial differential equation

We can compute P_E by solving a partial differential equation (PDE).

Define the generator

$$\mathcal{L} = \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2} + \alpha\frac{\partial}{\partial x} + \rho\sigma\eta\frac{\partial^2}{\partial x\partial y} + \frac{\eta^2}{2}\frac{\partial^2}{\partial y^2} + \kappa(\theta - y)\frac{\partial}{\partial y}.$$

The conversion value $P = P_E$ solves the Dirichlet problem

$$\frac{\partial P}{\partial t} + \mathcal{L}P + \lambda(x,y)(1+\gamma)Ge^x = [r + \lambda(x,y)]P,$$

for $(x, y, t) \in (-\infty, \infty) \times [y_B, \infty) \times [0, T]$. The boundary condition and terminal condition are

$$P(x, y_B, t) = Ge^x, \qquad P(x, y_B, T) = \begin{cases} 0, & y > y_B \\ Ge^x, & y = y_B \end{cases}.$$

This is an inhomogeneous PDE due to the non-linear term $\lambda(x,y)(1+\gamma)Ge^x$, which can be solved by standard finite-difference method for PDE.

State Dependent Intensity

Different forms of dependence of the stock price and/or capital ratio are possible.

- Stock price dependent intensity:

$$\lambda(x) = \exp(a_0 - a_1 x), \quad a_1 > 0,$$

  which prescribes an inverse relation between the intensity and stock price.

- Capital ratio dependent intensity:

$$\lambda(y) = b_0 \mathbf{1}_{\{y \le y_{RT}\}}, \quad b_0 > 0,$$

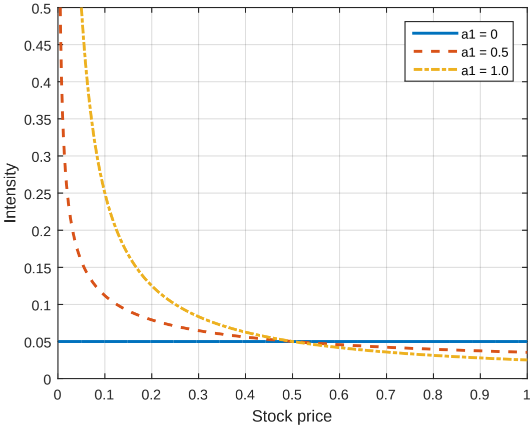
  where y_{RT} is a level that specifies the warning region for PONV trigger.

- We can combine the above two specifications as a sum of two terms:

$$\lambda(x, y) = \exp(a_0 - a_1 x) + b_0 \mathbf{1}_{\{y \le y_{RT}\}}, \quad a_1 > 0,\ b_0 > 0.$$

- One may link the PONV intensity with different economics variables.

State Dependent Intensity



Inverse relationship between conversion intensity and stock price.

Model Parameters

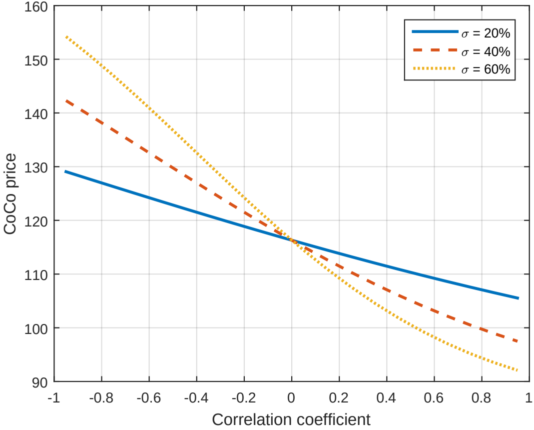
Since we directly model the two most important quantities for pricing of CoCos, the parametrization is easier than a bottom-up structural model.

We take the following model parameters:

r	q	σ	ρ	η	κ	θ	λ	γ
0.02	0.01	0.40	0.50	0.30	0.20	ln(0.10)	0.05	-0.70

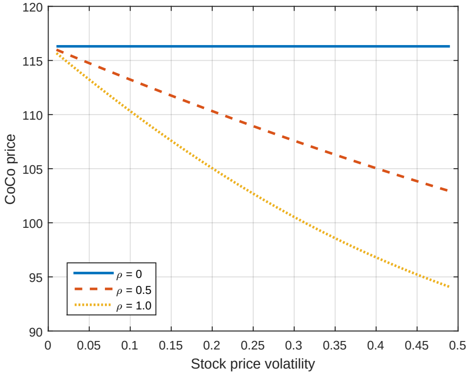
- Positive correlation coefficient ρ with moderate stock price volatility σ.
- Intensity λ of 5% can be translated to a credit spread of roughly 350 bps.
- Jump size $\gamma = -0.70$: high level of 70% write-down in stock price upon a PONV conversion.

Impact of correlation: higher CoCo bond price at more negative correlation



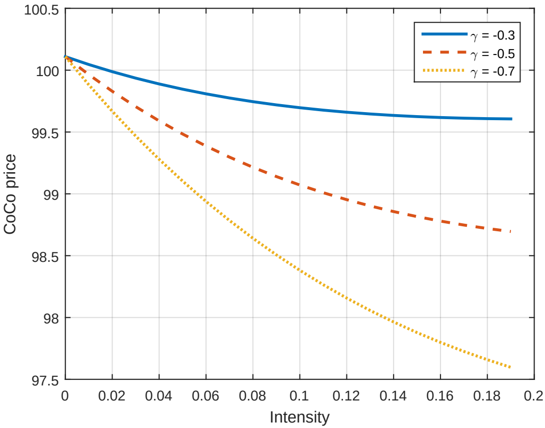
Strong impact of correlation coefficient ρ on the CoCo bond price at capital ratio of 8%.

Impact of Stock Price Volatility



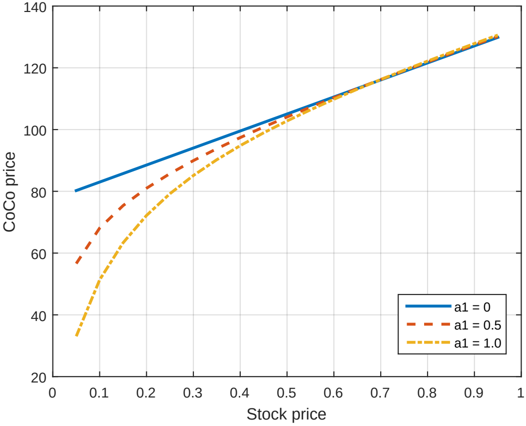
Impact of stock price volatility σ on the CoCo bond price at capital ratio of 8%. Since the conversion payoff resembles that of a forward, so the conversion value is independent of stock price volatility when $\rho = 0$.

Impact of Jump Intensity (PONV trigger)



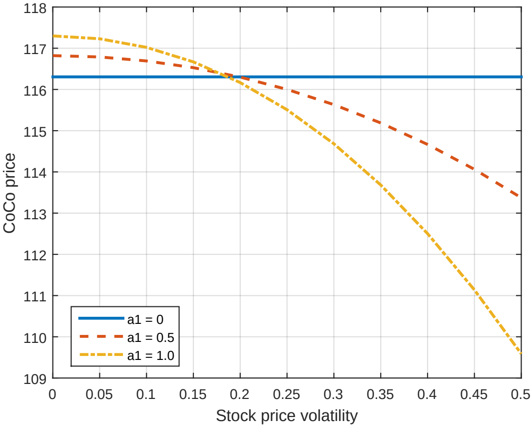
Impact of jump intensity λ on the CoCo bond price at capital ratio of 8%.

State Dependent Intensity



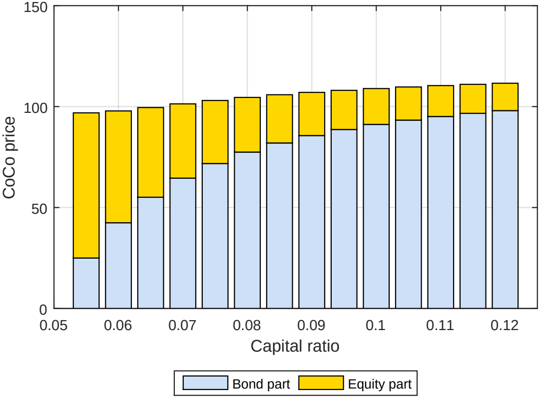
High delta risk under jump intensity with strong stock price dependence.

State Dependent Intensity



Effect of stock price dependent intensity on the vega risk (zero correlation).

Decomposition: Bond or Equity?



Bond component is high when the capital ratio is high.

Floored Payoff

The floored payoff limits the downside risk of a CoCo bond.

The conversion value with a floor K can be expressed as

$$P_E^{Floor} = G\mathbb{E}^Q\left[e^{-r\tau}\max\left(S_\tau, K\right)\mathbf{1}_{\{\tau\leq T\}}\right], \quad \tau = \tau_B \wedge \tau_R,$$

which can be readily computed using the PDE formulation.

- When the intensity λ is constant and $(1+\gamma)S_{\tau_B^-} < K$, we have

$$\begin{aligned} P_E^{Floor} &= G\int_0^T\int_{-\infty}^{\infty} q\left(x,u\right)e^{-(r+\lambda)u}\max\left(e^x, K\right)dxdu \\ &+GK\int_0^T \lambda e^{-(r+\lambda)u}Q\left(\tau_B > u\right)du. \end{aligned}$$

  in which the joint density $q\left(x,t\right)$ can be computed using the two-dimensional Fortet scheme.

Coupon Cancellation

- Coupon cancellation mechanism: the issuer has the discretionary right to cancel the coupon payments before the conversion event.
- Corcuera *et al.* (2014) discuss the coupon cancellation feature of the CoCo bond issued by the Spanish bank BBVA.
- We can model the coupon cancellation (CC) event in a reduced form manner and we have

$$P^{CC} = \sum_{i=1}^{n} \mathbb{E}^Q \left[c_i e^{-rt_i} \mathbf{1}_{\{\tau_C > t_i\}} \right] + \mathbb{E}^Q \left[F e^{-rT} \mathbf{1}_{\{\tau > T\}} \right] + \mathbb{E}^Q \left[e^{-r\tau} G S_\tau \mathbf{1}_{\{\tau \leq T\}} \right],$$

  where $\tau = \tau_B \wedge \tau_R$, and $\tau_C \sim \text{Exp}(\lambda_C)$ is an independent exponential variable.
- Exogenous feature of τ_C is consistent with the discretionary nature of coupon deferral or cancellation.

Perpetual CoCo Bond with Callable Feature

- The Perpetual Subordinated Contingent Convertible Securities by the HSBC bank, which involves perpetual maturity along with a callable feature (uncertain maturity).
- Approximate the call policy using a reduced form approach.
- We introduce an independent exponential random variable $\tau_C \sim \text{Exp}(\lambda_C)$ as the random time of issuer's call, such that

$$\begin{aligned} P^{Callable} &= \sum_{i=1}^{\infty} \mathbb{E}^Q \left[ce^{-rt_i} \mathbf{1}_{\{\tau_C \wedge \tau_B > t_i\}} \right] \\ &\quad + \mathbb{E}^Q \left[e^{-r\tau_c} GK \mathbf{1}_{\{\tau_B > \tau_C\}} \right] + \mathbb{E}^Q \left[e^{-r\tau_B} GS_{\tau_B} \mathbf{1}_{\{\tau_C > \tau_B\}} \right], \end{aligned}$$

  where K is the call price as specified in the contract.
- The intensity of issuer's call can be extended to be state-dependent too.

Concluding Remarks

- A hybrid equity-credit model is proposed with underlying bivariate process of the stock price and capital ratio.
- Forced equity conversion is subject to the accounting trigger and point-of-non-viability (PONV) trigger.
- We combine the structural approach for the capital ratio and reduced form approach for the PONV trigger.
- The analytical tractability is explored and the numerical schemes for various cases are proposed.
- The framework is versatile and can be used to cope with various complex contractual features.

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