

Real Option Signaling Game of Debt Financing Using Equity Guarantee Swaps under Asymmetric Information

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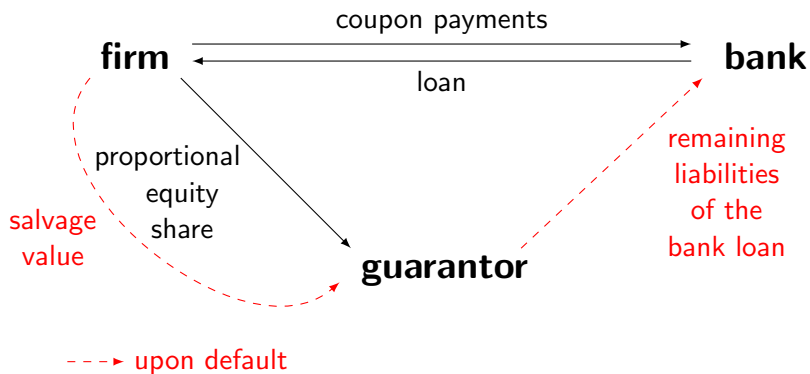
This is a joint work with Qiuqi Wang.

- 1 Product nature of equity guarantee swaps (EGS)
 - third party guarantor
 - hybrid of debt financing and equity participation
- 2 Real options signaling game models
 - private information on firm's stochastic revenue flow
 - investment timing as signal
 - separating and pooling equilibriums
 - information cost
- 3 Numerical results
 - time dependence of investment thresholds
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Equity guarantee swaps (EGS)

- In China, the **equity guarantee swaps** (EGS) are introduced to overcome the difficulties that private small- and medium-sized firms may not be able to secure bank loans.
- The EGS agreement introduces a **third party profit seeking guarantor** to hedge the default risk faced by the bank. The guarantor has the obligation to pay all the remaining liabilities to the bank upon default of the firm. In return, the guarantor obtains a proportional share of equity of the firm.
- The nature of EGS is a hybrid of **debt financing and equity participation**, similar to a convertible bond.

Equity guarantee swaps (EGS)



Revenue flow rate

- A firm is facing an investment opportunity with upfront cost I .
- Profit flow rate of the project (revenue - cost):

The diagram shows the equation $\lambda X_t - f$ where λ is in a red box, X_t is in a blue box, and f is in a green box. A red arrow points from λ to the word "multiplier". A blue arrow points from X_t to the phrase "stochastic revenue shock variable". A green arrow points from f to the phrase "operating expense rate".

$$\lambda X_t - f$$

multiplier stochastic revenue shock variable operating expense rate

$\lambda > 0$ is a multiplier that takes value of λ_h for high-type firm and λ_l for low-type firm.

- Stochastic state variable of revenue flow is modeled by the geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dZ_t.$$

Riskfree under EGS

- Let r be the riskfree interest rate and F be the present value of the future perpetual operating expenses, where

$$F = \int_t^{\infty} e^{-r(u-t)} f \, du = \frac{f}{r}.$$

- With the third party guarantor, loan becomes “risk-free” from prospective of the bank.
- Bank determines the perpetual coupon rate (coupon payment = upfront cost):

$$\frac{c_g}{r} = I \implies c_g = rI.$$

risk-free interest rate

upfront cost

risk-free coupon rate

Firm's default right

Under complete information and perpetual bank loan, the firm's default right is quantified as the option held by the firm to terminate the liabilities while forfeit the revenue flow.

- Similar to finding the exercise threshold of a perpetual American option, the default threshold is given by

$$\underline{X}_g(\lambda) = \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} (F + I),$$

where

$$\xi = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0, \quad \xi < 0.$$

Equity value

- Value of default right:

$$D_g(X; \lambda) = \left[F + I - \lambda \Pi(\underline{X}_g(\lambda)) \right] \left[\frac{X}{\underline{X}_g(\lambda)} \right]^\xi.$$

Diagram annotations:

- A red arrow points from the text "present value of operating expenses" to the F term in the equation.
- A blue arrow points from the text " $\lambda \Pi(X)$: present value of revenue flow" to the $\lambda \Pi(\underline{X}_g(\lambda))$ term.
- A green arrow points from the text "probability of default" to the $\left[\frac{X}{\underline{X}_g(\lambda)} \right]^\xi$ term.

- Equity value upon investment (revenue - cost + default right) under EGS agreement:

$$E_g(X; \lambda) = \lambda \Pi(X) - F - I + D_g(X; \lambda).$$

Liability value of guarantor

Upon default:

- Guarantor takes over the firm with a fractional bankruptcy loss α .
- Guarantor is obligated to pay remaining coupons to the bank.
- Value of liability borne by the guarantor:

$$L_g(X; \lambda) = [F + I - (1 - \alpha)\lambda\Pi(\underline{X}_g(\lambda))] \left[\frac{X}{\underline{X}_g(\lambda)} \right]^\xi.$$

fractional loss due to default



Equity share received by guarantor

- In return, the guarantor receives an equity share from firm.
- To be fair to both parties, the equity share received by guarantor equals value of liability:

$$\phi(X; \lambda) E_g(X; \lambda) = L_g(X; \lambda) \implies \phi(X; \lambda) = \frac{L_g(X; \lambda)}{E_g(X; \lambda)}.$$

proportional share demanded by guarantor

- Intrinsic value upon investment (reduced by the portion ϕ):

$$I_g^c(X; \lambda) = [1 - \phi(X; \lambda)] E_g(X; \lambda).$$

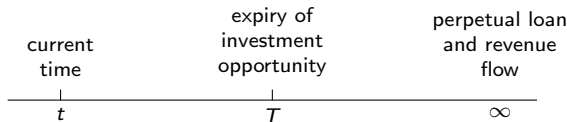
Real option value under finite time horizon

- In our model, investment opportunity expires at maturity date T (instead of perpetuity assumption).
- Real option value under finite time horizon (optimal stopping problem):

$$V_g^c(X, t; \lambda) = \sup_{t \leq u \leq T} \mathbb{E}_t \left[e^{-r(u-t)} I_g^c(X_u; \lambda) \mid X_t = X \right],$$

where u is the optimal stopping time.

- Optimal investment threshold: $X_g^*(t; \lambda)$.



Signaling game models

- Dynamic games:
Firm sends signals. The guarantor updates belief system and responds optimally.
- Firm quality type: high-type or low-type ($k = h$ or l).

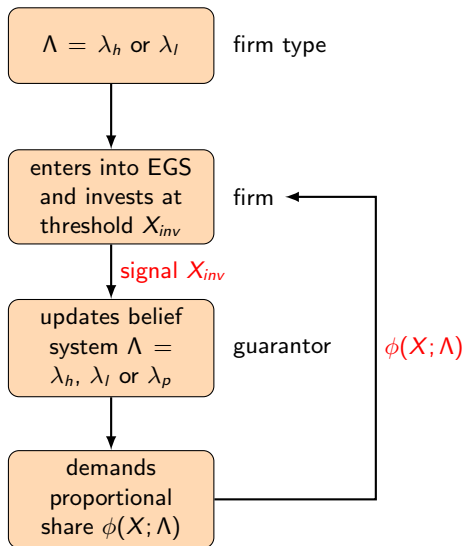
$$\Lambda X_t - f$$

multiplier Λ is not known to the guarantor

- Belief system of the guarantor:
 - $\Lambda = \lambda_h$ (true high type is revealed to the guarantor)
 - $\Lambda = \lambda_l$ (true low type is revealed to the guarantor)
 - $\Lambda = \lambda_p = p \lambda_h + (1 - p) \lambda_l$ (firm type is not revealed)

probability of being high-type

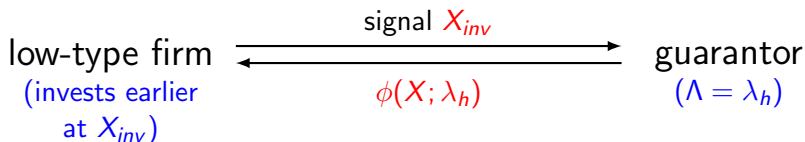
Signaling games



Mimicking strategy of the low-type firm

- Low-type firm has the incentive to mimic the high-type firm's strategy by investing earlier at some X , which is lower than the nonstrategic optimal threshold $X_g^*(t; \lambda_l)$.
- By mimicking, low-type firm enjoys the advantage to pay a lower equity share $\phi(X; \lambda_h) < \phi(X; \lambda_l)$ to the guarantor.
- Mimicking cost: Lowering of the firm value of the low-type firm due to exercising earlier investment $[X < X_g^*(t; \lambda_l)]$ under mimicking strategies.

Intrinsic value under mimicking strategy



Intrinsic value under mimicking strategy:

$$I_{g,l}^m(X) = [1 - \phi(X; \lambda_h)]E_g(X; \lambda_l).$$

- Comparing with $I_g^c(x) = [1 - \phi(x; \lambda)]E_g(x; \lambda)$

Incentive Compatibility Constraints (ICC) of low-type firm

The low-type firm chooses not to mimic when the mimicking cost of the low-type firm is higher than the benefit from the reduction of the equity share. This occurs when

$$I_{g,l}^m(X) = \underbrace{[1 - \phi(X; \lambda_h)] E_g(X; x; \lambda_l)}_{\text{mimicking}} \leq \underbrace{V_g^c(X, t; \lambda_l)}_{\text{complete information}}$$

Equality holds when $X = \bar{X}_{g,l}(t)$. $I_{g,l}^m(X)$ is less than $V_g^c(X, t; \lambda_l)$ when

$$X \leq \underbrace{\bar{X}_{g,l}(t)}_{\text{binding threshold}}$$

- High-type firm does not want to be perceived as low-type and being charged at higher proportional share $\phi(X; \lambda_l)$, so it would invest at lower X_{inv} to separate from low-type.

High-type firm fails to separate

Suppose the high-type firm fails to separate, then

- Guarantor's belief system: $\Lambda = \lambda_l$.
- Intrinsic value:

$$I_{g,h}^m(X) = [1 - \phi(X; \lambda_l)] E_g(X; \lambda_h).$$

- Real option value (optimal stopping problem):

$$V_{g,h}^m(X, t) = \sup_{t \leq u \leq T} \mathbb{E}_t \left[e^{-r(u-t)} I_{g,h}^m(X_u) \mid X_t = X \right].$$

Incentive Compatibility Constraint of high-type firm

- Benefit of separating: lower proportional equity share $\phi(X; \lambda_h)$.
- Information cost: Loss of real option value of the high-type firm when compared with that under complete information.

High-type firm prefers separating when

$$\underbrace{[1 - \phi(X; \lambda_h)]E_g(X; \lambda_h)}_{\text{separating}} \geq \underbrace{V_{g,h}^m(X, t)}_{\text{failure of separation}}$$

Equality holds when $X = \bar{X}_{g,h}(t)$. Provided

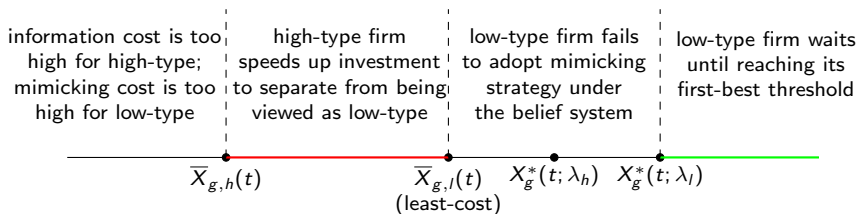
$$X \geq \underbrace{\bar{X}_{g,h}(t)}_{\text{binding threshold}},$$

the incentive compatibility constraint of the high-type firm is satisfied and the high-type firm may speed up investment to separate from being viewed as low-type.

Separating equilibrium

Under the assumption: $\bar{X}_{g,h}(t) < \bar{X}_{g,l}(t)$. Belief system:

$$\Lambda(X) = \begin{cases} \lambda_l, & \text{if } X > \min(\bar{X}_{g,l}(t), X_g^*(t; \lambda_h)) \\ \lambda_h, & \text{otherwise} \end{cases}$$



High-type firm invests at $\min(\bar{X}_{g,l}(t), X_g^*(t; \lambda_h))$, the lower of the least-cost threshold $\bar{X}_{g,l}(t)$ and non-strategic threshold $X_g^*(t; \lambda_h)$.

Pooling strategies

- Firm of either type sends the same signal of investment timing.
Pooling equilibrium exists only when the pooling strategies dominate the separating and mimicking strategies.
- Guarantor's belief is not updated after receiving the signal:

$$\Lambda = \lambda_p = p\lambda_h + (1 - p)\lambda_l.$$

- Expected equity value viewed by guarantor:

$$E_g^p(X) = pE_g(X; \lambda_h) + (1 - p)E_g(X; \lambda_l).$$

- Expected liability value viewed by guarantor:

$$L_g^p(X) = pL_g(X; \lambda_h) + (1 - p)L_g(X; \lambda_l).$$

- Proportional equity share: $\phi_p(X) = \frac{L_g^p(X)}{E_g^p(X)}.$

Pooling strategy of the high-type firm

- Intrinsic value:

$$I_{g,h}^p(X) = [1 - \phi_p(X)]E_g(X; \lambda_h).$$

- Real option value:

$$V_{g,h}^p(X, t) = \sup_{t \leq u \leq T} \mathbb{E}_t \left[e^{-r(u-t)} I_{g,h}^p(X_u) \mid X_t = X \right]$$

- Optimal pooling threshold: $X_g^{p*}(t)$; this is the Pareto-dominant investment threshold at which the high-type firm maximizes its value.

Incentive Compatibility Constraints for pooling equilibrium

- Low-type firm prefers pooling provided that

$$\underbrace{[1 - \phi_p(X)]E_g(X; \lambda_l)}_{\text{pooling}} \geq \underbrace{V_g^c(X, t; \lambda_l)}_{\text{complete information}} \implies X \geq \bar{X}_{g,l}^p(t).$$

$\bar{X}_{g,l}^p(t)$ is the threshold of pooling strategy adopted by the low-type firm.

- High-type firm:

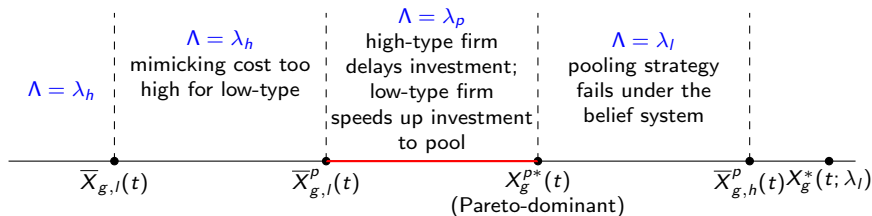
$$\underbrace{[1 - \phi(\bar{X}_{g,l}(t); \lambda_h)]E_g(\bar{X}_{g,l}(t); \lambda_h)}_{\text{separating}} \leq \underbrace{V_{g,h}^p(\bar{X}_{g,l}(t), t)}_{\text{pooling}} \\ \implies \bar{X}_{g,l}(t) \leq \bar{X}_{g,h}^p(t).$$

The high-type firm achieves a higher payoff by pooling with the low-type by lowering the negative effect of investment distortion, though it may be charged with higher share of equity by the guarantor.

Summary of pooling equilibrium

Belief system:

$$\Lambda(X) = \begin{cases} \lambda_h, & \text{if } X \leq \bar{X}_{g,l}(t) \\ \lambda_p, & \text{if } \bar{X}_{g,l}(t) < X \leq X_g^{p*}(t) \\ \lambda_l, & \text{otherwise} \end{cases}$$



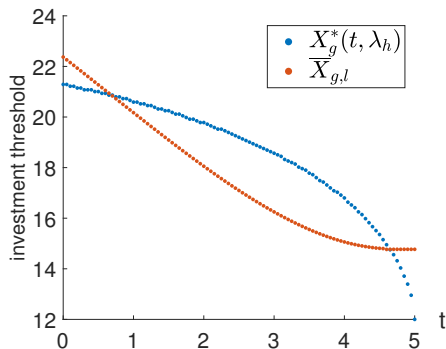
When the high-type waits too long beyond $X_g^{p*}(t)$, the guarantor may interpret the firm as low-type and $\Lambda = \lambda_l$.

The high-type firm always chooses the strategy with the lowest information cost (benchmarked with the real option value under complete information).

$$\text{information cost} = \frac{V_g^c(X, t; \lambda_h) - V}{V_g^c(X, t; \lambda_h)},$$

where V is the real option value under some specific strategy (separating, pooling or first-best).

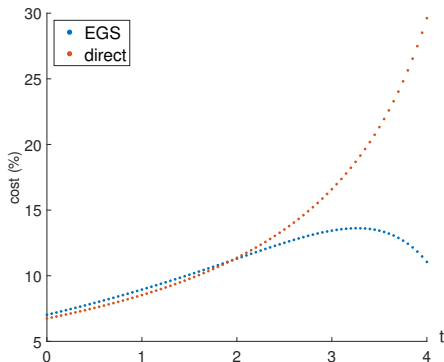
Investment thresholds under separating equilibrium



High-type firm invests at $\min(\bar{X}_{g,l}(t), X_g^*(t; \lambda_h))$ chooses its non-strategic threshold $X_g^*(t; \lambda_h)$ at time far from expiry, changes to the binding threshold $\bar{X}_{g,l}(t)$ at intermediate time and reverts to its non-strategic threshold when time is close to maturity.

Time dependence of financing choice

$$\lambda_h = 1.25, \lambda_l = 0.5$$



High-type firm prefers to separate through the direct bank loan when time is far from maturity and changes to EGS when time is closer to maturity.

Information costs

Impact of λ_h/λ_l on the financing choices among EGS separating, EGS pooling, direct bank loan separating and direct bank loan pooling:

information cost (expressed in percentage) ($\lambda_h = 1.25$)				
λ_l	EGS _s	EGS _p	loan _s	loan _p
0.125	0	78.67	6.68	99.08
0.250	0	64.55	6.68	87.83
0.375	0.12	52.75	6.66	69.09
0.500	7.14	42.42	6.67	51.93
0.625	18.80	33.22	6.94	38.62
0.750	21.36	25.00	7.76	28.62
0.875	19.28	17.61	8.34	20.98
1.000	14.47	11.06	7.85	15.15
1.125	7.99	5.19	6.68	10.46

Interpretation of financing choices

- When λ_l/λ_h is small, the high-type firm chooses to separate through EGS. High-type firm invests at its first-best threshold and the corresponding information cost is zero.
- When the multiplier λ_l is close to λ_h , the high-type firm resorts to the pooling strategy through EGS.
- At intermediate level of λ_l , the high-type firm chooses to separate through the direct bank loan.
- The information cost is always high at all levels of λ_l/λ_h under the pooling equilibrium through the direct bank loan.

Summary and conclusion

- Our signaling game real option models extend other similar signaling game models of corporate financing in two aspects
 - (i) new financing choice via the equity guarantee swap (EGS).
 - (ii) finite time horizon of the life span of the investment opportunity.
- The signals sent by the firm to outside investors involve investment timing and financial choice between the direct bank loan and EGS.
- Under the EGS arrangement, the low-type firm may have an incentive to mimic the investment timing of the high-type in order to take advantage of the lower proportional share of equity and coupon rate.
- We perform characterization of separating and pooling equilibriums under the EGS agreement.

- The high-type firm may have an incentive to separate from being perceived as low-type by speeding up investment and / or choosing different financing choice.
- Under the separating equilibrium, the high-type firm faces information costs due to investment distortion. Suppose the high-type firm fails to separate due to high information costs, pooling equilibrium is resulted where both firm types invest at the same investment threshold and adopt the same financing choice. In this case, the outsiders cannot differentiate the firm type quality from the investment decisions made by the firm.
- We examine the time dependent behaviors of the investment thresholds under separating equilibrium, and financing choice between direct bank loan and EGS.