Solutions to Homework 1 and 2

Problem 1, Homework 1. page 46, problems 11. 12. 13. 14. 15. 16. 17. 18. (just answer "yes" or "no", no reasons needed.)

11. Yes.
12. No, because some elements has no inverse.
13. Yes. 14. Yes.
15. No, because some elements has no inverse.
16. Yes. 17. Yes. 18. Yes

Problem 2, Homework 1. Let $(G, *)$ be a group. If $a, b \in G$ satisfies $abab = e$, prove that $baba = e$.

Proof. Let b' be the inverse of b, because $abab = e$, so $bababb' = beb'$, after using $bb' = e$, we get $baba = e$.

Problem 1. Homework 2. Compute the order of elements (just give the answers, no details needed):
(1). $-1, -i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, 3$ in $G = \mathbb{C}^*$,
(2). $5, 6, 8$ in $\mathbb{Z}_{12}$.
(3). $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$ in $GL(2, \mathbb{R})$.

(1). order of -1 is 2, order of $-i$ is 4. Notice that $\frac{1}{2} + \frac{\sqrt{3}}{2}i = e^{\frac{2\pi i}{6}}$. so its is 6. Order of 3 is infinity.

(2). order of 5 is 12, order of 6 is 2, order of 8 is 3.

(3). The first matrix has order 4, the second has order 6.

Problem 2. Homework 2. Let G be a group, suppose that $a^2 = e$ for all $a \in G$, prove that G is an abelian group.

Proof. For $a, b \in G$, we have $a^2 = e, b^2 = e$ also $(ab)^2 = e$, so $abab = e$. Multiple to the last equation by a from the left and b from the right, we get $aababb = ab$, so $ba = ab$. This proves G is abelian.

Problem 3. Homework 2. Page 55, problems 8, 9, 10, 11, 12, 13

8. No, because the subset does not contain the identity.

9. Yes. 10. Yes.

11. No, because the subset does not contain the identity.

12. Yes. 13. Yes.