Homework No.4 for Math 3121

Due Date: Nov. 8

Problem 1. Determine if the following maps are homomorphisms of groups (No reasons needed).

(1). $\Phi : \mathbb{R} \to \mathbb{R}, \quad \Phi(a) = 2018a$

(2). $\Phi : \mathbb{R}^* \to \mathbb{R}^*, \quad \Phi(a) = 2018a$

(3). $\Phi : \mathbb{R}^* \to \mathbb{R}^*, \quad \Phi(a) = a^{2018}$

(4). $\Phi : GL(n, \mathbb{R}) \to \mathbb{R}^*, \quad \Phi(A) = Det(A)^{10}$.

(5). $\Phi : \mathbb{R} \to \mathbb{R}^*, \quad \Phi(a) = 10^a$.

(6). $\Phi : \mathbb{R}^* \to \mathbb{R}, \quad \Phi(a) = 10^a$.

(7). $\Phi : S_5 \to S_5, \quad \Phi(\sigma) = \sigma^{120}$.

Problem 2. Find a homomorphism $\Phi : \mathbb{R}^* \to \mathbb{R}$ such that $\Phi(2) = 3$.

Problem 3. Let G be a group, H_1 and H_2 be finite subgroups of G. Suppose that $|H_1|$ and $|H_2|$ are relatively prime, prove that $H_1 \cap H_2$ has only one element (hint: use the Lagrange Theorem).

Problem 4. Let G, G' be finite groups. Suppose that $|G|$ and $|G'|$ are relatively prime. Prove that a homomorphism $\Phi : G \to G'$ must be trivial, i.e., $\Phi(a) = e'$ for all $a \in G$ (hint: Use the Lagrange Theorem).